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PLANE AND SPHERICAL TRIGONOMETRY

BY

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AND

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PREFACE

THE plan and scope of this work may be indicated briefly by the following characteristic features:

Directed lines and Cartesian coordinates are introduced as a working basis.

Each subject, when first introduced, is treated in a general manner and is presented as fully as the character of the work demands. This avoids the necessity of treating the same subjects several times for the purpose of modifying and extending certain conceptions.

Statements in the form of theorems and problems are used freely to indicate the aim of various articles and to define the data clearly.

Inverse functions are treated more fully than is customary.

The general principles governing the solution of triangles, the solution of trigonometric equations, and the proof of identities are carefully presented.

Special attention is given to the arrangement of computations.

The sine and cosine series are obtained from De Moivre's Theorem, thus completing the line of development which leads to the calculation of the trigonometric functions.

The work on spherical trigonometry contains the development of all the formulas that are generally used in practical astronomy.

The right spherical triangle is treated from two points of view: as a special case of the oblique triangle, and directly

from geometric figures. The work is so arranged that either view may be presented independently.

The solutions of the oblique spherical triangle by means of auxiliary quantities, characteristic of astronomy, are included as interesting mathematical problems and as preparation for astronomical work.

PREFACE TO THE SECOND REVISED EDITION

The second edition embodies such modifications, rearrangements, and additions as have been suggested by experience in the classroom.

Due to numerous requests tables have been added. Simple three-place tables have been included. By their use the numerical work is reduced to a minimum, thus leaving the student free to give more attention to the principles involved in the solution of a triangle. Accordingly a few triangles, suitable for three-place table or slide rule computation, have been introduced at the beginning of the list of problems under the right triangle and also under the oblique triangle.

In order to give a wider range to computational work four-place tables as well as five-place tables have been included.

G. N. B.
W. E. B.

Minneapolis, Minnesota,
June, 1917.

CONTENTS

PLANE TRIGONOMETRY

CHAPTER I

RECTANGULAR COORDINATES AND ANGLES

ART. PAGE

1. Introduction 1

RECTANGULAR COORDINATES

2. Directed lines 1
3. Lines of reference 2
4. Quadrants 2
5. Coordinates of a point 2
6. Signs of coordinates 3
7. Exercises 3

ANGLES

8. Magnitude of angles 4
9. Direction of rotation. Positive and negative angles . . 4
10. Initial and terminal lines 5
11. Sign of terminal line 5
12. Algebraic sum of two angles 5
13. Measurement of angles 6
14. Circular or radian measure 7
15. Value of radian 7
16. Relation between degree and radian 7
17. Relation between angle, radius, and arc 9
18. Linear and angular velocity 9
19. Examples 10

CHAPTER II

TRIGONOMETRIC FUNCTIONS

ART.		PAGE
20.	Trigonometric functions introduced	12
21.	Definitions of the trigonometric functions	12
22.	Values of the trigonometric functions of 30°, 45°, and 60°	13
23.	Values of the trigonometric functions of 120°, 135°, and 150°	15
24.	Signs of the trigonometric ratios	17
25.	Trigonometric functions are single valued	18
26.	A given value of a trigonometric function determines an infinite number of angles	19
27.	Examples	20

CHAPTER III

RIGHT TRIANGLES

28.	Statement of problem	22
29.	Application of the definitions of the trigonometric functions to the right triangle	22
30.	Trigonometric tables	23
31.	Formulas used in the solution of right triangles	24
32.	Selection of formulas	25
33.	Check formulas	25
34.	Suggestions on solving a triangle	25
35.	Illustrative examples	26
36.	Examples	29
37.	Oblique triangles	30
38.	Applications	31

CHAPTER IV

VARIATIONS OF THE TRIGONOMETRIC FUNCTIONS REDUCTION OF FUNCTIONS OF $n\,90° \pm \alpha$

39.	Values of functions at 0°, 90°, 180°, 270°, and 360°	34
40.	Variation of the functions	35
41.	Graphical representation	39
42.	Periodicity of the trigonometric functions	40
43.	Examples	41

ART. PAGE
44. Use of formulas 42
45. Functions of $-\alpha$ in terms of functions of α 42
46. Functions of $90° + \alpha$ in terms of functions of α . . . 43
47. Functions of $90° - \alpha$ in terms of functions of α . . . 44
48. Functions of $180° - \alpha$ in terms of functions of α . . . 45
49. Laws of reduction 46
50. Examples 46

CHAPTER V

FUNDAMENTAL RELATIONS. LINE VALUES

51. General statement 47

Fundamental Relations

52. Development of formulas 47
53. The use of exponents 48
54. Trigonometric identities 49
55. Trigonometric equations 49
56. Examples 51

Line Values

57. Representation of the trigonometric functions by lines . 53
58. Variations of the trigonometric functions as shown by line values 55
59. Fundamental relations by line values 56
60. Examples 57

CHAPTER VI

FUNCTIONS OF THE SUM OF TWO ANGLES DOUBLE ANGLES. HALF ANGLES

61. Statement of problem 61
62. The sine of the sum of two acute angles expressed in terms of the sines and cosines of the angles 61
63. The cosine of the sum of two acute angles 62
64. Importance of formulas 62
65. Generalization of formulas 63
66. Tangent of the sum of two angles 65
67. Cotangent of the sum of two angles 65
68. Addition formulas 65

ART. PAGE
69. Sine, cosine, tangent, and cotangent of the difference of two angles 66
70. Exercises 66
71. Double angles 67
72. Half angles 68
73. Sum and difference of two sines and of two cosines . . 69
74. Equations and identities 70
75. Examples 71

CHAPTER VII

INVERSE FUNCTIONS

76. Statement of problem 74
77. Fundamental idea of an inverse function 74
78. Multiple values of an inverse function 75
79. Principal values 77
80. Interpretation of $\sin \sin^{-1} a$ and $\sin^{-1} \sin a$ 77
81. Application of the fundamental relations to angles expressed as inverse functions 78
82. The value of any function of an inverse function . . . 79
83. Some inverse functions expressed in terms of other inverse functions 81
84. Relations between inverse functions derived from the formulas for double angles, half angles, and the addition formulas 82
85. Examples 82

CHAPTER VIII

OBLIQUE TRIANGLE

86. General statement 84
87. Law of sines 84
88. Law of tangents 85
89. Cyclic interchange of letters 85
90. Law of cosines 85
91. Sine of half angle in terms of sides of triangle . . . 86
92. Cosine of half angle in terms of sides of triangle . . . 87
93. Tangent of half angle in terms of sides 88
94. Area of plane triangle in terms of two sides and included angle 89
95. Area of triangle in terms of a side and two adjacent angles . 89

ART. PAGE
96. Area in terms of sides 90
97. Formulas for solving an oblique triangle 90
98. Check formulas 91
99. Illustrative problems 92
100. The ambiguous case 95
101. Examples 98
102. Miscellaneous Exercises 102

CHAPTER IX

DE MOIVRE'S THEOREM WITH APPLICATIONS

103. Introduction to chapter 113
104. Geometric representation of a complex number . . . 113
105. De Moivre's theorem 114
106. Geometric interpretation 115
107. Applications of De Moivre's theorem 116
108. Cube roots of unity 116
109. Fifth roots of unity 117
110. Square root of a complex number 118
111. Any root of a complex number 119
112. Sin $n\alpha$ and cos $n\alpha$ expressed in terms of sin α and cos α . 120
113. Comparison of the values of sin α, α, and tan α, α being any acute angle 121
114. Value of $\frac{\sin \alpha}{\alpha}$ for small values of α 121
115. Sine and cosine series 122
116. Examples 124

SPHERICAL TRIGONOMETRY

CHAPTER X

FUNDAMENTAL FORMULAS

117. The spherical triangle 127
118. Law of sines 128
119. Law of cosines 129
120. Law of cosines extended 130
121. Relation between one side and three angles . . . 131
122. The sine-cosine law 131

ART. PAGE
123. Relation between two sides and three angles . . . 132
124. Relation between two sides and two angles 132
125. Formulas independent of the radius of the sphere . . 133

CHAPTER XI

SPHERICAL RIGHT TRIANGLE

126. Definition of spherical right triangle 134
127. Formulas for the solution of right triangles 134
128. Direct geometric derivation of formulas 134
129. Sufficiency of formulas 137
130. Comparison of formulas of plane and spherical right triangles 137
131. Napier's rules 138
132. Side and angle opposite terminate in same quadrant . . 139
133. Two sides determine quadrant of a third 139
134. Two parts determine a triangle 139
135. The quadrant of any required part 140
136. Check formula 140
137. Solution of a right triangle 141
138. Two solutions 141
139. Examples 143
140. Quadrantal triangles 143
141. Isosceles triangles 143
142. Examples 144

CHAPTER XII

OBLIQUE SPHERICAL TRIANGLE

143. Statement of aim of chapter 145

General Solution

144. Angles determined from the three sides 145
145. Sides found from the three angles 146
146. Delambre's or Gauss's formulas 147
147. Napier's analogies 147
148. Formulas collected 148
149. All formulas excepting law of sines determine quadrant . 148

ART. PAGE
150. Theorem to determine quadrant 149
151. Second theorem to determine quadrant 149
152. Third theorem to determine quadrant 150
153. Illustrative examples 150
154. Two solutions 153
155. Area of spherical triangle 154
156. Examples 154

SOLUTION WHEN ONLY ONE PART IS REQUIRED

157. Statement of problem 156
158. From two sides and the included angle, to find any one of the remaining parts 156
159. Two parts required 158
160. Problems 159
161. Each unknown part found from two angles and the included side 160
162. Each unknown part found from two sides and an angle opposite one of them 161
163. Each unknown part found from two angles and a side opposite one of them 163
164. The general triangle 164

ANSWERS 165

PLANE TRIGONOMETRY

CHAPTER I

RECTANGULAR COORDINATES AND ANGLES

1. Introduction. The word *trigonometry* is derived from the two Greek words for triangle (τρίγωνον) and measurement (μετρία). Originally trigonometry was concerned chiefly with the solution of triangles. At present this is but one part of the subject.

Certain preliminary considerations, concerning directed lines and angles, are necessary before introducing the fundamental definitions of trigonometry.

RECTANGULAR COORDINATES

2. Directed lines. A positive and a negative direction may be assigned arbitrarily to every line.

If the direction from A to C is *positive*, the opposite direction from C to A is *negative*.

If we let the order of the letters indicate the direction in which a segment is measured, it is evident that AC and CA represent the same segment measured in opposite directions; hence

$$AC = -CA \text{ or } -AC = CA.$$

Also, if B be a third point on the line, the segments AB and BA are opposite in sign; likewise the segments BC and CB. Hence

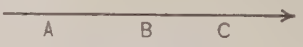
$$AB = -BA \text{ or } -AB = BA,$$

and

$$BC = -CB \text{ or } -BC = CB.$$

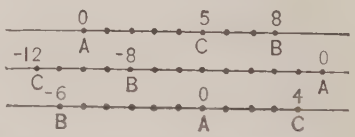
Then for all positions of A, B, and C on a line it follows that

$$AC = AB + BC.$$

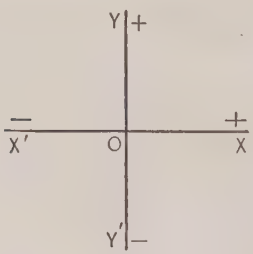


Thus for the following figures:

$$5 = 8 + (-3) = 5$$
$$-12 = -8 + (-4) = -12$$
$$4 = -6 + (+10) = 4$$



3. Lines of reference. Two directed lines, perpendicular to each other, may be taken as **lines of reference** or **axes.** They are usually designated by $X'X$ and $Y'Y$ and are called the X-axis and the Y-axis respectively.



The X-axis is positive from left to right and negative from right to left.

The Y-axis is positive upward and negative downward.

The point of intersection of the axes is the **origin.** The origin serves as a convenient starting point from which to measure distances.

4. Quadrants. The axes produced divide the plane into four parts called **quadrants.** The quadrants are designated by number. The first quadrant is indicated by XOY, the second by YOX', the third by $X'OY'$, and the fourth by $Y'OX$.

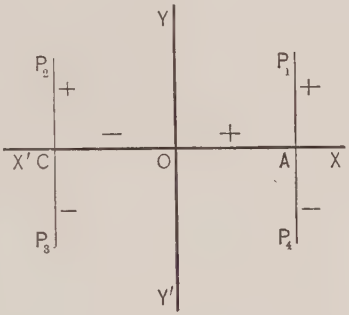
5. Coordinates of a point. From any given point P_1 in the plane, draw a line parallel to the Y-axis intersecting the X-axis in some point A.

Then the distance from the origin to the point of intersection, or OA, is the **abscissa** of the given point P_1.

The distance from the point of intersection to the given point, or AP_1, is the **ordinate** of the given point P_1.

The abscissa and the ordinate of the point P_2 are OC and CP_2 respectively.

The abscissa and the ordinate of the point P_3 are OC and CP_3 respectively.



The abscissa and the ordinate of the point P_4 are OA and AP_4 respectively.

6. Signs of coordinates. Abscissas are positive when measured from the origin to the right, and negative when measured from the origin to the left.

Ordinates are positive when measured upward from the X-axis and negative when measured downward.

Thus OA, AP_1, and CP_2 are positive; OC, CP_3, and AP_4 are negative.

7. EXERCISES

1. Locate the point whose abscissa is 4 and whose ordinate is 7. This point is designated (4, 7).

2. Locate the point whose abscissa is 2 and whose ordinate is 5, *i.e.* the point (2, 5).

3. Locate the points $(-3, 4)$, $(-6, -3)$, $(5, -1)$, $(0, 4)$, $(-7, 0)$, and $(-8, 10)$.

4. Locate the points $(1\frac{1}{2}, 3)$, $(-\frac{1}{2}, 0)$, (m, n), (x, y), $(x, 0)$.

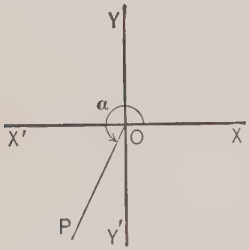
5. What is the locus of the points whose abscissas are 6?

6. What is the locus of the points whose ordinates are -3?

7. What is the locus of the points whose abscissas are twice their ordinates?

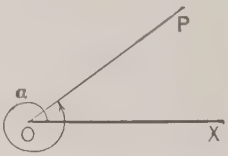
ANGLES

8. Magnitude of angles. In elementary geometry the angles considered are usually less than two right angles; but in trigonometry it is necessary to introduce angles of *any* magnitude, positive or negative.

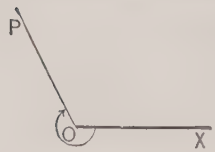


To extend the conception of an angle, suppose a line to revolve in a fixed plane about a fixed point O, from the initial position OX to the successive positions OY, OX', and OP, generating α*, an angle greater than two right angles.

If the line continues to revolve, making more than one complete revolution, it generates an angle α which is greater than four right angles. Evidently by continuing the rotation an angle of any magnitude may be generated. Thus the size of the angle α depends upon the amount of rotation of OP and is designated by an arc.



9. Direction of rotation. Positive and negative angles. As a positive and negative direction may be assigned arbitrarily to a line, so a positive and negative sense of generation may be assigned arbitrarily to an angle.



The direction of rotation indicated by the arrows in the figures of Art. 8 is the positive direction; the opposite direction, indicated by the arrow in the adjoining figure, is the negative direction.

* Angles will usually be designated by Greek letters:

α . . . Alpha	δ . . . Delta
β . . . Beta	θ . . . Theta
γ . . . Gamma	ϕ . . . Phi

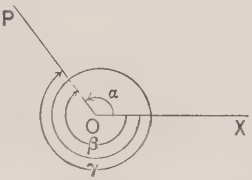
A **positive angle** is an angle generated by a line rotating in the positive direction.

A **negative angle** is an angle generated by a line rotating in the negative direction.

Negative angles, like positive angles, may have unlimited magnitude.

10. Initial and terminal lines. The fixed line from which both positive and negative angles are measured is the **initial line.** It usually coincides with that part of the X-axis lying to the right of the origin. Thus, in the last three figures, OX is the initial line.

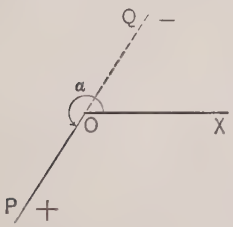
The final position of the revolving line, marking the termination of the angle, is the **terminal line.** Thus, in the last three figures, OP is the terminal line.



Two or more unequal angles may have the same terminal line. Thus the positive angle α and the negative angles β and γ have the same terminal line.

Angles having the same terminal line, and the same initial line, are called **coterminal angles.**

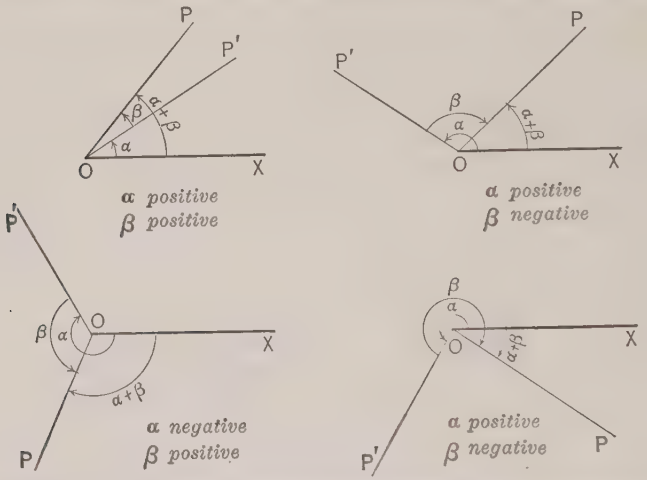
11. Sign of terminal line. The terminal line, drawn from the origin in the direction of the extremity of the measuring arc, is positive; the terminal line produced, drawn from the origin in the opposite direction, is negative.



Thus the terminal line OP, of the angle α, is positive, and the terminal line produced, OQ, is negative. As the terminal line OP revolves it retains its positive sign.

12. The algebraic sum of two angles. To construct the algebraic sum of two angles, conceive OP to rotate from the

position OX, through the angle α, to OP'; then from this



position let it rotate through the angle β, whether positive or negative, to OP. Then XOP is the desired angle $\alpha + \beta$.

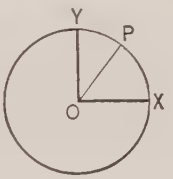
13. Measurement of angles. Various units may be employed in the measurement of angles. In elementary geometry the right angle is frequently used. Two other units in common use are the degree and the radian. The degree is generally used in practical problems involving numerical computations, while the radian is essential in many theoretical considerations.

The degree is defined as one ninetieth of a right angle. The degree is divided into sixty equal parts called minutes. The minute is divided into sixty equal parts called seconds.

Then 60 seconds $(60'') = 1$ minute.
60 minutes $(60') = 1$ degree.
90 degrees $(90°) = 1$ right angle.

The angle 26 degrees, 39 minutes, and 57 seconds is written $26°\ 39'\ 57''$.

14. Circular or radian measure. A **radian** is an angle of such magnitude that, if placed with its vertex at the center of any circle, it will intercept an arc equal in length to the radius of the circle.



Thus, if the arc XP is equal to the radius OX, the angle XOP is a radian, or $\angle XOP = 1^r$, ^{r} being used to designate radian.

15. Value of the radian. Since, in the same circle or in equal circles, angles at the center are proportional to their intercepted arcs, it follows that

$$\frac{\angle XOP}{\angle XOY} = \frac{\text{arc } XP}{\text{arc } XY},$$

or
$$\frac{\text{one radian}}{\text{one right angle}} = \frac{r}{\frac{1}{4}(2\pi r)} = \frac{2}{\pi}.$$

Therefore,
$$\textbf{one radian} = \frac{2}{\pi} \cdot \textbf{one right angle.}$$

or
$$\pi \textbf{ radians} = 180°.$$

It is clear that the value of the radian is independent of the radius, depending only upon the constant π and the right angle, and hence is an invariable unit.

16. Relation between degree and radian. From the preceding article it follows that

$$1^r = \frac{180°}{\pi}, \tag{1}$$

or
$$1^r = \frac{180°}{3.1416} = 57° 17' 44''. \tag{2}$$

Also, from equation (1),

$$1° = \frac{\pi^r}{180}, \tag{3}$$

or
$$1° = 0.01745^r. \tag{4}$$

Equations (1) and (2) are used to convert radians into

degrees, and equations (3) and (4) are used to convert degrees into radians. Thus,

from (1), $\frac{2}{3}\pi^r = \frac{2}{3}\pi\left(\frac{180°}{\pi}\right) = 120°,$

from (2), $4^r = 4\,(57° 17' 44'') = 229° 10' 56'',$

from (3), $20° = 20\left(\frac{\pi^r}{180}\right) = \frac{\pi^r}{9},$

from (4), $3° = 3\,(0.01745^r) = 0.05235^r.$

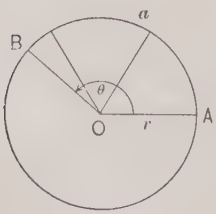
Using equation (3) to convert degrees into radians introduces π into the numerical value of the angle. Thus π becomes associated with radian measure. Since the radian is the angular unit with which π is commonly associated, no ambiguity arises by omitting to state with each angle, expressed in terms of π, that the radian is the unit.

Thus, $90° = \frac{\pi}{2}$ radians, $180° = \pi$ radians, $29° = \frac{29\,\pi}{180}$ radians are usually written

$$90° = \frac{\pi}{2}, \qquad 180° = \pi, \qquad 29° = \frac{29\,\pi}{180}.$$

It must be especially noted that when no unit is specified the radian is always understood. The constant π is always equal to 3.1416, and can never equal 180°, but π radians are equal to 180°.

17. Relation between angle, radius, and arc. It is evident from the definition of a radian that if any arc of a circle AB be divided by the radius, the quotient indicates the number of radians contained in the central angle subtended by the given arc, hence



$$\frac{\text{arc } AB}{\text{radius}} = \text{angle } AOB$$

where angle AOB is expressed in radians. Representing

the length of the arc AB by a, the radius by r, and the angle AOB by θ, the relation above may be written

$$\frac{a}{r} = \theta$$

or

$$a = r\theta \tag{1}$$

It should be especially noted that the angle θ is expressed in terms of radians.

In the above figure θ is approximately equal to $2\frac{1}{3}$ radians.

Equation (1) expresses the relation between a, r, and θ, and determines any one of these elements when the other two are known.

Problem. What is the length of the arc subtended by a central angle of 112° in a circle whose radius is 15 feet?

Solution. Reducing the angle to radians it is seen that

$$112° = \frac{28\,\pi^r}{45} = 1.95^r.$$

Hence $$\theta = 1.95.$$

Substituting in Equation (1) we have

$$a = 15 \times 1.95 = 29.3$$

Hence the length of the arc is 29.3 ft.

18. Linear and angular velocity. Equation (1) of Art. 17 leads directly to the relation between *linear velocity* and *angular velocity* in uniform motion in a circle. Suppose a point P moves along the circumference of a circle at a constant velocity v, describing an arc a in time t; then a/t is called the *linear velocity* and is represented by v. During the same time t, the angle θ is generated; and θ/t is called the *angular velocity*, and is represented by the Greek letter ω (omega).

Dividing Equation (1), Art. 17, by t gives

$$\frac{a}{t} = \frac{r\theta}{t} \text{ or } v = r\omega$$

i.e. the *linear velocity* is equal to r *times the angular velocity.*

While in general angular velocity may be expressed in any units whatsoever, in the equation $v = r\omega$ the angular velocity must be expressed in radians per unit of time.

Problem 1. If a point moves 26 feet in the arc of a circle of radius 7 feet in 3 seconds, what is its angular velocity?

Solution. The linear velocity of the point is $\frac{26}{3}$ feet per second.

Substituting in equation (1) we have

$$\frac{26}{3} = 7\,\omega$$

$$\therefore \omega = \frac{26}{21} = 1.238 \text{ radians per second.}$$

Problem 2. A flywheel makes 200 revolutions per minute. Show that its angular velocity is 72,000° per minute or $\frac{20\,\pi}{3}$ radians per second.

If the radius of the flywheel is 3 feet, show that the velocity of a point on the rim is 42.84 miles per hour.

19. EXAMPLES

1. Construct the following angles: 30°, 45°, 60°, 135°, 300°, − 60°, − 90°, − 390°, − 420°.

2. Construct approximately the following angles: 2 radians, $3\frac{1}{2}$ radians, $-\frac{1}{2}$ radian, − 4 radians, 9 radians.

3. Construct the following angles:

$$\frac{\pi}{2},\ -\frac{\pi}{3},\ \frac{\pi}{4},\ \pi,\ -\frac{5\,\pi}{4},\ \frac{5\,\pi}{2}.$$

4. Reduce the following angles to radian or circular measure: 10°, 30°, 45°, 60°, 135°, 225°, − 270°, − 12°, − 18°, 24° 15′, − 612° 19′ 25″.

5. Reduce to degree measure: 2 radians, 5 radians, -3 radians, $\frac{1}{2}$ radian, $-\frac{1}{4}$ radian, b radians.

6. Reduce to degrees:

$$\frac{\pi}{3}, \frac{3\pi}{4}, -\frac{5\pi}{3}, \frac{3.1416}{\pi}, \frac{2}{\pi}, \frac{\pi+1}{2}, \frac{2}{\pi+3}.$$

7. If an arc of 30 ft. subtends an angle of 4 radians, find the radius of the circle.

8. In a circle whose radius is 5, the length of an intercepted arc is 12. Find the angle (a) in radians, (b) in degrees.

9. If two angles of a plane triangle are respectively equal to 1 radian and $\frac{1}{2}$ radian, express the third angle in degrees.

10. In a circle whose radius is 12 ft., find the length of the arc intercepted by a central angle of 16°.

11. Find the angle between the tangents to a circle at two points whose distance apart measured on the arc of the circle is 378 ft., the radius of the circle being 900 ft.

12. An automobile whose wheels are 34 inches in diameter travels at the rate of 25 miles per hour. How many revolutions per minute does a wheel make? What is its angular velocity in radians per second?

13. Assuming the earth's orbit to be a circle of radius 92,000,000 miles, what is the velocity of the earth in its path in miles per second?

14. The rotor of a steam turbine is two feet in diameter and makes 25,000 revolutions per minute. The blades of the turbine, situated on the circumference of the rotor, have one-half the velocity of the steam which drives them. What is the velocity of the steam in feet per second?

15. A belt travels around two pulleys whose diameters are 3 feet and 10 inches respectively. The larger pulley makes 80 revolutions per minute. Find the angular velocity of the smaller pulley in radians per second, also the speed of the belt in feet per minute.

CHAPTER II

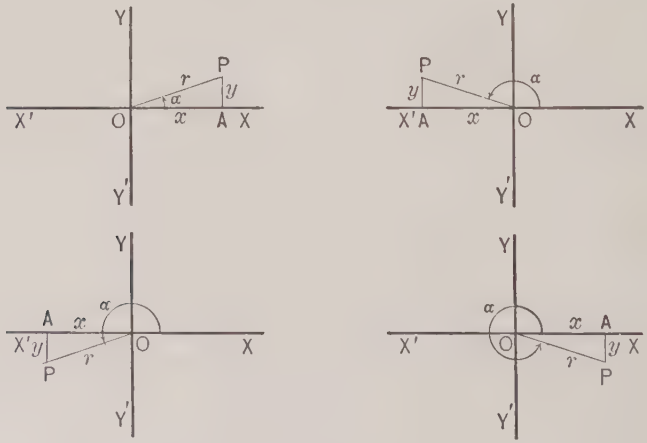
TRIGONOMETRIC FUNCTIONS

20. The trigonometric functions, upon which trigonometry is based, are functions of an angle.

These functions are the sine, cosine, tangent, cotangent, secant, cosecant, versed sine, and coversed sine of an angle.

For any angle α they are written sin α, cos α, tan α, cot α, sec α, csc α, vers α, and covers α.

21. Definitions of the trigonometric functions. Let OX and OP be the initial and terminal lines respectively of any angle α.



Let P be any point on the terminal line,

OP or r the **distance** from the origin to the point P,
OA or x the **abscissa** of the point P, and
AP or y the **ordinate** of the point P.

It should be noted that OP, OA, and AP are directed lines and hence

$$r = OP \text{ and not } PO$$
$$x = OA \text{ and not } AO$$
$$y = AP \text{ and not } PA.$$

Then the trigonometric functions are defined as follows:

$$\sin \alpha = \frac{y}{r} = \frac{\text{ordinate}}{\text{distance}}$$

$$\cos \alpha = \frac{x}{r} = \frac{\text{abscissa}}{\text{distance}}$$

$$\tan \alpha = \frac{y}{x} = \frac{\text{ordinate}}{\text{abscissa}}$$

$$\cot \alpha = \frac{x}{y} = \frac{\text{abscissa}}{\text{ordinate}}$$

$$\sec \alpha = \frac{r}{x} = \frac{\text{distance}}{\text{abscissa}}$$

$$\csc \alpha = \frac{r}{y} = \frac{\text{distance}}{\text{ordinate}}$$

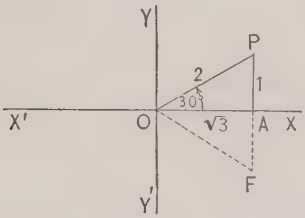
$$\text{vers } \alpha = 1 - \cos \alpha$$
$$\text{covers } \alpha = 1 - \sin \alpha$$

It will be observed that each of the first six functions is defined as a **ratio** between two line segments. These are the fundamental **trigonometric functions**, and the ratios defining them are called the **trigonometric ratios.**

The first six functions are called **trigonometric ratios.** The expression **trigonometric functions** is more general and embraces the versed sine, coversed sine, and the trigonometric ratios. It is evident, from the definitions, that the values of the trigonometric functions are *abstract numbers.*

22. Values of the trigonometric functions of 30°, 45°, and 60°. A few concrete illustrations serve to show the nature of the trigonometric functions, to fix ideas, and to prepare the way for more general considerations.

Functions of 30°. Let OPF be an equilateral triangle having its sides equal to 2 units. Place the triangle with a vertex at the origin so that OX bisects the angle POF. Then, by geometry, the angle $AOP = 30°$, the ordinate $AP = 1$, and the abscissa $OA = \sqrt{3}$.



Hence, applying definitions,

$$\sin 30° = \frac{1}{2} = .500$$

$$\cos 30° = \frac{\sqrt{3}}{2} = .866$$

$$\tan 30° = \frac{1}{\sqrt{3}} = \frac{1}{3}\sqrt{3} = .577$$

$$\cot 30° = \frac{\sqrt{3}}{1} = \sqrt{3} = 1.732$$

$$\text{vers } 30° = 1 - \frac{1}{2}\sqrt{3} = .134$$

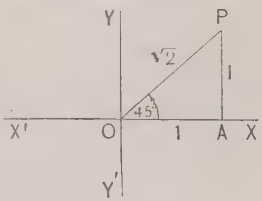
$$\sec 30° = \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3} = 1.155$$

$$\text{covers } 30° = 1 - \frac{1}{2} = \frac{1}{2} = .500$$

$$\text{csc } 30° = \frac{2}{1} = 2.000$$

PROBLEM. Find the values of the trigonometric functions of 30°, as above, taking $OP = 1$.

Functions of 45°. Let OAP be a right-angled isosceles triangle having its sides OA and AP each equal to 1. Then $\angle AOP = 45°$ and the distance $OP = \sqrt{2}$.



Hence, applying definitions,

$$\sin 45° = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$$

$$\sec 45° = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\cos 45° = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$$

$$\text{csc } 45° = \frac{\sqrt{2}}{1} = \sqrt{2}$$

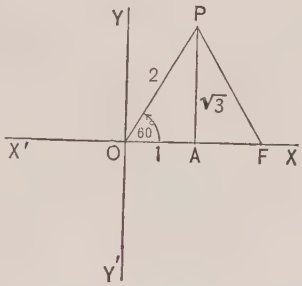
$$\tan 45° = \frac{1}{1} = 1$$

$$\text{vers } 45° = 1 - \frac{1}{2}\sqrt{2}$$

$$\cot 45° = \frac{1}{1} = 1$$

$$\text{covers } 45° = 1 - \frac{1}{2}\sqrt{2}$$

PROBLEM. Find the values of the trigonometric functions of 45°, taking $OA = 3$.



Functions of 60°. Let OPF, an equilateral triangle having each side equal to 2, be placed as in the figure. Then the abscissa and ordinate of P are 1 and $\sqrt{3}$, respectively.

Hence, applying definitions,

$$\sin 60° = \frac{\sqrt{3}}{2} \qquad \sec 60° = \frac{2}{1} = 2$$

$$\cos 60° = \frac{1}{2} \qquad \csc 60° = \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3}$$

$$\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3} \qquad \text{vers } 60° = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\cot 60° = \frac{1}{\sqrt{3}} = \frac{1}{3}\sqrt{3} \qquad \text{covers } 60° = 1 - \frac{1}{2}\sqrt{3}$$

PROBLEM. Find the values of the functions of 60°, as above, taking $OP = 4a$.

The values of the sines and cosines of 30°, 45°, and 60° are used frequently and should be memorized. The following table may be found helpful:

$$\sin 30° = \frac{1}{2}\sqrt{1} = \cos 60°$$

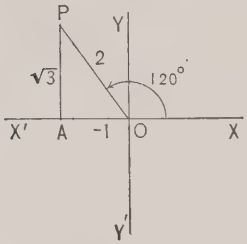
$$\sin 45° = \frac{1}{2}\sqrt{2} = \cos 45°$$

$$\sin 60° = \frac{1}{2}\sqrt{3} = \cos 30°$$

23. Values of the trigonometric functions of 120°, 135°, and 150°. By using the magnitudes of the figures of Art. 22 and properly placing them with respect to the axes, the

values of the trigonometric functions of various angles may be obtained.

Functions of 120°. From the figure and definitions it is evident that



$$\sin 120° = \frac{\sqrt{3}}{2}$$

$$\cos 120° = \frac{-1}{2} = -\frac{1}{2}$$

$$\tan 120° = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

$$\cot 120° = \frac{-1}{\sqrt{3}} = -\frac{1}{3}\sqrt{3}$$

$$\text{vers } 120° = 1 - \left(-\frac{1}{2}\right) = \frac{3}{2} \qquad \sec 120° = \frac{2}{-1} = -2$$

$$\text{covers } 120° = 1 - \frac{\sqrt{3}}{2} \qquad \csc 120° = \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3}$$

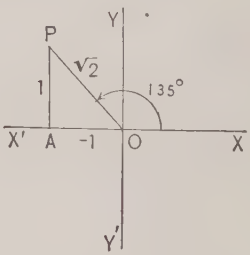
Functions of 135°. From the figure and definitions it is evident that

$$\sin 135° = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$$

$$\cos 135° = \frac{-1}{\sqrt{2}} = -\frac{1}{2}\sqrt{2}$$

$$\tan 135° = \frac{1}{-1} = -1$$

$$\cot 135° = \frac{-1}{1} = -1$$



$$\sec 135° = \frac{\sqrt{2}}{-1} = -\sqrt{2} \qquad \text{vers } 135° = 1 + \frac{1}{2}\sqrt{2}$$

$$\csc 135° = \frac{\sqrt{2}}{1} = \sqrt{2} \qquad \text{covers } 135° = 1 - \frac{1}{2}\sqrt{2}$$

Functions of 150°. From the figure and definitions it is evident that

$$\sin 150° = \frac{1}{2}$$

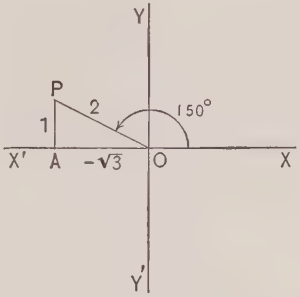
$$\cos 150° = \frac{-\sqrt{3}}{2} = -\frac{1}{2}\sqrt{3}$$

$$\tan 150° = \frac{1}{-\sqrt{3}} = -\frac{1}{3}\sqrt{3}$$

$$\cot 150° = \frac{-\sqrt{3}}{1} = -\sqrt{3}$$

$$\sec 150° = \frac{2}{-\sqrt{3}} = -\frac{2}{3}\sqrt{3}$$

$$\csc 150° = \frac{2}{1} = 2$$



$$\text{vers } 150° = 1 + \frac{1}{2}\sqrt{3}$$

$$\text{covers } 150° = 1 - \frac{1}{2} = \frac{1}{2}$$

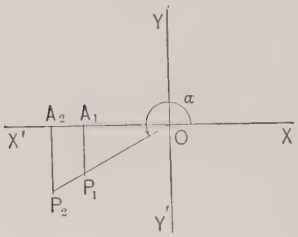
PROBLEM. Find the values of the trigonometric functions of 210°, 225°, 240°, 300°, 315°, and 330°.

24. Signs of the trigonometric ratios. The signs of the trigonometric ratios of any angle depend upon the signs of the ordinate, the abscissa, and the distance of any point on its terminal line. As the terminal line passes from one quadrant to another, there is always a change of sign in either the abscissa or the ordinate of any point on that line, but the distance remains positive. When a coördinate changes its sign, every trigonometric ratio dependent upon it must also change its sign.

The following table is constructed by taking account of the signs of the abscissa x and of the ordinate y, and remembering that the distance r is always positive. It gives the sign of each trigonometric ratio of an angle terminating in any quadrant.

	1st Quadrant	2d Quadrant	3d Quadrant	4th Quadrant
sin α	$\frac{+}{+}=+$	$\frac{+}{+}=+$	$\frac{-}{+}=-$	$\frac{-}{+}=-$
cos α	$\frac{+}{+}=+$	$\frac{-}{+}=-$	$\frac{-}{+}=-$	$\frac{+}{+}=+$
tan α	$\frac{+}{+}=+$	$\frac{+}{-}=-$	$\frac{-}{-}=+$	$\frac{-}{+}=-$
cot α	$\frac{+}{+}=+$	$\frac{-}{+}=-$	$\frac{-}{-}=+$	$\frac{+}{-}=-$
sec α	$\frac{+}{+}=+$	$\frac{+}{-}=-$	$\frac{+}{-}=-$	$\frac{+}{+}=+$
csc α	$\frac{+}{+}=+$	$\frac{+}{+}=+$	$\frac{+}{-}=-$	$\frac{+}{-}=-$

25. Theorem. *For every given angle there is one and only one value of each trigonometric function.*



The theorem is demonstrated for the sine of an angle. The same method is applicable to each of the remaining functions.

Let α be any angle. Referring to Art. 21, it is clear that if it be possible to obtain two or more values for sin α they must be obtained by taking different points on the terminal line.

Let P_1 and P_2 be any two points on the terminal line. Then by definition

$$\sin\alpha = \frac{A_1P_1}{OP_1}, \text{ or } \sin\alpha = \frac{A_2P_2}{OP_2}.$$

But since the right triangles OA_1P_1 and OA_2P_2 are similar,

and have their corresponding sides in the same direction, it follows that

$$\frac{A_1P_1}{OP_1} = \frac{A_2P_2}{OP_2}.$$

Hence the value of sin α is independent of the position of the point chosen on the terminal line, but depends solely upon the position of the terminal line, *i.e.* upon the angle.

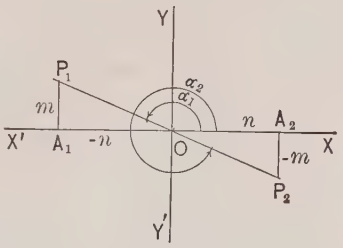
The above theorem may be stated as follows: *The trigonometric functions are single-valued functions of the angle.*

26. Theorem. *Every given value of a trigonometric function determines an unlimited or infinite number of positive and negative angles, among which there are always two positive angles less than* 360°.

The theorem is demonstrated for a given tangent. A similar method is applicable to the remaining functions.

Let $\tan \alpha = -\frac{m}{n}$ where m and n are positive numbers. Then

$$\tan \alpha = \frac{+m}{-n} = \frac{-m}{+n}.$$



Locate the point P_1, whose abscissa is $-n$, and whose ordinate is m. This point and the origin determine the terminal line of the angle α_1, whose tangent is $-\frac{m}{n}$.

Likewise the point P_2 is located by using $-m$ and n as ordinate and abscissa respectively. Drawing the terminal line OP_2, a second angle α_2 is found, which also has the given tangent.

The angles α_1 and α_2, are evidently less than 360°, and have the given tangent $-\frac{m}{n}$. There is an unlimited number of positive and negative angles coterminal with α_1 and α_2, all of which have the given tangent. Hence the theorem.

27. EXAMPLES

In what quadrants does α terminate when

1. $\sin \alpha = -\frac{1}{2}$.
2. $\cos \alpha = \frac{1}{3}$.
3. $\tan \alpha = 5$.
4. $\cot \alpha = -8$.
5. $\sin \alpha = \frac{2}{3}$.
6. $\cos \alpha = -\frac{3}{5}$.
7. $\sin \alpha$ is positive and $\cos \alpha$ is negative.
8. $\tan \alpha$ is positive and $\cos \alpha$ is negative.
9. $\operatorname{cosec} \alpha$ is negative and $\cos \alpha$ is negative.
10. $\tan \alpha$ is negative and $\sin \alpha$ is positive.
11. $\cos \alpha$ is negative and $\sin \alpha$ is negative.

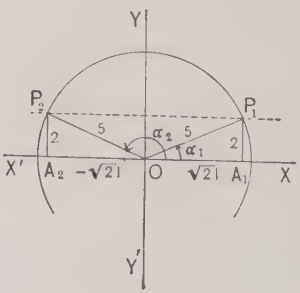
Give the signs of the trigonometric functions of the following angles:

12. $750°$.
13. $\frac{8\pi}{3}$.
14. $560°$.
15. $5\frac{2}{3}\pi$.
16. $-15°$.
17. $-\frac{5\pi}{4}$.
18. $-470°$.
19. $-\frac{7\pi}{6}$.

Find the negative angles, numerically less than $360°$, that are coterminal with the following angles:

20. $\frac{10\pi}{3}$.
21. $\frac{2\pi}{3}$.
22. $300°$.
23. π.
24. $\frac{3\pi}{2}$.
25. $-495°$

26. Construct the positive angles, less than $360°$, for which the sine is equal to $\frac{2}{5}$, and find the values of the other functions of both angles.



Solution. Determine the points whose ordinates are 2 and whose distances are 5, as follows:

With O as center and a radius 5, describe a circle. Through a point on the Y-axis, 2 units above the origin, draw a line parallel to the X-axis, intersecting the circle in the two required points P_1 and P_2. Draw the terminal lines OP_1 and OP_2, giving the angles α_1 and α_2, for which we have

$$\sin\alpha_1 = \frac{2}{5} \qquad \text{and} \qquad \sin\alpha_2 = \frac{2}{5}$$

$$\cos\alpha_1 = \frac{\sqrt{21}}{5} \qquad \cos\alpha_2 = \frac{-\sqrt{21}}{5}$$

$$\tan\alpha_1 = \frac{2}{\sqrt{21}} = \frac{2}{21}\sqrt{21} \qquad \tan\alpha_2 = \frac{2}{-\sqrt{21}} = -\frac{2}{21}\sqrt{21}$$

$$\cot\alpha_1 = \frac{\sqrt{21}}{2} \qquad \cot\alpha_2 = \frac{-\sqrt{21}}{2}$$

$$\sec\alpha_1 = \frac{5}{\sqrt{21}} = \frac{5}{21}\sqrt{21} \qquad \sec\alpha_2 = \frac{5}{-\sqrt{21}} = -\frac{5}{21}\sqrt{21}$$

$$\csc\alpha_1 = \frac{5}{2} \qquad \csc\alpha_2 = \frac{5}{2}$$

27. Construct the positive angles, less than 360°, for which the cosine is $\frac{3}{5}$, and find the values of the other functions of both angles.

Find the values of the functions of all angles less than 360° determined by

28. $\tan\alpha = -\frac{6}{7}$.

29. $\cot\alpha = 1\frac{1}{2}$.

30. $\csc\alpha = 3\frac{1}{2}$.

31. $\cot\alpha = -5$.

32. $\sin\alpha = -\frac{2}{3}$.

33. $\sin\alpha = \frac{a}{b}$.

34. $\cos\alpha = .4$.

35. Given $\tan\alpha = -4$, find the value of $\dfrac{\sin\alpha\cos\alpha}{\cot\alpha}$.

36. Given $\sec\alpha = 6$, find the value of $\dfrac{\sin^2\alpha + \cos^2\alpha}{\cos^2\alpha}$.

37. Given $\sin\alpha = .3$, find the value of $\tan\alpha\sec\alpha\cos\alpha$.

38. Given $\csc\alpha = 8$ and $\tan\beta = 3$, find the value of $\sin\alpha\cos\beta + \cos\alpha\sin\beta$.

39. Given $\tan\alpha = \frac{1}{2}$ and $\cos\beta = -\frac{1}{5}\sqrt{5}$, α terminating in the first quadrant and β in the second, show that the angle between the terminal lines of α and β is a right angle.

CHAPTER III

RIGHT TRIANGLES

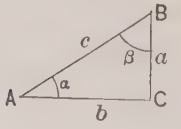
28. In every triangle there are six **elements** or **parts.** These are the three sides and the three angles.

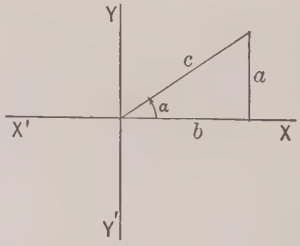
When three elements are given, one of which is a side, the other three elements can be determined.

The solution of a triangle is the process of determining the unknown parts from the given parts.

In the present chapter it will be shown how the trigonometric functions can be employed to solve the right triangle.

29. Applications of definitions of the trigonometric functions to the right triangle. Let ABC be any right triangle. Place the triangle in the first quadrant with the vertex of one acute angle coinciding with the origin, and one side, not the hypotenuse, coinciding with the X-axis, as in the figure.





Then, by definitions,

$$\sin \alpha = \frac{a}{c} = \frac{\text{side opposite}}{\text{hypotenuse}}$$

$$\cos \alpha = \frac{b}{c} = \frac{\text{side adjacent}}{\text{hypotenuse}}$$

$$\tan \alpha = \frac{a}{b} = \frac{\text{side opposite}}{\text{side adjacent}}$$

$$\cot \alpha = \frac{b}{a} = \frac{\text{side adjacent}}{\text{side opposite}}$$

$$\sec \alpha = \frac{c}{b} = \frac{\text{hypotenuse}}{\text{side adjacent}}$$

$$\csc \alpha = \frac{c}{a} = \frac{\text{hypotenuse}}{\text{side opposite}}$$

It is seen that the functions of any acute angle of a right triangle are expressed in terms of the *side adjacent*, the *side opposite*, and the *hypotenuse*, *without reference to the coordinate axes.*

It follows that

$$\sin \beta = \frac{b}{c} \qquad \cot \beta = \frac{a}{b}$$

$$\cos \beta = \frac{a}{c} \qquad \sec \beta = \frac{c}{a}$$

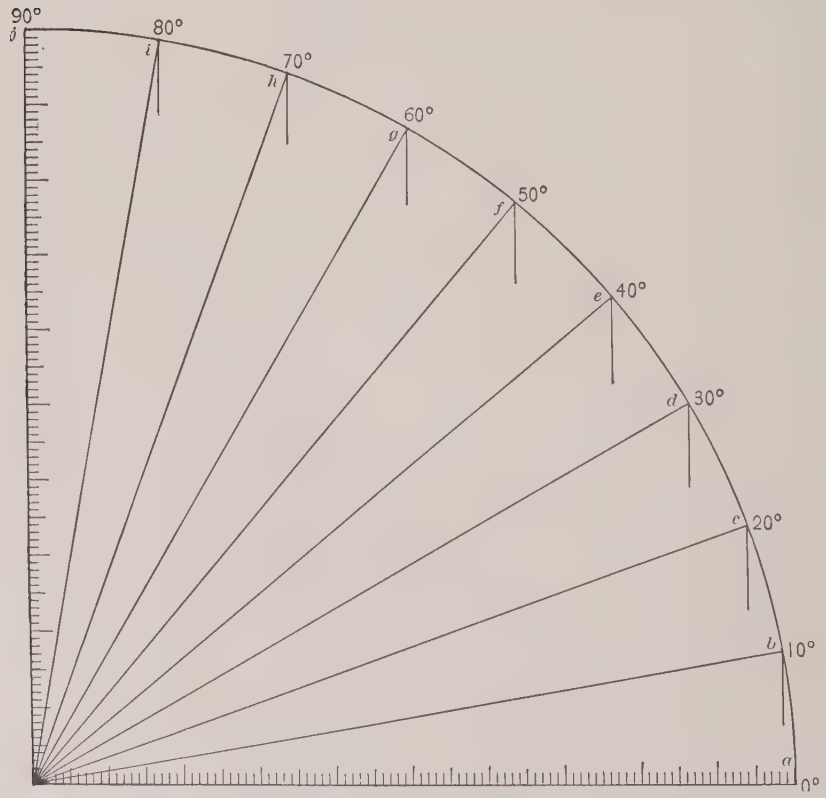
$$\tan \beta = \frac{b}{a} \qquad \csc \beta = \frac{c}{b}$$

30. Trigonometric tables. In the preceding chapter the trigonometric functions of 30°, 45°, and 60° were calculated. By processes too complicated to introduce here, tables have been computed, giving the values of the trigonometric functions of acute angles. These tables generally contain two parts. In one part the values of the functions, called **natural functions**, are given; in the other part the logarithms of the trigonometric functions, called **logarithmic functions**, are given.

When an angle is given its trigonometric functions can be taken from the tables, and *vice versa.* The functions of known angles thus become known numbers and can be used in problems of computation.

Approximate values of the trigonometric functions can be obtained by graphical methods.

Problem. Measure the distance, abscissas and ordinates of the points $a, b, c, \cdots, j$. From these measurements compute to two figures the sine, cosine, and tangent of 0°, 10°, 20°, $\cdots$, 90°. By arranging the results in tabular form, a two place table is constructed.



This table may be used to solve Examples 1 to 6, Art. 36. On account of its extreme simplicity the use of this table allows the attention to be focused upon the fundamental processes involved in the solution of triangles.

31. Formulas used in the solution of right triangles. A right triangle can always be solved when, in addition to the right angle γ, two independent parts are given. The formulas usually employed are:

$$\sin\alpha = \frac{a}{c} \qquad \sin\beta = \frac{b}{c} \qquad \alpha + \beta = 90°$$

$$\cos\alpha = \frac{b}{c} \qquad \cos\beta = \frac{a}{c} \qquad a^2 + b^2 = c^2$$

$$\tan\alpha = \frac{a}{b} \qquad \tan\beta = \frac{b}{a}$$

When the computations are made without logarithms, the formulas involving the cotangent, secant, and cosecant may sometimes be used advantageously.

32. Selection of formulas.

(*a*) If an angle and a side are given, it is always possible to find the unknown parts directly from the given parts without the use of the formula $a^2 + b^2 = c^2$. To find the unknown side, select that formula which contains the given parts and the desired unknown side. The unknown angle can always be found from $\alpha + \beta = 90°$.

(*b*) If two sides are given, the third side would naturally be found by the use of $a^2 + b^2 = c^2$, but in practice it is generally preferable first to compute an angle by the use of a formula involving the given sides and an angle. To find the third side, select a formula containing one of the given sides, the angle already computed, and the required side.

33. Check formulas. In all computations it is necessary constantly to guard against numerical errors. However carefully the computations are made, errors may still occur, and therefore computed parts should be checked by means of check formulas. Any formula which was not used in the solution of the triangle may be used for this purpose.

For the right triangle the formula

$$a^2 = c^2 - b^2 = (c - b)(c + b)$$

may conveniently be used, and in general it is sufficient.

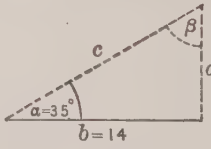
34. Suggestions on solving a triangle. Make a careful free-hand construction of the required triangle, and write down an estimate of the values of the unknown parts. Large errors will be detected readily, without the use of check formulas, when the computed parts are compared with the estimates.

Before entering the tables, and before making any computations, select all the formulas to be used, solve the formulas for the required parts, and make an outline in which a place is provided for every number to be used in the computation. This will often lessen the actual work, for frequently several required numbers are found on the same page of the table.

The arrangement of the work is of considerable importance in every extended computation.

35. Illustrative examples.

1. In a right triangle, given $b = 14$, $\alpha = 35°$, to find a, c, and β.



SOLUTION. Approximate construction.

Estimate $a = 9$, $c = 17$.

By natural functions

$\tan \alpha = \dfrac{a}{b}$ $\qquad$ $\cos \alpha = \dfrac{b}{c}$

$\therefore a = b \tan \alpha$ $\qquad$ $\therefore c = \dfrac{b}{\cos \alpha}$ $\qquad$ $\beta = 90° - \alpha$

$a = 14 \times .7002$ $\qquad$ $c = 14 \div .8192$ $\qquad$ $\beta = 55°$

$a = 9.803$ $\qquad$ $c = 17.09$

Check

$c^2 = a^2 + b^2$

$a^2 = 96.10$

$b^2 = 196.0$

292.1

$c^2 = (17.09)^2 = 292.1$

By logarithms

$a = b \tan \alpha$

b	14
α	35°
$\log b$	
$\log \tan \alpha$	
$\log a$	
a	

$c = \dfrac{b}{\cos \alpha}$

$\log b$	
$\log \cos \alpha$	
$\log c$	
c	

$\beta = 90° - \alpha$

$\beta =$

Check

$a^2 = c^2 - b^2 = (c - b)(c + b)$

c		$\log (c - b)$	
b		$\log (c + b)$	
$c - b$		$2 \log a$	
$c + b$		$\log a$	

Filling in the above outline, the completed work appears as follows:

$a = b \tan \alpha$ $\qquad$ $c = \dfrac{b}{\cos \alpha}$ $\qquad$ $\beta = 90° - \alpha$

b	14	$\log b$	1.14613	$\beta = 55°$
α	35°	$\log \cos \alpha$	9.91336 − 10	
$\log b$	1.14613	$\log c$	1.23277	
$\log \tan \alpha$	9.84523 − 10	c	17.091	
$\log a$	0.99136			
a	9.8030			

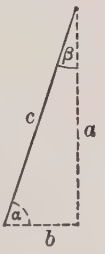
Check

$a^2 = c^2 - b^2 = (c - b)(c + b)$

c	17.091	$\log (c - b)$	0.49010
b	14.	$\log (c + b)$	1.49263
$c - b$	3.091	$2 \log a$	1.98273
$c + b$	31.091	$\log a$	0.99136

Since the check formula gives the same value for $\log a$ as that found in the solution, the computation is in all probability correct.

2. In a right triangle, given $c = 6.275$, $\beta = 18° 47'$, to find a, b, and α.



SOLUTION. Approximate construction.

Estimated $a = 5$, $\quad b = 2$.

By natural functions		*Check*
$a = c \cos \beta$	$b = c \sin \beta$	$c^2 = a^2 + b^2$
$a = 6.275 \times .9468$	$b = 6.275 \times .3220$	$a^2 = 35.30$
$a = 5.941$	$b = 2.021$	$b^2 = \underline{4.084}$
	$\alpha = 90° - \beta$	39.384
	$\alpha = 71° 13'$	$c^2 = (6.275)^2 = 39.38$

By logarithms

$a = c\cos\beta$	
c	6.275
β	18° 47′
$\log c$	0.79761
$\log\cos\beta$	9.97623 − 10
$\log a$	0.77384
a	5.9407

$b = c\sin\beta$	
$\log c$	0.79761
$\log\sin\beta$	9.50784 − 10
$\log b$	0.30545
b	2.0205

$\alpha = 90° - \beta$

$\alpha = 71° 13'$

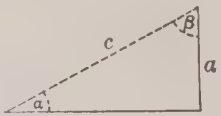
Check

$a^2 = c^2 - b^2 = (c - b)(c + b)$

c	6.275
b	2.0205
$c - b$	4.2545
$c + b$	8.2955

$\log(c - b)$	0.62885
$\log(c + b)$	0.91884
$2\log a$	1.54769
$\log a$	0.77384

3. Given $a = .064873$, $b = .12574$, to find α, β, and c.



Solution. Approximate construction.
Estimate $\alpha = 30°$, $\beta = 60°$, $c = .15$.

$$\tan\alpha = \frac{a}{b} \qquad c = \frac{a}{\sin\alpha}$$

a	.064873
b	.12574
$\log a$	8.81206 − 10
$\log b$	9.09948 − 10
$\log\tan\alpha$	9.71258 − 10
α	27° 17′ 23″
β	62° 42′ 37″

$\log a$	8.81206 − 10
$\log\sin\alpha$	9.66133 − 10
$\log c$	9.15073 − 10
c	.14149

Check

$a^2 = (c - b)(c + b)$

c	.14149
b	.12574
$c - b$	.01575
$c + b$	.26723

$\log(c - b)$	8.19728 − 10
$\log(c + b)$	9.42689 − 10
$2\log a$	17.62417 − 20
$\log a$	8.81208 − 10

36. EXAMPLES

Solve the following right triangles, γ being the right angle. It is recommended that the first ten problems be solved by the use of a three-place table of natural functions, or by means of a slide rule.

1. $a = 6$, $\alpha = 20°$

2. $c = 2.5$, $\alpha = 35°$

3. $b = .84$, $\beta = 75°$

4. $a = 25$, $b = 60$

5. $c = 82$, $a = 37$

6. $c = .0091$, $a = .0029$

7. $b = 371$, $\alpha = 43°$

8. $c = 7.72$, $b = 6.87$

9. $\alpha = 18°$, $c = .0938$

10. $\beta = 49° 30'$, $c = 12.47$

The following problems should be solved by the use of a four-place * or a five-place table.

11. $a = 1870$, $\alpha = 19° 55'$

12. $c = .3194$, $\alpha = 25° 41'$

13. $b = .9292$, $\beta = 32° 43'$

14. $a = .00006$, $b = .000019$

15. $c = 1200.7$, $a = 885.6$

16. $c = 12.145$, $a = 9.321$

17. $b = 78.545$, $\alpha = 31° 41' 6''$

18. $b = 3.4572$, $\beta = 57° 57' 57''$

19. $c = 20.082$, $b = 16.174$

20. $\alpha = 78° 0' 3''$, $a = 271.82$

* Whenever sides are given to four significant figures and angles to minutes.

21. $a = 5987.2$, $\beta = 88° 53' 2''$

22. $c = .09008$, $a = .07654$

23. $\alpha = 46° 39' 50''$, $a = 26.434$

24. $a = 30.008$, $b = 29.924$

25. $a = 111.45$, $b = 121.69$

26. $\beta = 83° 15' 6''$, $c = 7000$

27. $\alpha = 66° 6' 18''$, $c = 8070.6$

28. $a = 978.45$, $b = 1067.2$

29. $a = 5280$, $b = 5608$

30. $\alpha = 17° 26' 34''$, $c = 46.474$

31. Solve the isosceles triangle, one of the equal sides being 690.13, and one of the base angles being $15° 20' 25''$.

32. Solve the isosceles triangle whose altitude is 606.6, one of the equal sides being 955.7.

33. Solve the isosceles triangle whose base is 2558, and whose vertical angle is $104° 0' 46''$.

34. Solve the isosceles triangle whose base is 161.4, and whose altitude is 204.4.

35. Find the length of a side of a regular octagon inscribed in a circle whose radius is 49.

37. Oblique triangles. When any three independent parts of an oblique triangle are given it can be solved by means of right triangles. The oblique triangle may be divided into two right triangles by drawing a perpendicular from one vertex to the opposite side, or the opposite side produced.

When one of the given parts is an angle, the perpendicular must be selected so that one of the resulting right triangles will contain two of the given parts.

In case the three sides are given, a second part of one of

the right triangles can be obtained by equating the expressions for the length of the perpendicular obtained from each right triangle.

Thus $a^2 - x^2 = b^2 - (c - x)^2$,

from which $x = \dfrac{a^2 - b^2 + c^2}{2c}$.

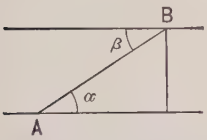
38. APPLICATIONS

1. To find the distance from B to C, two points on opposite sides of a river, a line BA, 200 feet long, was laid off at right angles to the line BC, and the angle BAC was measured and found to equal 55° 29′. What was the required distance?

2. A railway is inclined 4° 23′ 20″ to the horizontal. How many feet does it rise per mile, measured along the horizontal?

3. From the top of a tower 120 feet high the angle of depression of an object in the horizontal plane of the base of the tower is 24° 27′. How far is the object from the foot of the tower? How far from the observer?

Given any point A and a second point B at a greater elevation than A.



The angle of *elevation* of B from A is the angle that the line AB makes with its orthogonal projection upon the horizontal plane through A. In the figure it is the angle α.

The angle of *depression* of A from B is the angle that the line BA makes with its orthogonal projection upon the horizontal plane through B. In the figure it is the angle β. It is clear that $\alpha = \beta$.

4. Find the length of one side of a regular pentagon inscribed in a circle whose radius is 18.24 feet.

5. Find the perimeter of a regular polygon of n sides inscribed in a circle whose radius is r.

6. Find the perimeter of a regular pentagon circumscribed about a circle whose radius is 18.24 feet.

7. Find the perimeter of a regular polygon of n sides circumscribed about a circle whose radius is r.

8. Find the length of a chord subtending a central angle of $63° 14' 20''$ in a circle whose radius is 124.93 feet.

9. A straight road, PR, makes an angle of $19° 27' 30''$ with another straight road, PS. Having given $PR = 640$ feet, find RS, the perpendicular distance from R to PS.

10. At a certain point the angle of elevation of a mountain is $34° 28'$. At a second point 500 feet farther away, the angle of elevation is $31° 12'$. Find the height of the mountain above the table-land.

11. One side of the square base of a right pyramid is 15 inches, and the altitude is 20 inches. Find the slant height and the edge. Find the inclination of a face to the base of the pyramid.

12. How far can you see from an elevation of 2000 feet, assuming the earth to be a sphere with a radius of 3960 miles?

13. Two forces of 95.75 pounds and 120.25 pounds, at right angles to each other, act at a point; find the magnitude of their resultant and the angle it makes with the greater force.

14. Find the velocity of a point, whose latitude is $44° 30' 20''$, due to the rotation of the earth. Assume the radius of the earth to be 3960 miles.

15. Find the length of a belt running around two pulleys whose radii are 12 inches and 4 inches, respectively, the distance between the centers of the pulleys being 6 feet.

16. A diagonal of a cube and a diagonal of a face of a cube intersect at a vertex. Find the angle between them.

17. A force of 2000 pounds applied at the origin makes an angle of 33° 25′ with the positive X-axis. Find its components along the X and Y axes respectively.

18. A force of 185 pounds applied at the origin makes an angle of 82° 12′ with the positive X-axis. Another force of 327 pounds applied at the origin makes an angle of 11° 32′ with the positive X-axis. Find the X and Y components of each of these forces. Add the X-components and also the Y-components and then find the resultant in magnitude and direction.

19. A cylindrical tank, whose axis is horizontal, is 8 feet in diameter and 12 feet long. The tank is partly filled with water, so that the depth of the water at the deepest point is 3 feet. How many gallons of water are in the tank, there being $7\frac{1}{2}$ gallons in a cubic foot?

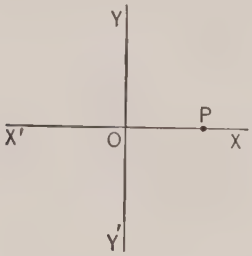
20. A man who can paddle his canoe 5 miles per hour in still water paddles at his usual rate directly across a river one-half mile wide. If the river flows 4 miles per hour, where will the canoe land and what is its speed in the water?

CHAPTER IV

VARIATIONS OF THE TRIGONOMETRIC FUNCTIONS REDUCTION OF FUNCTIONS OF $n\,90° \pm \alpha$

39. In the study of the variations of the trigonometric functions, their values at 0°, 90°, 180°, 270°, and 360° are of special importance.

Values of functions of 0°. The terminal line of 0° coincides with the positive X-axis. For a point on this line, at a distance r from the origin, we have



$$x = r, \qquad y = 0$$

Hence

$$\sin 0° = \frac{0}{r} = 0 \qquad \cot 0° = \frac{r}{0}^{*} = \infty$$

$$\cos 0° = \frac{r}{r} = 1 \qquad \sec 0° = \frac{r}{r} = 1$$

$$\tan 0° = \frac{0}{r} = 0 \qquad \csc 0° = \frac{r}{0} = \infty$$

Values of functions of 90°. The terminal line of 90° coincides with the positive Y-axis, therefore

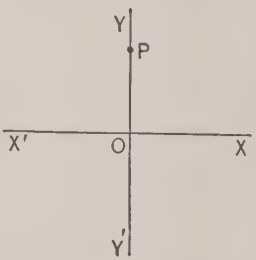
$$x = 0, \qquad y = r.$$

Hence

$$\sin 90° = \frac{r}{r} = 1 \qquad \cot 90° = \frac{0}{r} = 0$$

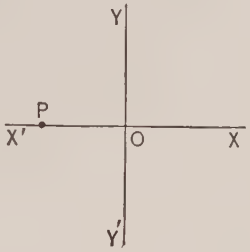
$$\cos 90° = \frac{0}{r} = 0 \qquad \sec 90° = \frac{r}{0} = \infty$$

$$\tan 90° = \frac{r}{0} = \infty \qquad \csc 90° = \frac{r}{r} = 1$$



* For the interpretation of $\frac{r}{0}$ see any College Algebra.

Values of functions of 180°. The terminal line of 180° coincides with the negative X-axis, therefore



$$x = -r, \qquad y = 0.$$

Hence

$$\sin 180° = \frac{0}{r} = 0$$

$$\cos 180° = \frac{-r}{r} = -1$$

$$\tan 180° = \frac{0}{-r} = 0 \qquad \sec 180° = \frac{r}{-r} = -1$$

$$\cot 180° = \frac{-r}{0} = \infty \qquad \csc 180° = \frac{r}{0} = \infty$$

Values of functions of 270°. The terminal line of 270° coincides with the negative Y-axis, therefore

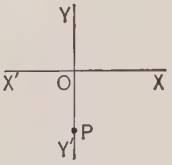
$$x = 0, \qquad y = -r$$

Hence

$$\sin 270° = \frac{-r}{r} = -1 \qquad \cot 270° = \frac{0}{-r} = 0$$

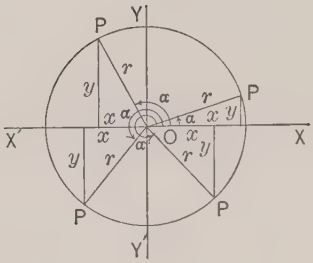
$$\cos 270° = \frac{0}{r} = 0 \qquad \sec 270° = \frac{r}{0} = \infty$$

$$\tan 270° = \frac{-r}{0} = \infty \qquad \csc 270° = \frac{r}{-r} = -1$$



The values of the functions of 360° are identical with the values of the functions of 0°, since these angles are coterminal.

40. Variation of the functions. It has been shown that the trigonometric functions are functions of an angle. As the angle varies, the trigonometric functions depending upon the angle also vary.



Let the line OP, of fixed length r, revolve about O from the initial position OX. Then the angle XOP or α increases from 0° to 360°. The variations of the trigonometric functions can be traced by observing the changes in the abscissa and ordinate of the point P.

a. As the angle α increases from 0° to 90°,
y increases from 0 to r,
and x decreases from r to 0.

Hence

$\sin\alpha$ or $\frac{y}{r}$ is positive and increases from 0 to 1

$\cos\alpha$ or $\frac{x}{r}$ is positive and decreases from 1 to 0

$\tan\alpha$ or $\frac{y}{x}$ is positive and increases from 0 to ∞

$\cot\alpha$ or $\frac{x}{y}$ is positive and decreases from ∞ to 0

$\sec\alpha$ or $\frac{r}{x}$ is positive and increases from 1 to ∞

$\csc\alpha$ or $\frac{r}{y}$ is positive and decreases from ∞ to 1

As the angle α increases through 90°, x passes through zero, changing from a positive number to a negative number. Then, immediately before α becomes 90°, $\cos\alpha$ or $\frac{x}{r}$ is a very small positive number; while immediately after α has passed 90°, $\cos\alpha$ is a very small negative number. This may be expressed by saying that $\cos\alpha$ passes through zero and changes sign as α passes through 90°.

Likewise, immediately before α becomes 90°, $\tan \alpha$ or $\frac{y}{x}$ is a very large positive number; while immediately after α has passed 90°, $\tan \alpha$ is a very large negative number. It has been seen that $\tan 90° = \infty$. Hence $\tan \alpha$ passes through ∞ and changes sign as α passes through 90°. Hence we may say that $\tan 90° = \pm \infty$, choosing the positive sign when associating 90° with the first quadrant and the negative sign when associating 90° with the second quadrant.

Similarly, whenever a trigonometric function passes through ∞ it changes sign.

b. As the angle α increases from 90° to 180°,
y decreases from r to 0,
and x decreases from 0 to $-r$.

Hence

$\sin \alpha$ or $\frac{y}{r}$ is positive and decreases from 1 to 0

$\cos \alpha$ or $\frac{x}{r}$ is negative and decreases from 0 to -1

$\tan \alpha$ or $\frac{y}{x}$ is negative and increases from $-\infty$ to 0

$\cot \alpha$ or $\frac{x}{y}$ is negative and decreases from 0 to $-\infty$

$\sec \alpha$ or $\frac{r}{x}$ is negative and increases from $-\infty$ to -1

$\csc \alpha$ or $\frac{r}{y}$ is positive and increases from 1 to $+\infty$

c. As the angle α increases through 180°, y passes through zero, changing from a positive number to a negative number.

As the angle α increases from 180° to 270°,
y decreases from 0 to $-r$,
and x increases from $-r$ to 0.

Hence

$\sin \alpha$ or $\frac{y}{r}$ is negative and decreases from 0 to -1

$\cos\alpha$ or $\frac{x}{r}$ is negative and increases from -1 to **0**

$\tan\alpha$ or $\frac{y}{x}$ is positive and increases from 0 to $+\infty$

$\cot\alpha$ or $\frac{x}{y}$ is positive and decreases from $+\infty$ to 0

$\sec\alpha$ or $\frac{r}{x}$ is negative and decreases from -1 to $-\infty$

$\csc\alpha$ or $\frac{r}{y}$ is negative and increases from $-\infty$ to -1

d. As the angle α increases through 270°, x passes through zero, changing from a negative number to a positive number.

As the angle α increases from 270° to 360°,
y increases from $-r$ to 0,
and x increases from 0 to r.

Hence

$\sin\alpha$ or $\frac{y}{r}$ is negative and increases from -1 to 0

$\cos\alpha$ or $\frac{x}{r}$ is positive and increases from 0 to 1

$\tan\alpha$ or $\frac{y}{x}$ is negative and increases from $-\infty$ to **0**

$\cot\alpha$ or $\frac{x}{y}$ is negative and decreases from 0 to $-\infty$

$\sec\alpha$ or $\frac{r}{x}$ is positive and decreases from $+\infty$ to 1

$\csc\alpha$ or $\frac{r}{y}$ is negative and decreases from -1 to $-\infty$

The above results are presented in tabular form.

	1st Quadrant	2d Quadrant	3d Quadrant	4th Quadrant
	0°	90°	180°	270° 360°
$\sin \alpha = \frac{y}{r}$	+ 0 inc. + 1	+ 1 dec. + 0	− 0 dec. − 1	− 1 inc. − 0
$\cos \alpha = \frac{x}{r}$	+ 1 dec. + 0	− 0 dec. − 1	− 1 inc. − 0	+ 0 inc. + 1
$\tan \alpha = \frac{y}{x}$	+ 0 inc. + ∞	− ∞ inc. − 0	+ 0 inc. + ∞	− ∞ inc. − 0
$\cot \alpha = \frac{x}{y}$	+ ∞ dec. + 0	− 0 dec. − ∞	+ ∞ dec. + 0	− 0 dec. − ∞
$\sec \alpha = \frac{r}{x}$	+ 1 inc. + ∞	− ∞ inc. − 1	− 1 dec. − ∞	+ ∞ dec. + 1
$\csc \alpha = \frac{r}{y}$	+ ∞ dec. + 1	+ 1 inc. + ∞	− ∞ inc. − 1	− 1 dec. − ∞

It is thus seen that the sine and cosine can never be greater than $+1$ nor less than -1, while the secant and cosecant have no values between $+1$ and -1, but have values ranging from $+1$ to $+\infty$ and from -1 to $-\infty$.

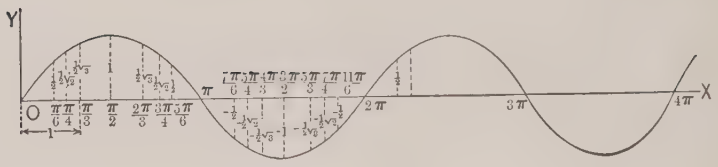
The tangent and cotangent may have any value from $+\infty$ to $-\infty$.

41. Graphical representation. A graphical representation of the trigonometric functions is effected by first locating points using the different values of the angle as abscissas and the corresponding function-values as ordinates, and then drawing a smooth curve through these points taken in the order of increasing angles.

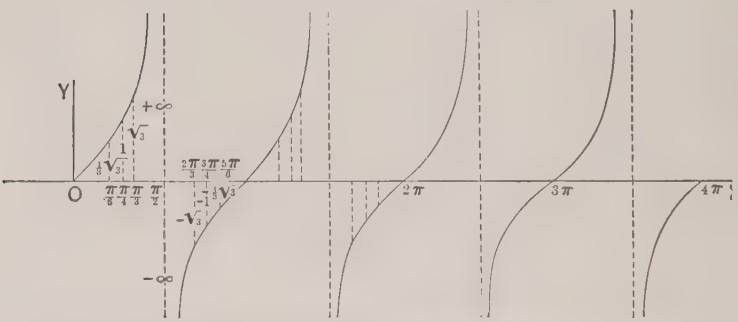
The values of the functions of the angles previously calculated are sufficient to determine an approximate graph. For greater accuracy the values of the functions may be taken from the table of natural functions.

α	$\sin \alpha$	α	$\tan \alpha$
0	0	0	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\pi}{6}$	$\frac{1}{3}\sqrt{3}$
$\frac{\pi}{4}$	$\frac{1}{2}\sqrt{2}$	$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	$\frac{1}{2}\sqrt{3}$	$\frac{\pi}{3}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	$\frac{\pi}{2}$	∞
$\frac{2\pi}{3}$	$\frac{1}{2}\sqrt{3}$	$\frac{2\pi}{3}$	$-\sqrt{3}$
$\frac{3\pi}{4}$	$\frac{1}{2}\sqrt{2}$	$\frac{3\pi}{4}$	−1
$\frac{5\pi}{6}$	$\frac{1}{2}$	$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{3}$
π	0	π	0
$\frac{7\pi}{6}$	$-\frac{1}{2}$	$\frac{7\pi}{6}$	$\frac{1}{3}\sqrt{3}$
etc.		etc.	

Sine Curve $y = \sin x$



Tangent Curve $y = \tan x$



The sine and tangent curves illustrate the truth of the theorems of Arts. 25 and 26.

Thus for any given value of the angle, as $\frac{3\pi}{4}$, there is but one value of each function, the curves showing the sine and tangent to be $\frac{1}{2}\sqrt{2}$ and -1 respectively.

But for any given value of a trigonometric function, as $\sin \alpha = \frac{1}{2}$, there are an unlimited number of angles, as

$$\frac{\pi}{6},\ \frac{5\pi}{6},\ 2\tfrac{1}{6}\pi,\ 2\tfrac{5}{6}\pi,\ \text{etc.}$$

Also for $\tan \alpha = \sqrt{3}$ we see that α may have the values

$$\frac{\pi}{3},\ 1\tfrac{1}{3}\pi,\ 2\tfrac{1}{3}\pi,\ 3\tfrac{1}{3}\pi,\ \text{etc.}$$

42. Periodicity of the trigonometric functions. From the sine curve it is readily seen that the sine is 0 when the

angle is 0, and that it increases to 1 as the angle increases to $\frac{\pi}{2}$. At this point the sine begins to decrease and has the value 0 when the angle is π, and finally reaches the value -1 when the angle is $\frac{3\pi}{2}$; then the sine again begins to increase and has the value 0 when the angle has the value 2π.

When the angle increases from 2π to 4π, the sine repeats its values of the interval from 0 to 2π. If the angle were increased indefinitely, the sine would repeat its values for each interval of 2π. For this reason the sine is called a **periodic function**, and 2π is its period.

A study of the tangent curve shows that the tangent has the same values between 0 and π that it has between π and 2π or between 2π and 3π. Hence the tangent is also a periodic function having the period π.

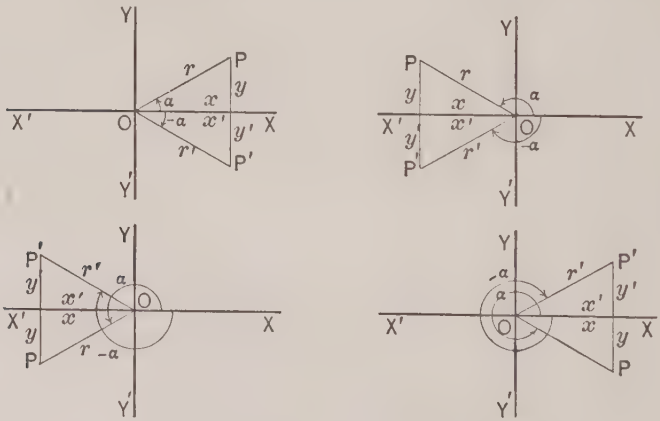
43. EXAMPLES

1. Plot $y = \cos x$ and give the period of the cosine.
2. Plot $y = \cot x$ and give the period of the cotangent.
3. Plot $y = \sec x$ and give the period of the secant.
4. Plot $y = \csc x$ and give the period of the cosecant.
5. Plot $y = \sin x + \cos x$.
6. Plot $y = x + \sin x$.

REDUCTION OF FUNCTIONS OF $n\,90° \pm \alpha$

44. The formulas to be developed in this section enable us to express any function of any angle in terms of a function of an angle differing from the given angle by any multiple of 90°. They may be used to express any function of any angle in terms of a function of an angle less than 90° or less than 45°.

45. Functions of $-\alpha$ in terms of functions of $\alpha(n = 0)$. Let XOP be any positive angle and XOP' a numerically equal negative angle.



Then taking $OP' = OP$ we have, for each figure,

$$x' = x,\ y' = -y,\ r' = r,$$

where x, y, r and x', y', r' are associated with P and P' respectively.

Then in each quadrant

$$\sin(-\alpha) = \frac{y'}{r'} = \frac{-y}{r} = -\sin\alpha$$

$$\cos(-\alpha) = \frac{x'}{r'} = \frac{x}{r} = \cos\alpha$$

$$\tan(-\alpha) = \frac{y'}{x'} = \frac{-y}{x} = -\tan\alpha$$

$$\cot(-\alpha) = \frac{x'}{y'} = \frac{x}{-y} = -\cot\alpha$$

$$\sec(-\alpha) = \frac{r'}{x'} = \frac{r}{x} = \sec\alpha$$

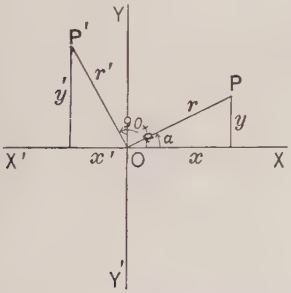
$$\csc(-\alpha) = \frac{r'}{y'} = \frac{r}{-y} = -\csc\alpha$$

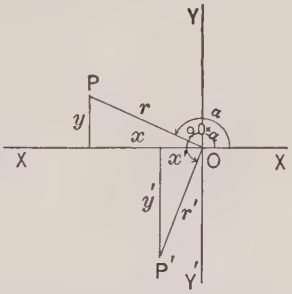
46. Functions of $90° + \alpha$ in terms of functions of $\alpha(n = 1)$. Let XOP be any positive angle α and XOP' the angle $90° + \alpha$.

Then taking $OP' = OP$ we have, for each figure,

$$x' = -y,\ y' = x,\ r' = r,$$

where x, y, r and x', y', r' are associated with P and P' respectively.





Similar figures can be constructed for the other quadrants. Then in each quadrant

$$\sin(90° + \alpha) = \frac{y'}{r'} = \frac{x}{r} = \cos\alpha$$

$$\cos(90° + \alpha) = \frac{x'}{r'} = \frac{-y}{r} = -\sin\alpha$$

$$\tan(90° + \alpha) = \frac{y'}{x'} = \frac{x}{-y} = -\cot\alpha$$

$$\cot(90° + \alpha) = \frac{x'}{y'} = \frac{-y}{x} = -\tan\alpha$$

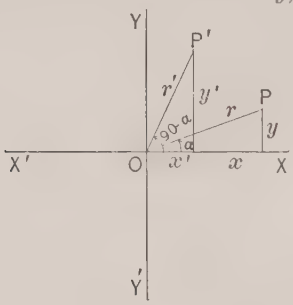
$$\sec(90° + \alpha) = \frac{r'}{x'} = \frac{r}{-y} = -\csc\alpha$$

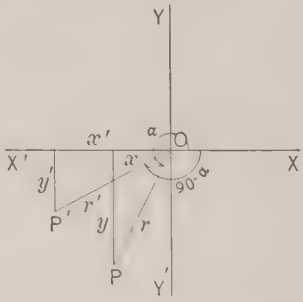
$$\csc(90° + \alpha) = \frac{r'}{y'} = \frac{r}{x} = \sec\alpha$$

47. Functions of $90° - \alpha$ in terms of functions of α $(n = 1)$. Let XOP be any positive angle and XOP' the angle $90° - \alpha$.

Then taking $OP' = OP$ we have, for each figure,

$$x' = y,\ y' = x,\ r' = r.$$





Similar figures can be constructed for the other quadrants. Then in each quadrant

$$\sin(90° - \alpha) = \frac{y'}{r'} = \frac{x}{r} = \cos\alpha$$

$$\cos(90° - \alpha) = \frac{x'}{r'} = \frac{y}{r} = \sin\alpha$$

$$\tan(90° - \alpha) = \frac{y'}{x'} = \frac{x}{y} = \cot\alpha$$

$$\cot(90° - \alpha) = \frac{x'}{y'} = \frac{y}{x} = \tan\alpha$$

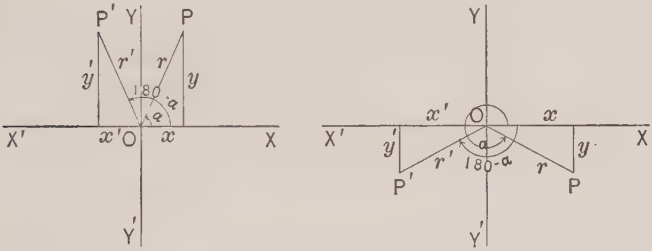
$$\sec(90° - \alpha) = \frac{r'}{x'} = \frac{r}{y} = \csc\alpha$$

$$\csc(90° - \alpha) = \frac{r'}{y'} = \frac{r}{x} = \sec\alpha$$

48. Functions of $180° - \alpha$ in terms of functions of α (n=2). Let XOP be any positive angle and XOP' the angle $180° - \alpha$.

Then taking $OP' = OP$ we have, for each figure,

$$x' = -x,\ y' = y,\ r' = r.$$



Similar figures can be constructed for the other quadrants. Then in each quadrant

$$\sin(180° - \alpha) = \frac{y'}{r'} = \frac{y}{r} = \sin\alpha$$

$$\cos(180° - \alpha) = \frac{x'}{r'} = \frac{-x}{r} = -\cos\alpha$$

$$\tan(180° - \alpha) = \frac{y'}{x'} = \frac{y}{-x} = -\tan\alpha$$

$$\cot(180° - \alpha) = \frac{x'}{y'} = \frac{-x}{y} = -\cot\alpha$$

$$\sec(180° - \alpha) = \frac{r'}{x'} = \frac{r}{-x} = -\sec\alpha$$

$$\csc(180° - \alpha) = \frac{r'}{y'} = \frac{r}{y} = \csc\alpha$$

49. Laws of reduction. The method of the last four articles may be applied to any angle of the form $n\,90° \pm \alpha$ to obtain formulas of reduction for all positive and negative integral values of n, α being any angle. A complete investigation would show the following laws to be true:

When n is even, any function of $n\,90° \pm \alpha$ is numerically equal to the same function of α; when n is odd, any function of $n\,90° \pm \alpha$ is numerically equal to the cofunction of α.

If the function of $n\,90° \pm \alpha$ is positive, α being considered acute, the members of the equation have like signs; if the function of $n\,90° \pm \alpha$ is negative, α being considered acute, the members of the equation have unlike signs. The results thus obtained are valid for all positive and negative values of α.

50. EXAMPLES

Express as functions of a positive angle less than 90°:

1. $\sin 130°$.

SUGGESTION. $130° = 90° + 40°$, or $130° = 180° - 50°$.

2. $\cos 170°$.

3. $\tan 110°$.

4. $\cot 160°$.

5. $\cos(-20°)$.

6. $\tan(-80°)$.

7. $\sin(-120°)$.

Express as functions of θ:

8. $\sin(810° - \theta)$.

9. $\tan(360° - \theta)$.

10. $\cot(270° + \theta)$.

11. $\sin(\theta - 90°)$.

12. $\tan(\theta - 180°)$.

13. $\sec(-180° - \theta)$.

14. $\csc(-630° + \theta)$.

15. $\cos(990° - \theta)$.

Express each of the trigonometric functions of the following angles as functions of α without using the laws of reduction.

16. $180° + \alpha$.

17. $270° - \alpha$.

18. $270° + \alpha$.

19. $360° - \alpha$.

CHAPTER V

FUNDAMENTAL RELATIONS. LINE VALUES

51. In the present chapter eight fundamental relations between the trigonometric functions are developed. It is then shown that each trigonometric function may be represented geometrically by a single line, giving the so-called line values. This gives a second view of the trigonometric functions. The line values offer a simple method of demonstrating the fundamental relations, and of developing the properties already derived by means of the trigonometric ratios. In fact the line values might serve as the fundamental definitions of the trigonometric functions and thus trigonometry could be based upon these values. Incidentally the line values suggest the origin of some of the terms used to designate the several trigonometric functions.

FUNDAMENTAL RELATIONS

52. Certain relations of fundamental importance exist between the trigonometric functions of an angle. These will now be developed.

From definitions,

$$\sin \alpha = \frac{y}{r},$$

$$\csc \alpha = \frac{r}{y}. \qquad \therefore \sin \alpha = \frac{1}{\csc \alpha}. \tag{1}$$

$$\cos \alpha = \frac{x}{r},$$

$$\sec \alpha = \frac{r}{x}. \qquad \therefore \cos \alpha = \frac{1}{\sec \alpha}. \tag{2}$$

$$\tan \alpha = \frac{y}{x},$$

$$\cot \alpha = \frac{x}{y}. \qquad \therefore \tan \alpha = \frac{1}{\cot \alpha}. \tag{3}$$

$$\sin \alpha = \frac{y}{r},$$

$$\cos \alpha = \frac{x}{r},$$

$$\tan \alpha = \frac{y}{x}. \qquad \therefore \frac{\sin \alpha}{\cos \alpha} = \tan \alpha. \tag{4}$$

$$\cot \alpha = \frac{x}{y}, \quad \text{and} \frac{\cos \alpha}{\sin \alpha} = \cot \alpha. \tag{5}$$

From geometry, $y^2 + x^2 = r^2$.

Dividing successively by r^2, x^2, and y^2,

$$\frac{y^2}{r^2} + \frac{x^2}{r^2} = 1, \quad \therefore \sin^2 \alpha + \cos^2 \alpha = 1. \tag{6}$$

$$\frac{y^2}{x^2} + 1 = \frac{r^2}{x^2}, \quad \therefore \tan^2 \alpha + 1 = \sec^2 \alpha. \tag{7}$$

$$\frac{x^2}{y^2} + 1 = \frac{r^2}{y^2}, \quad \therefore \cot^2 \alpha + 1 = \csc^2 \alpha. \tag{8}$$

These eight formulas are called the **fundamental relations.** They are used frequently, and should be memorized. In doing this, it is well to have the method of derivation clearly in mind.

53. The use of exponents. In affecting trigonometric functions with exponents, the exponent is usually placed immediately after the function. Thus, in the formulas of the preceding article, $\sin^2 \alpha$ is identical in meaning with $(\sin \alpha)^2$, and $\tan^2 \alpha$ is identical with $(\tan \alpha)^2$.

An exception to the above is made for the exponent -1, in which case it is always necessary to make use of paren-

theses. Thus, $(\cos \alpha)^{-1}$ is used to express $\frac{1}{\cos \alpha}$, while $\cos^{-1} \alpha$ has an entirely different meaning, as will be explained later.

54. Trigonometric identities. An identical equation, or an **identity**, is an equation which is satisfied for *all* values of the unknown quantities. The eight fundamental relations are trigonometric identities. There are many other identities depending upon these.

The truth of an identity can be established in two ways:

First. Begin with one of the fundamental relations, and produce the given identity by means of the fundamental relations and algebraic principles.

Second. Begin with the given identity and transform it to one of the fundamental relations, or reduce one member of the equation to the other by means of the fundamental relations and algebraic principles.

The choice of the trigonometric transformations necessary to effect the reductions is often suggested by the functions involved in the given problem. When no transformation is thus suggested, the problem may generally be simplified by expressing each of the functions in terms of sines and cosines, and making use of the relation $\sin^2 \alpha + \cos^2 \alpha = 1$.

Avoid the use of radicals whenever possible.

55. Trigonometric equations. A conditional equation is an equation which is not satisfied for all values of the unknown quantity, but is satisfied only for particular values.

(*a*) Trigonometric equations involving different trigonometric functions of the same angle may often be solved by simplifying the equation by the use of the fundamental relations. Thus

$$\sin \alpha = \cos \alpha$$

may be written

$$\frac{\sin \alpha}{\cos \alpha} = 1 \text{ or } \tan \alpha = 1$$

$$\therefore \alpha = 45^\circ \text{ or } 225^\circ.$$

(*b*) Some equations may conveniently be solved by transposing all of the terms to the first member of the equation, factoring, and then placing each factor equal to zero. Thus

$$2 \sin^2 \alpha + \sqrt{3} \cos \alpha = 2\sqrt{3} \sin \alpha \cos \alpha + \sin \alpha$$

may be written

$$\sin \alpha (2 \sin \alpha - 1) + \sqrt{3} \cos \alpha (1 - 2 \sin \alpha) = 0$$

or

$$(\sin \alpha - \sqrt{3} \cos \alpha)(2 \sin \alpha - 1) = 0.$$

This equation is satisfied if either

$$\sin \alpha - \sqrt{3} \cos \alpha = 0 \text{ or } 2 \sin \alpha - 1 = 0$$

whence $\tan \alpha = \sqrt{3}$ or $\sin \alpha = \frac{1}{2}$.

$$\therefore \alpha = 60°, 240° \text{ or } \alpha = 30°, 150°.$$

(*c*) When no other method suggests itself, the equation may be transformed, by the use of the fundamental relations, into an equation containing only one function. The equation thus obtained may be solved algebraically for the function involved, from which the values of the angle may be obtained. Thus

$$10 \cos^3 \alpha \tan \alpha - 9 \cos^2 \alpha - \frac{2}{\csc \alpha} - 10 \sin \alpha + 9 = 0$$

$$10 \sin \alpha (1 - \sin^2 \alpha) - 9 (1 - \sin^2 \alpha) - 12 \sin \alpha + 9 = 0$$

$$10 \sin \alpha - 10 \sin^3 \alpha + 9 \sin^2 \alpha - 12 \sin \alpha = 0$$

$$\therefore \sin \alpha = 0 \text{ or } 10 \sin^2 \alpha - 9 \sin \alpha + 2 = 0$$

from which $\sin \alpha = 0$, $\frac{2}{5}$, or $\frac{1}{2}$.

$$\therefore \alpha = 0°, 23° 35', 30°, 150°, 156° 25', \text{ or } 180°.$$

A trigonometric equation is considered completely solved when every positive angle less than 360° which satisfies it has been determined. All other angles coterminal with these angles also satisfy the equation.

In the solution of trigonometric equations extraneous roots may occur. These may be detected by substitution in the original equation.

56. EXAMPLES

Prove the following trigonometric identities:

1. $\sin\theta = \cos\theta\tan\theta$.

2. $\tan\theta = \sin\theta\sec\theta$.

3. $\cos\theta = \dfrac{\cot\theta}{\csc\theta}$.

4. $(1-\sin x)(1+\sin x) = \cos^2 x$.

5. $(\sec x - \tan x)(\sec x + \tan x) = 1$.

6. $(\sin x + \cos x)^2 = 2\sin x\cos x + 1$.

7. $\dfrac{\sqrt{1+\cot^2 x}}{\cot x} = \sec x$.

8. $\cos^2\alpha\tan^2\alpha + \sin^2\alpha\cot^2\alpha = 1$.

9. Given $\cos\alpha = \frac{5}{9}$, α being in the fourth quadrant, find the values of the remaining trigonometric functions.

SOLUTION. Since $\sin^2\alpha + \cos^2\alpha = 1$,

$$\sin^2\alpha = 1 - \cos^2\alpha = 1 - \tfrac{25}{81} = \tfrac{56}{81}.$$

$$\therefore \sin\alpha = -\sqrt{\tfrac{56}{81}} = -\tfrac{2}{9}\sqrt{14}.$$

Also $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{-\frac{2}{9}\sqrt{14}}{\frac{5}{9}} = -\tfrac{2}{5}\sqrt{14}$;

$\cot\alpha = \dfrac{1}{\tan\alpha} = \dfrac{1}{-\frac{2}{5}\sqrt{14}} = -\dfrac{5}{2\sqrt{14}} = -\tfrac{5}{28}\sqrt{14}$;

$\sec\alpha = \dfrac{1}{\cos\alpha} = \dfrac{1}{\frac{5}{9}} = \dfrac{9}{5}$;

$\csc\alpha = \dfrac{1}{\sin\alpha} = \dfrac{1}{-\frac{2}{9}\sqrt{14}} = -\dfrac{9}{2\sqrt{14}} = -\tfrac{9}{28}\sqrt{14}$.

Compare with the method of examples 26 and 27, Art. 27.

10. Given $\tan y = 4$, y being in the third quadrant, find the values of the remaining trigonometric functions.

SOLUTION. Since $\sec^2 y = 1 + \tan^2 y$,

$$\sec y = -\sqrt{1+16} = -\sqrt{17}.$$

Also $\cot y = \dfrac{1}{\tan y} = \dfrac{1}{4}$;

$\csc^2 y = 1 + \cot^2 y = 1 + \frac{1}{16} = \frac{17}{16}$;

$\csc y = -\frac{1}{4}\sqrt{17}$;

$$\sin y = \frac{1}{\csc y} = \frac{1}{-\frac{1}{4}\sqrt{17}} = -\frac{4}{17}\sqrt{17};$$

$$\cos y = \frac{1}{\sec y} = \frac{1}{-\sqrt{17}} = -\frac{1}{17}\sqrt{17}.$$

11. Given $\sin\theta = \frac{2}{3}$, θ terminating in the second quadrant, find the values of the remaining trigonometric functions.

12. Given $\cot\theta = \frac{7}{10}$, θ terminating in the first quadrant, find the values of the remaining trigonometric functions.

13. Given $\sec\theta = -2$, θ being in the third quadrant, find the values of the remaining trigonometric functions.

14. Given $\tan\alpha = -\frac{4}{7}$, find the remaining functions of the angles less than 360° which satisfy the equation.

Solve the following trigonometric equations for angles less than 90°:

15. $5\sin x + 8 = 3(4 - \sin x)$.

16. $\sin u - \cos u = 0$.

17. $2\cos^2 x - 3\cos x + 1 = 0$.

18. $(\tan\theta - 1)(\tan\theta - \sqrt{3}) = 0$.

19. Solve $\sin x = \cot x$ for $\sin x$.

Some of the following examples are identities and others are equations. Establish the identities and solve the equations for angles less than 180°.

20. $\sin^4 x = 1 - 2\cos^2 x + \cos^4 x$.

21. $\sin^4 x = 2 - 6\cos^2 x + \cos^4 x$.

22. $(\sqrt{3}+1)\dfrac{\sin\theta}{\tan\theta} + \dfrac{2\cos\theta}{\cot\theta} = \sin\theta + \cos\theta$.

23. $2\dfrac{\sin\theta}{\tan\theta} + \dfrac{\cos\theta}{\cot\theta} = \sin\theta + \cos\theta + \dfrac{\sqrt{3}}{2}$.

24. $\tan u + \cot u = \sec u \csc u$.

25. $2\sin^2 y \csc y + 3\csc y = 7$.

Prove the following trigonometric identities:

26. $\tan^2\theta + \cot^2\theta = \sec^2\theta \csc^2\theta - 2.$

27. $\sec^2\beta + \cos^2\beta = \tan^2\beta \sin^2\beta + 2.$

28. $\csc^2\gamma - \sec^2\gamma = \cos^2\gamma \csc^2\gamma - \sin^2\gamma \sec^2\gamma.$

29. $\sin\alpha \tan\alpha = \dfrac{\sin\alpha + \tan\alpha}{\cot\alpha + \csc\alpha}.$

30. $\cot^2\alpha - \cos^2\alpha = \cot^2\alpha \cos^2\alpha.$

31. $\dfrac{1 - 2\cos^2\theta}{\sin\theta \cos\theta} = \tan\theta - \cot\theta.$

32. $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2.$

33. $\sin^2 x + \operatorname{vers}^2 x = 2(1 - \cos x).$

34. $\sec^2\alpha \csc^2\alpha - \dfrac{(1 - \tan^2\alpha)^2}{\tan^2\alpha} = 4.$

Solve the following trigonometric equations for angles less than 180°:

35. $2\cos^2\alpha - 3\sin\alpha = 0.$

36. $\sec^2\alpha + \cot^2\alpha = 4\frac{1}{3}.$

37. $1 + \tan^2 x - 4\cos^2 x = 0.$

38. $\tan x + \cot x = 2.$

39. $2\sin^2 x + 3\cos x = 0.$

40. $\sqrt{3}\cos x + \sin x = \sqrt{2}.$

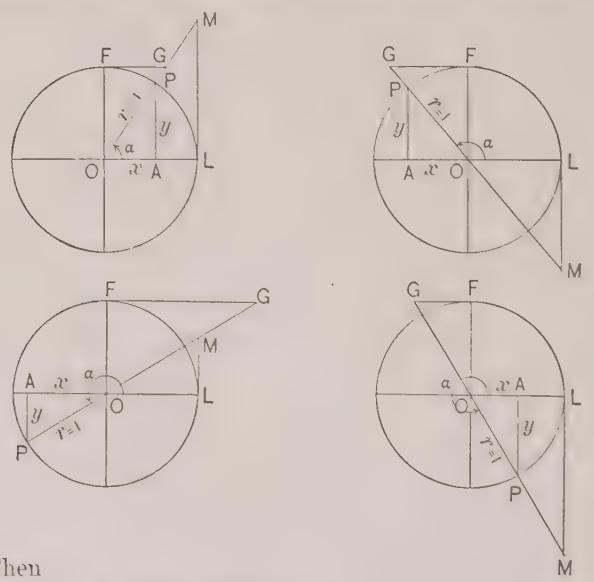
41. $\csc^2 x - 4\sin^2 x = 0.$

42. $\cot x + \csc^2 x = 3.$

LINE VALUES

57. Representation of the trigonometric functions by lines. The trigonometric functions of an angle have been studied solely from the standpoint of ratios; but each function can also be represented in magnitude and sign by a single line.

Let LOP be any angle. Take $OP = 1$. Draw the ordinate and abscissa of the point P. About O as the center construct a circle with OP as a radius. Draw the tangents to this circle at L and F, the *beginning of the first and second quadrants* respectively, and produce them to intersect the terminal line, or the terminal line produced, in M and G.



Then

$$\sin\alpha = \frac{y}{r} = \frac{AP}{OP} = \frac{AP}{1} = AP \quad \text{or} \quad \sin\alpha = AP$$

$$\cos\alpha = \frac{x}{r} = \frac{OA}{OP} = \frac{OA}{1} = OA \quad \text{or} \quad \cos\alpha = OA$$

$$\tan\alpha = \frac{y}{x} = \frac{AP}{OA} = \frac{LM}{OL} = LM \quad \text{or} \quad \tan\alpha = LM$$

$$\cot\alpha = \frac{x}{y} = \frac{OA}{AP} = \frac{FG}{OF} = FG \quad \text{or} \quad \cot\alpha = FG$$

$$\sec\alpha = \frac{r}{x} = \frac{OP}{OA} = \frac{OM}{OL} = OM \quad \text{or} \quad \sec\alpha = OM$$

$$\csc \alpha = \frac{r}{y} = \frac{OP}{AP} = \frac{OG}{OF} = OG \qquad \text{or} \qquad \csc \alpha = OG$$

Thus the trigonometric functions are represented by the segments AP, OA, LM, FG, OM and OG. It is evident that these segments represent the trigonometric functions *in magnitude*. They also represent the trigonometric functions *in sign*. In accordance with the conventions of Arts. 6 and 11, the segments AP and LM are positive when drawn upward from the X-axis and negative when drawn downward; the segments OA and FG are positive when drawn from the Y-axis to the right and negative when drawn to the left; the segments OM and OG are positive when they coincide with the terminal line of the angle and negative when they coincide with the terminal line produced.

It is evident that the sign of each trigonometric function is determined by the segment representing the function, since in each quadrant the segment is positive whenever the ratio defining the function is positive, and negative whenever the ratio is negative. Thus in each quadrant LM has the same sign as the ratio $\frac{y}{x}$, and similarly for the other functions. Therefore the segments represent the trigonometric functions in sign as well as in magnitude.

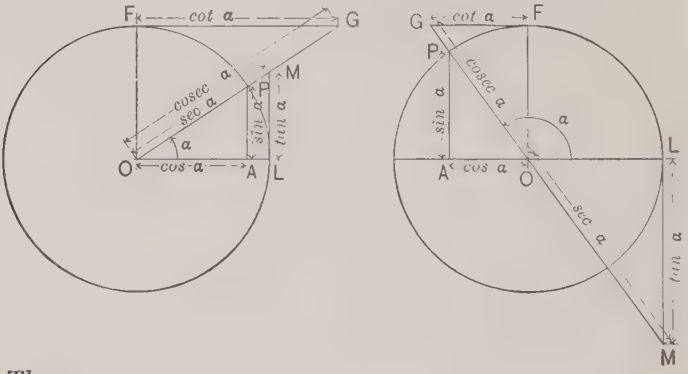
The segments which represent the trigonometric functions are called the **line values** of the functions.

Problem. Represent vers α and covers α by line values.

58. Variations of the trigonometric functions as shown by line values. The line values of the trigonometric functions give a simple method of tracing the variations in the functions as the angle varies from 0° to 360°. Thus as the point P, in the figures of Art. 57, describes the circle of radius unity, the changes in the segments AP, OA, LM, etc., represent the variations in the sine, cosine, tangent, etc., respectively.

Numerous figures, with three or four values of the angle in each quadrant, serve to suggest these variations.

59. Fundamental relations by line values. By the use of line values the fundamental relations of Art. 52 may be simply obtained and easily memorized.



Thus

From the triangle OAP,

$$\overline{AP}^2 + \overline{OA}^2 = \overline{OP}^2 \quad \text{or} \quad \sin^2 \alpha + \cos^2 \alpha = 1.$$

From the triangle OLM,

$$\overline{LM}^2 + \overline{OL}^2 = \overline{OM}^2 \quad \text{or} \quad \tan^2 \alpha + 1 = \sec^2 \alpha.$$

From the triangle OGF,

$$\overline{FG}^2 + \overline{OF}^2 = \overline{OG}^2 \quad \text{or} \quad \cot^2 \alpha + 1 = \csc^2 \alpha.$$

From definitions,

$$\tan \alpha = \frac{AP}{OA} = \frac{\sin \alpha}{\cos \alpha} \quad \text{or} \quad \tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\cot \alpha = \frac{OA}{AP} = \frac{\cos \alpha}{\sin \alpha} \quad \text{or} \quad \cot \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$\sec \alpha = \frac{OP}{OA} = \frac{1}{\cos \alpha} \quad \text{or} \quad \sec \alpha = \frac{1}{\cos \alpha}$$

$$\csc \alpha = \frac{OP}{AP} = \frac{1}{\sin \alpha} \quad \text{or} \quad \csc \alpha = \frac{1}{\sin \alpha}$$

$$\cot \alpha = \frac{OA}{AP} = \frac{OL}{LM} = \frac{1}{\tan \alpha} \quad \text{or} \quad \cot \alpha = \frac{1}{\tan \alpha}$$

The process of derivation of these formulas applies equally well to an angle of the third or fourth quadrant; hence the formulas are true for all values of α.

To memorize these formulas most easily, form a clear mental picture of the figure for the first quadrant and read each formula directly from the figure, applying the ratio definitions and the Pythagorean theorem.

60. EXAMPLES

1. Construct the line values of an angle terminating in the third quadrant.

2. Construct the line values of an angle terminating in the fourth quadrant.

3. Obtain the fundamental relations for an angle in the third quadrant, by the use of line values.

4. Obtain the fundamental relations for an angle terminating in the fourth quadrant, by the use of line values.

5. Deduce the relations between the functions of $90° + x$ and the functions of x by means of line values.

SUGGESTION. Construct two figures, one for the angle $90° + x$ and the other for the angle x.

6. Deduce the relations between the functions of $180° - x$ and the functions of x by means of line values.

7. Deduce the relations between the functions of $90° - x$ and the functions of $270° + x$ by means of line values.

By means of line values, trace the variations in the following functions as α varies from 0° to 360°:

8. $\sin \alpha$.

9. $\cos \alpha$.

10. $\tan \alpha$.

11. $\cot \alpha$.

12. $\sec \alpha$.

13. $\csc \alpha$.

Prove the following trigonometric identities:

14. $\dfrac{1}{\cot x + \tan x} = \sin x \cos x.$

15. $\cos^2 \alpha (1 + \cot^2 \alpha) = \cot^2 \alpha.$

16. $\sin \theta \cot^2 \theta = \csc \theta - \sin \theta.$

17. $\tan \alpha - \tan \alpha \sin^2 \alpha = \sin \alpha \cos \alpha.$

18. $\cos \alpha \tan^2 \alpha = \sec \alpha - \cos \alpha.$

19. $\dfrac{\cos \alpha}{\tan \alpha} + \dfrac{\sin \alpha}{\cot \alpha} = \dfrac{\sin^3 \alpha + \cos^3 \alpha}{\sin \alpha \cos \alpha}.$

20. $\dfrac{\sin^3 \alpha + \cos^3 \alpha}{\sin \alpha + \cos \alpha} = 1 - \sin \alpha \cos \alpha.$

21. $\dfrac{\sin x}{\cos x} + \dfrac{\tan x}{\cot x} + \dfrac{\sec x}{\csc x} = \dfrac{2 \cot x + 1}{\cot^2 x}.$

22. $\dfrac{1 - \cos x}{1 + \cos x} = \dfrac{\sec x - 1}{\sec x + 1}.$

23. $\dfrac{1 - \cos x}{1 + \cos x} = \dfrac{\sin^2 x}{(1 - \cos x)^2}.$

24. $\dfrac{1 - \cos x}{1 + \cos x} = \dfrac{(1 - \cos x)^2}{\sin^2 x}.$

25. $2 \sin y \cos^2 y + (2 \cos^2 y - 1) \sin y = 3 \sin y - 4 \sin^3 y.$

26. $\cos^4 \alpha - \sin^4 \alpha = \cos^2 \alpha - \sin^2 \alpha.$

27. $\sec^2 \alpha + \operatorname{cosec}^2 \alpha = \sec^2 \alpha \csc^2 \alpha.$

28. $\dfrac{\tan x + \tan y}{\sec x - \sec y} = \dfrac{\sec x + \sec y}{\tan x - \tan y}.$

29. $\tan^4 u - \cot^4 u = (\tan^2 u + \cot^2 u)(\sec^2 u - \csc^2 u).$

30. $\dfrac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y.$

31. $\dfrac{\tan x - \tan y}{\cot x - \cot y} = -\tan x \tan y.$

32. $\dfrac{\tan x + \cot y}{\cot x + \tan y} = \dfrac{\tan x}{\tan y}$.

33. Express $\sin^2 x - \cos^2 x$ in terms of $\tan x$.

34. Express $\operatorname{vers} x - \operatorname{covers} x$ in terms of $\sin x$.

35. Express $\cot^2 x + \csc^2 x$ in terms of $\cos x$.

36. Express $\dfrac{\operatorname{vers} x}{\operatorname{covers} x}$ in terms of $\tan x$.

37. Express $\tan^2 x + \sec^2 x$ in terms of $\cot x$.

38. Express $(4 \sin^3\theta - 3 \sin\theta)(2 \cos^2\theta - 1) + (3 \cos\theta - 4 \cos^3\theta)(2 \sin\theta \cos\theta)$ in terms of $\sin\theta$.

39. Given $\sin\theta = a$, find the remaining functions of θ.

40. Given $\cot\theta = b$, find the remaining functions of θ.

41. Express each trigonometric function of θ in terms of $\cos\theta$.

42. Express each trigonometric function of θ in terms of $\tan\theta$.

43. Simplify $(a + b)\sin 30° - b \cos 60° + a \tan 180°$.

44. Simplify $l \sin(270° - x) + m \cos(180° + x) + n \sin(90° - x)$.

45. Simplify $a \sin 135° + (a - b)\cos 225° + b \cos 315°$.

46. Simplify $\tan(-120°) + \cot 150° - \tan 210° + \cot 240°$.

Find the positive values of θ, less than 180°, that satisfy the following equations:

47. $\sin\theta \cos\theta = 0$.

48. $\sin\theta + \cos\theta = 0$.

49. $\sin\theta\,(2 \sin\theta - 1)\,(2 \cos\theta - 1) = 0$.

50. $\sin\theta + \cos\theta \cot\theta = 2$.

51. $\sin\theta\cos\theta=\frac{1}{2}$.

52. $2\sin\theta\cos\theta+\sin\theta-2\cos\theta=1$.

53. $2\cos\theta+\sec\theta=3$.

54. $\sec\theta+\tan\theta=2$.

55. $2\sin\theta+5\cos\theta=2$.

56. $1+\sin^2\theta=3\sin\theta\cos\theta$.

57. $\tan^4\theta-4\tan^2\theta-5=0$.

58. What negative angles, numerically less than 180°, satisfy the equation $\sin^2 x+\sin x=\cos^2 x-1$?

59. Given $9\cos^2 u+9\sin u=11$, find $\tan u$.

60. Given $\tan^2 x-5\sec x+7=0$, find $\sin x$.

Find the positive values of θ, less than 360°, that satisfy the following equations:

61. $\sin\theta=-\cos 285°$.

62. $\tan\theta=\cot(-144°)$.

63. $\cos(-\theta)=\sin 190°$.

64. $\sin\theta=-\sin 50°$.

65. If $\sin 122°=k$, prove that $\tan 32°=\dfrac{\sqrt{1-k^2}}{k}$.

66. If $\cot 255°=a$, prove that $\cos 345°=\dfrac{1}{\sqrt{1+a^2}}$.

67. If $\cos(-100°)=k$, prove that $\tan 80°=-\dfrac{\sqrt{1-k^2}}{k}$.

Solve the following equations for *any trigonometric function* of α.

68. $2\sin\alpha+\csc\alpha=1$.

69. $\csc\alpha\cot\alpha=\frac{5}{3}$.

70. $2\sin\alpha+\cos^2\alpha=1$.

71. $\tan^2\alpha+4\sec\alpha=5$.

72. $\tan\alpha+\cot\alpha=2$.

73. Given $\sin u=k\sin v$ and $\tan u=l\tan v$, find $\sin u$ and $\sin v$.

CHAPTER VI

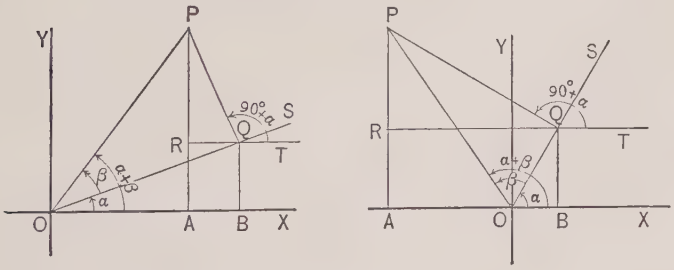
FUNCTIONS OF THE SUM OF TWO ANGLES

DOUBLE ANGLES. HALF ANGLES

61. We have thus far considered the properties and relations of the functions of a single angle. We have also shown that the functions of the angles $n\,90 \pm \alpha$ depend upon the functions of the angle α.

In the present chapter we consider the relations between the functions of the sum of two independent angles and the functions of the separate angles, and develops some related formulas.

62. The sine of the sum of two acute angles expressed in terms of the sines and cosines of the angles.



Let α and β be any given acute angles.

Construct the angle $(\alpha + \beta)$, Art. 12.

Then $(\alpha + \beta)$ may be acute as in Fig. 1, or obtuse as in Fig. 2.

From any point P, in the terminal line of the angle $(\alpha + \beta)$, draw PA perpendicular to OX, and PQ perpendicular to OS. Through Q draw RQT parallel to OX, and QB perpendicular to OX.

Then the angle TQP is equal to $90° + \alpha$.

By definition

$$\sin(\alpha+\beta)=\frac{AP}{OP}=\frac{AR+RP}{OP}=\frac{BQ}{OP}+\frac{RP}{OP}=$$

$$\frac{BQ}{OQ}\cdot\frac{OQ}{OP}+\frac{RP}{QP}\cdot\frac{QP}{OP}.$$

But $\frac{BQ}{OQ}=\sin\alpha$, $\frac{OQ}{OP}=\cos\beta$, $\frac{RP}{QP}=\sin(90°+\alpha)=\cos\alpha$, and $\frac{QP}{OP}=\sin\beta$,

therefore $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$. (1)

Problem 1. Show that formula (1) is true when either angle is 90°.

Problem 2. Show that formula (1) is true when each angle is 90°.

63. The cosine of the sum of two acute angles expressed in terms of the sines and cosines of the angles. Referring to the figures of the previous article, we have by definition

$$\cos(\alpha+\beta)=\frac{OA}{OP}=\frac{OB+BA}{OP}\text{ (Art. 2)}=\frac{OB}{OP}+\frac{QR}{OP}=$$

$$\frac{OB}{OQ}\cdot\frac{OQ}{OP}+\frac{QR}{QP}\cdot\frac{QP}{OP}.$$

But $\frac{OB}{OQ}=\cos\alpha$, $\frac{OQ}{OP}=\cos\beta$,

$\frac{QR}{QP}=\cos(90°+\alpha)=-\sin\alpha$, and $\frac{QP}{OP}=\sin\beta$,

therefore $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$. (1)

Problem 1. Show that the formula (1) is true when either angle is 90°.

Problem 2. Show that formula (1) is true when each angle is 90°.

64. The formulas developed in the last two articles are of the utmost importance, since many other formulas are

derived from them. They form the basis of the present chapter. In developing these formulas α and β were considered positive acute angles, but the formulas are true for all values of α and β, as may be shown either geometrically or analytically.

It is shown geometrically by constructing figures in which α and β are of any magnitude, and following the method of proof given above for acute angles.

We present the analytic demonstration as the more satisfactory.

65. *To prove that*

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta \qquad (1)$$

and $$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta \qquad (2)$$

are true for all values of α and β.

First. *To show that α can be replaced by α', where $\alpha'=\alpha+90°$.*

Let $\alpha'=\alpha+90°$ where α is acute.

As α varies from 0° to 90°, α' varies from 90° to 180°.

$$\text{Also } \sin\alpha'=\sin(\alpha+90°)=\cos\alpha, \qquad (3)$$

$$\cos\alpha'=\cos(\alpha+90°)=-\sin\alpha. \qquad (4)$$

$$\text{Now } \sin(\alpha'+\beta)=\sin(90°+\alpha+\beta)=\cos(\alpha+\beta).$$

Since α and β are both acute, $\cos(\alpha+\beta)$ may be expanded by (2), hence

$$\sin(\alpha'+\beta)=\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta.$$

Therefore, by (3) and (4),

$$\sin(\alpha'+\beta)=\sin\alpha'\cos\beta+\cos\alpha'\sin\beta. \qquad (5)$$

Similarly,

$$\cos(\alpha'+\beta)=\cos(90°+\alpha+\beta)=-\sin(\alpha+\beta)$$
$$=-\sin\alpha\cos\beta-\cos\alpha\sin\beta.$$

Therefore, by (3) and (4),

$$\cos(\alpha' + \beta) = \cos\alpha' \cos\beta - \sin\alpha' \sin\beta. \tag{6}$$

Formulas (5) and (6) show that one angle, α, in (1) and (2) may be extended to 180°.

Second. *To show that α can be replaced by α'' where $\alpha'' = \alpha + 180°$.*

Let $\alpha'' = \alpha + 180°$.

As α varies from 0° to 180°, α'' varies from 180° to 360°.

$$\text{Also } \sin\alpha'' = \sin(\alpha + 180°) = -\sin\alpha, \tag{7}$$

$$\cos\alpha'' = \cos(\alpha + 180°) = -\cos\alpha. \tag{8}$$

$$\text{Now } \sin(\alpha'' + \beta) = \sin(180° + \alpha + \beta) = -\sin(\alpha + \beta).$$

Since α is less than 180° and β acute, $\sin(\alpha + \beta)$ may be expanded by (1), as extended in the first part of the proof, hence

$$\sin(\alpha'' + \beta) = -\sin(\alpha + \beta) = -\sin\alpha\cos\beta - \cos\alpha\sin\beta.$$

Therefore, by (7) and (8),

$$\sin(\alpha'' + \beta) = \sin\alpha''\cos\beta + \cos\alpha''\sin\beta. \tag{9}$$

Similarly,

$$\cos(\alpha'' + \beta) = \cos(180° + \alpha + \beta) = -\cos(\alpha + \beta)$$
$$= -\cos\alpha\cos\beta + \sin\alpha\sin\beta.$$

Therefore, by (7) and (8),

$$\cos(\alpha'' + \beta) = \cos\alpha''\cos\beta - \sin\alpha''\sin\beta. \tag{10}$$

Formulas (9) and (10) show that one angle, α, of (1) and (2) may be extended to 360°.

Third. In a similar manner it may be shown that (1) and (2) are true when β varies from 0° to 360°, α having any value from 0° to 360°, from which it easily follows that (1) and (2) are true for *all positive* values of α and β.

Fourth. Formulas (1) and (2) may be shown to be true when either α or β or both are negative, by the use of the substitutions

$$\alpha' = \alpha - n\,360°$$
$$\beta' = \beta - n\,360°;$$

hence (1) and (2) are true for all positive and negative values of α and β.

66. *To find the tangent of the sum of any two given angles in terms of the tangents of the given angles.*

Let α and β be the given angles.

Then $$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}.$$

Dividing numerator and denominator of the last fraction by $\cos\alpha\cos\beta$ and simplifying, we have

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}.$$

67. *To find the cotangent of the sum of any two given angles in terms of the cotangents of the given angles.*

Let α and β be the given angles.

Then $$\cot(\alpha+\beta) = \frac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)} = \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\sin\alpha\cos\beta + \cos\alpha\sin\beta}.$$

Dividing numerator and denominator of the last fraction by $\sin\alpha\sin\beta$ and simplifying we have

$$\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\beta + \cot\alpha}.$$

68. Addition formulas. Collecting the formulas for the sum of two angles for convenience of reference, we have

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad (1)$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad (2)$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \tag{3}$$

$$\cot(\alpha+\beta)=\frac{\cot\alpha\cot\beta-1}{\cot\beta+\cot\alpha}. \tag{4}$$

69. *To find the sine, cosine, tangent, and cotangent of the difference of two given angles.*

Since the formulas of Art. 68 are true for all values of α and β, they are true when $-\beta$ is substituted for β.

Hence

$$\sin(\alpha-\beta)=\sin\alpha\cos(-\beta)+\cos\alpha\sin(-\beta)$$

$$\cos(\alpha-\beta)=\cos\alpha\cos(-\beta)-\sin\alpha\sin(-\beta)$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha+\tan(-\beta)}{1-\tan\alpha\tan(-\beta)}$$

$$\cot(\alpha-\beta)=\frac{\cot\alpha\cot(-\beta)-1}{\cot(-\beta)+\cot\alpha}$$

But remembering that

$$\sin(-\beta)=-\sin\beta,\ \cos(-\beta)=\cos\beta,$$

$$\tan(-\beta)=-\tan\beta,\text{ and }\cot(-\beta)=-\cot\beta,$$

these formulas become

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta \tag{1}$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta \tag{2}$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta} \tag{3}$$

$$\cot(\alpha-\beta)=\frac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}. \tag{4}$$

70. EXERCISES

1. Find $\sin 75°$.

SOLUTION. $75°=\sin(45°+30°)=\sin 45°\cos 30°+\cos 45°\sin 30°$

$$=\tfrac{1}{2}\sqrt{2}\cdot\tfrac{1}{2}\sqrt{3}+\tfrac{1}{2}\sqrt{2}.$$

$$=\tfrac{1}{4}\sqrt{2}(\sqrt{3}+1).$$

2. Find $\cos 75°$.
3. Find $\sin 15°$.
4. Find $\cos 15°$.
5. Find $\tan 75°$.
6. Find $\cot 15°$.
7. Find $\sin (90° - \beta)$.
8. Find $\tan (90° + \alpha)$.
9. Find $\cos (180° - \alpha)$.
10. Find $\sec 15°$.
11. Find $\csc (90° + \alpha)$.

71. Double angles. *To find the sine, cosine, tangent, and cotangent of twice a given angle in terms of the functions of the given angle.*

From the formulas of Art. 68, letting $\beta = \alpha$, we have, after reduction,

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha \tag{1}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha \tag{2}$$

$$= 1 - 2 \sin^2 \alpha \tag{3}$$

$$= 2 \cos^2 \alpha - 1 \tag{4}$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \tag{5}$$

$$\cot 2\alpha = \frac{\cot^2 \alpha - 1}{2 \cot \alpha}. \tag{6}$$

To express clearly the real significance of these formulas, we may state them from two points of view.

If α be the angle under consideration, the formula

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

may be stated: *The sine of twice an angle is equal to twice the sine of the angle times the cosine of the angle.*

If 2α be the angle under consideration, the same formula may be stated: *The sine of an angle is equal to twice the sine of half the angle times the cosine of half the angle.*

It then follows that

$$\sin \alpha = 2 \sin \tfrac{1}{2}\alpha \cos \tfrac{1}{2}\alpha. \tag{7}$$

Similarly, if α be the angle under consideration, the formula $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$

may be stated: *The cosine of twice an angle is equal to the square of the cosine of the angle minus the square of the sine of the angle.*

If 2α be the angle under consideration, the same formula may be stated: *The cosine of an angle is equal to the square of the cosine of half the angle minus the square of the sine of half the angle.*

It then follows that

$$\cos \alpha = \cos^2 \tfrac{1}{2}\alpha - \sin^2 \tfrac{1}{2}\alpha. \tag{8}$$

Similarly,

from $\cos 2\alpha = 1 - 2\sin^2 \alpha$ we have

$$\cos \alpha = 1 - 2\sin^2 \tfrac{1}{2}\alpha, \tag{9}$$

from $\cos 2\alpha = 2\cos^2 \alpha - 1$ we have

$$\cos \alpha = 2\cos^2 \tfrac{1}{2}\alpha - 1, \tag{10}$$

from $\tan 2\alpha = \dfrac{2\tan \alpha}{1 - \tan^2 \alpha}$ we have

$$\tan \alpha = \frac{2\tan \tfrac{1}{2}\alpha}{1 - \tan^2 \tfrac{1}{2}\alpha}, \tag{11}$$

from $\cot 2\alpha = \dfrac{\cot^2 \alpha - 1}{2\cot \alpha}$ we have

$$\cot \alpha = \frac{\cot^2 \tfrac{1}{2}\alpha - 1}{2\cot \tfrac{1}{2}\alpha}. \tag{12}$$

72. Half angles. *To find the sine, cosine, tangent, and cotangent of one half a given angle in terms of functions of the given angle.*

From formula (9), Art. 71, we have

$$\cos \alpha = 1 - 2\sin^2 \frac{\alpha}{2},$$

or $$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}. \tag{1}$$

From formula (10), Art. 71, we have

$$\cos\alpha=2\cos^2\frac{\alpha}{2}-1,$$

or $$\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}. \tag{2}$$

From (1) and (2),

$$\tan\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}, \tag{3}$$

and $$\cot\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}. \tag{4}$$

In each of these formulas the positive or negative sign is chosen to agree with the sign of the function of $\frac{\alpha}{2}$, depending on the quadrant in which $\frac{\alpha}{2}$ lies.

These formulas may be considered from two view points. For example, formula (1) may be stated: *The sine of half an angle is equal to the square root of the fraction whose numerator is one minus the cosine of the angle, and whose denominator is two;* or, *The sine of an angle is equal to the square root of the fraction whose numerator is one minus the cosine of twice the angle, and whose denominator is two.*

73. *To find the sum and difference of the sines of any two angles, also the sum and difference of the cosines of any two angles.*

From the formulas of Arts. 68 and 69 we have, by addition and subtraction,

$$\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta$$
$$\sin(\alpha+\beta)-\sin(\alpha-\beta)=2\cos\alpha\sin\beta$$
$$\cos(\alpha+\beta)+\cos(\alpha-\beta)=2\cos\alpha\cos\beta$$
$$\cos(\alpha+\beta)-\cos(\alpha-\beta)=-2\sin\alpha\sin\beta.$$

Let $\alpha+\beta=\gamma$ and $\alpha-\beta=\delta$. Solving for α and β, we have $\alpha=\frac{1}{2}(\gamma+\delta)$ and $\beta=\frac{1}{2}(\gamma-\delta)$. Substituting these values in the above equation, it follows that

$$\sin\gamma+\sin\delta=2\sin\tfrac{1}{2}(\gamma+\delta)\cos\tfrac{1}{2}(\gamma-\delta), \quad (1)$$

$$\sin\gamma-\sin\delta=2\cos\tfrac{1}{2}(\gamma+\delta)\sin\tfrac{1}{2}(\gamma-\delta), \quad (2)$$

$$\cos\gamma+\cos\delta=2\cos\tfrac{1}{2}(\gamma+\delta)\cos\tfrac{1}{2}(\gamma-\delta), \quad (3)$$

$$\cos\gamma-\cos\delta=-2\sin\tfrac{1}{2}(\gamma+\delta)\sin\tfrac{1}{2}(\gamma-\delta). \quad (4)$$

Equations (1), (2), (3), and (4) may be read from two view points.

Regarding γ and δ as the given angles, (1) may be stated: *The sum of the sines of two angles is equal to twice the product of the sine of half the sum of the given angles into the cosine of half the difference of the given angles.*

Thus, $\sin 6x+\sin 4x=2\sin 5x\cos x$.

Regarding $\frac{1}{2}(\gamma+\delta)$ and $\frac{1}{2}(\gamma-\delta)$ as the given angles, it is clear that their sum is γ and their difference δ. Then, by reading the second member first, equation (1) may be stated: *Twice the sine of any angle times the cosine of any other angle is equal to the sine of the sum of the angles plus the sine of the difference of the angles.*

Thus, $2\sin 20^\circ\cos 5^\circ=\sin 25^\circ+\sin 15^\circ$.

74. Equations and identities. The formulas developed in the present chapter are true for all values of the angles involved; hence they are trigonometric identities. By the use of these identities many others may be established. The remarks of Art. 54, concerning the use of the fundamental relations in establishing identities, apply here.

The identities of the present chapter are also useful in solving trigonometric equations. By their aid an equation involving functions of multiple angles may be transformed into an equation containing functions of a single angle (see Ex. 33, Art. 75). This transformed equation can then be solved as indicated in Art. 55. Frequently equations

may be much simplified by reducing sums or differences of sines and cosines to products by the relations of Art. 73 (see Ex. 41, Art. 75).

75. EXAMPLES

1. Find $\sin 22\frac{1}{2}°$, $\cos 22\frac{1}{2}°$, $\tan 22\frac{1}{2}°$, and $\cot 22\frac{1}{2}°$.

2. Given $\cos\alpha = \frac{1}{4}$; find $\sin 2\alpha$, $\cos 2\alpha$, $\tan 2\alpha$, and $\cot 2\alpha$.

3. Given $\tan\alpha = 3$; find $\sin\frac{1}{2}\alpha$, $\cos\frac{1}{2}\alpha$, $\tan\frac{1}{2}\alpha$, and $\cot\frac{1}{2}\alpha$.

Prove the following identities:

4. $(\cos\alpha + \sin\alpha)(\cos\alpha - \sin\alpha) = \cos 2\alpha$.

5. $(\sin\alpha + \cos\alpha)^2 = 1 + \sin 2\alpha$.

6. $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$.

SUGGESTION: $\sin 3\alpha = \sin(2\alpha + \alpha)$.

7. $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$.

8. $\tan 3\alpha = \dfrac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$.

9. $\tan\frac{1}{2}x = \dfrac{\sin x}{1 + \cos x}$.

10. $\cot\frac{1}{2}x = \dfrac{\sin x}{1 - \cos x}$.

11. $\dfrac{1 + \tan\frac{1}{2}x}{1 - \tan\frac{1}{2}x} = \dfrac{1 + \sin x}{\cos x}$.

12. $\tan^2\alpha = \dfrac{1 - \cos 2\alpha}{1 + \cos 2\alpha}$.

13. $\sec 2\alpha = \dfrac{\sec^2\alpha}{2 - \sec^2\alpha}$.

14. $4\sin^2\frac{1}{2}\alpha\cos^2\frac{1}{2}\alpha = 1 - \cos^2\alpha$.

15. $\sin 4\alpha + \sin 2\alpha = 2\sin 3\alpha\cos\alpha$. (See Art. 73.)

16. $\cos 6\alpha - \cos 2\alpha = -2\sin 4\alpha\sin 2\alpha$.

17. $\sin(45° + \alpha) - \sin(45° - \alpha) = \sqrt{2}\sin\alpha.$

18. $\sin(30° + \alpha) + \sin(30° - \alpha) = \cos\alpha.$

19. Express $\cos 4\alpha \sin 3\alpha$ as the difference of two sines.

20. Express $\cos 5\alpha \cos\alpha$ as the sum of two cosines.

Prove the following identities:

21. $\cos^2\alpha - \cos^2\beta = -\sin(\alpha+\beta)\sin(\alpha-\beta).$

22. $\dfrac{\sin 2\alpha + \sin\alpha}{\cos 2\alpha + \cos\alpha} = \tan\dfrac{3\alpha}{2}.$

23. $\dfrac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\dfrac{\alpha+\beta}{2}.$

24. $\sin\theta = \dfrac{2\tan\frac{1}{2}\theta}{1+\tan^2\frac{1}{2}\theta}.$

25. $\sin\frac{1}{2}x + 2\sin^2\frac{1}{2}x\cos\frac{1}{2}x\cot x = \sin x\cos\frac{1}{2}x.$

26. $2\tan\frac{1}{2}x\csc x = 1 + \tan^2\frac{1}{2}x.$

27. $\cos(\alpha+\beta)\cos\beta - \cos(\alpha+\gamma)\cos\gamma =$
$\sin(\alpha+\gamma)\sin\gamma - \sin(\alpha+\beta)\sin\beta.$

28. $\cos 2\alpha = 8\sin^4\frac{1}{2}\alpha - 8\sin^2\frac{1}{2}\alpha + 1.$

29. $\cos 2\alpha - \cos\alpha = 2(4\sin^4\frac{1}{2}\alpha - 3\sin^2\frac{1}{2}\alpha).$

30. $(1 - \cos 2\alpha)\cot^2\frac{1}{2}\alpha = \cos 2\alpha + 4\cos\alpha + 3.$

31. $2\sin^2\frac{1}{2}\theta\cot\theta - \csc 2\theta + \csc\theta = \cot 2\theta + \sin\theta.$

32. Given $\tan x = a\tan\frac{1}{3}x$; find $\tan\frac{1}{3}x$ in terms of a.

Solve the following equations for all positive values of α less than 180°:

33. $\sin 2\alpha - 4\cos^2\frac{1}{2}\alpha - \sin\alpha + 3 = 0.$

SOLUTION. $2\sin\alpha\cos\alpha - 4\left(\dfrac{1+\cos\alpha}{2}\right) - \sin\alpha + 3 = 0,$

or $2\sin\alpha\cos\alpha - 2\cos\alpha - \sin\alpha + 1 = 0,$

or $(2\cos\alpha - 1)(\sin\alpha - 1) = 0,$

$\therefore$ $\cos\alpha = \frac{1}{2}$ or $\sin\alpha = 1.$

and $\alpha = 60°$ or $90°.$

34. $\sin 2\alpha - \cos \alpha = 0.$

35. $\sin 2\alpha + \sin \alpha = 0.$

36. $\sin 2\alpha + \cos 2\alpha = 0.$

37. $\sin \alpha + \cos 2\alpha = 1.$

38. $\cos 3\alpha + 2\cos 2\alpha - 8\cos^2\alpha + 2\sin^2\alpha + 5\cos\alpha = 0.$

39. $\cos 2\alpha + 2\sin^2\tfrac{1}{2}\alpha - 1 = 0.$

40 $\sin 2\alpha + \cos 2\alpha + \sin \alpha = 1.$

41. $\sin 5\alpha - \sin 3\alpha + \sin \alpha = 0.$

SOLUTION. $2\sin 3\alpha\cos 2\alpha - \sin 3\alpha = 0.$ Art. 73.

or $\sin 3\alpha(2\cos 2\alpha - 1) = 0.$

$\therefore$ $\sin 3\alpha = 0$, or $\cos 2\alpha = \tfrac{1}{2}.$

$\therefore$ $3\alpha = 0°, 180°, 360°,$ and $2\alpha = 60°, 300°.$

$\therefore$ $\alpha = 0°, 60°, 120°, 30, 150°.$

42. $\cos 3\alpha + \sin 2\alpha - \cos \alpha = 0.$

43. $\sin 3\alpha + \sin 2\alpha + \sin \alpha = 0.$

44. $\cos 3\alpha - \sin 2\alpha + \cos \alpha = 0.$

45. $\sin 3\alpha + \cos 3\alpha - \sin \alpha - \cos \alpha = 0.$

46. $\sin x + \sin 2x + \sin 3x = 1 + \cos x + \cos 2x.$

47. $\dfrac{1 - \tan x}{1 + \tan x} = 2\cos 2x.$

Prove the following identities:

48. $\tan(30° + \alpha)\tan(30° - \alpha) = \dfrac{2\cos 2\alpha - 1}{2\cos 2\alpha + 1}.$

49. $\tan \alpha + \cot \alpha = 2\csc 2\alpha.$

50. $\sin 80° = \sin 40° + \sin 20°.$

51. $\dfrac{1 + \sin\alpha - \cos\alpha}{1 + \sin\alpha + \cos\alpha} = \tan\dfrac{\alpha}{2}.$

52. $\cos^2\alpha - \sin(30° + \alpha)\sin(30° - \alpha) = \tfrac{3}{4}.$

53. $\dfrac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = \tan\dfrac{\alpha + \beta}{2}.$

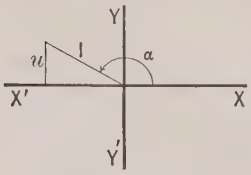
CHAPTER VII

INVERSE FUNCTIONS

76. Inverse trigonometric functions are closely related to the trigonometric functions previously studied.

After introducing the fundamental idea of an inverse function, it will be shown that they lead to new relations, closely allied to the relations already developed.

77. Fundamental idea of an inverse function. From the equation $\sin \alpha = u$, it is evident that α is an angle whose sine is u. The statement *α is an angle whose sine is u* is abbreviated into



$$\alpha = \sin^{-1} u.$$

The equation $u = \sin \alpha$ expresses u in terms of α.

The equation $\alpha = \sin^{-1} u$ expresses α in terms of u.

We thus have two methods of expressing the relation between an angle and its sine.

The symbol $\sin^{-1} u$ is the **inverse sine of *u***. It may be read the *inverse sine of u*, *the anti sine of u*, *arc sine u*, or **an angle whose sine is *u***. The last form should be used until the conception of an inverse function is perfectly clear.

Corresponding to each direct function there is an inverse function. Thus,

$\alpha = \sin^{-1} u$ corresponds to $\sin \alpha = u$,
$\alpha = \cos^{-1} u$ corresponds to $\cos \alpha = u$,
$\alpha = \tan^{-1} u$ corresponds to $\tan \alpha = u$, etc.

Caution. Since $\sin^{-1} u$ is an inverse function, the -1

cannot be an exponent. It is simply part of a symbol for an inverse function. When a function is affected by -1 as an exponent, it *must* be written with parentheses, thus $(\sin x)^{-1}$.

78. Multiple values of an inverse function. It has been shown that the trigonometric functions are *single-valued* functions. Thus, if α is given, there is only one value for $\sin \alpha$. See Arts. 25 and 41.

On the contrary, the inverse functions are *multiple-valued* functions. Thus, if u is given there are an infinite number of values for $\sin^{-1} u$. See Arts. 26 and 41. A few examples will illustrate this property of inverse functions, and show that all the values of any given inverse function may be combined in a single expression.

To find all the values of $\tan^{-1} \frac{1}{\sqrt{3}}$.

$$\begin{aligned}
\tan^{-1} \frac{1}{\sqrt{3}} &= 30° \\
&= 180° + 30° \\
&= 360° + 30° \\
&\quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
&= -180° + 30°, \text{ or } -150° \\
&= -360° + 30°, \text{ or } -330° \\
&= -540° + 30°, \text{ or } -510° \\
&\quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot
\end{aligned}$$

All these angles may be expressed by $n\,180° + 30°$, n being any positive or negative integer. Hence

$$\tan^{-1} \frac{1}{\sqrt{3}} = n\pi + \frac{\pi}{6}.$$

In general, if α is an angle whose tangent is u, it may be shown that

$$\tan^{-1} u = n\pi + \alpha.$$

To find all the values of $\cos^{-1}\frac{1}{2}$.

$$\begin{aligned}
\cos^{-1}\tfrac{1}{2} &= 60° \\
&= -60° \\
&= 360° + 60° \\
&= 360° - 60° \\
&= 720° + 60° \\
&= 720° - 60° \\
&\cdots\cdots \\
&= -360° + 60° \\
&= -360° - 60° \\
&= -720° + 60° \\
&= -720° - 60° \\
&\cdots\cdots
\end{aligned}$$

All these angles may be expressed by $n\,360° \pm 60°$, n being any positive or negative integer. Hence

$$\cos^{-1}\tfrac{1}{2} = 2n\pi \pm \frac{\pi}{3}.$$

In general, if α is an angle whose cosine is u, it may be shown that $\cos^{-1} u = 2n\pi \pm \alpha$.

To find all the values of $\sin^{-1}\frac{1}{2}$.

$$\begin{aligned}
\sin^{-1}\tfrac{1}{2} &= 30° \\
&= 180° - 30° \\
&= 360° + 30° \\
&= 540° - 30° \\
&= 720° + 30° \\
&= 900° - 30° \\
&\cdots\cdots \\
&= -180° - 30° \\
&= -360° + 30° \\
&= -540° - 30° \\
&= -720° + 30° \\
&\cdots\cdots
\end{aligned}$$

All these angles may be expressed by $n\,180° + (-1)^n 30°$, n being any positive or negative integer. Hence

$$\sin^{-1}\tfrac{1}{2} = n\pi + (-)1^n\frac{\pi}{6}.$$

In general, if α is an angle whose sine is u, it may be shown that

$$\sin^{-1} u = n\pi + (-1)^n \alpha.$$

In a similar manner it may be shown that

$$\cot^{-1} u = n\pi + \alpha,$$

$$\sec^{-1} u = 2\,n\pi \pm \alpha,$$

$$\csc^{-1} u = n\pi + (-1)^n \alpha.$$

79. Principal values. The smallest numerical value of an inverse function is called its **principal value**, preference being given to positive angles in case of ambiguity.

The principal values of the inverse sine and the inverse cosecant lie between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$; of the inverse cosine and the inverse secant, between 0 and π; of the inverse tangent and the inverse cotangent, between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

The principal values of an inverse function are sometimes distinguished from the general values by the use of a capital letter.

Thus $\quad \mathrm{Sin}^{-1}\tfrac{1}{2} = \frac{\pi}{6},$

while $\quad \sin^{-1}\tfrac{1}{2} = n\pi + (-1)^n\frac{\pi}{6}.$

80. *To interpret sin sin^{-1}u and sin^{-1} sin α.*

The expression sin sin^{-1}u is read: the sine of the angle whose sine is u. This sine is evidently u, hence

$$\sin \sin^{-1} u = u.$$

The expression $\sin^{-1}\sin\alpha$ is read: the angle whose sine is the sine of α. This angle is evidently α, hence

$$\sin^{-1}\sin\alpha = \alpha,$$

or more generally,

$$\sin^{-1}\sin\alpha = n\pi + (-1)^n\alpha.$$

Similar relations exist between any direct function and the corresponding inverse function.

Thus $\cos\cos^{-1}u = u$;

$\cos^{-1}\cos\alpha = \alpha$, or $\cos^{-1}\cos\alpha = 2n\pi \pm \alpha$;

$\tan\tan^{-1}u = u$;

$\tan^{-1}\tan\alpha = \alpha$, or $\tan^{-1}\tan\alpha = n\pi + \alpha$, etc.

81. Application of the fundamental relations to angles expressed as inverse functions. The fundamental relations, being true for all angles, must necessarily be true when the angles are expressed as inverse functions.

Thus, letting $\alpha = \tan^{-1}u$ in the identity

$$\sin^2\alpha + \cos^2\alpha = 1,$$

we have $(\sin\tan^{-1}u)^2 + (\cos\tan^{-1}u)^2 = 1.$

Similarly $(\sin\csc^{-1}u)^2 + (\cos\csc^{-1}u)^2 = 1;$

$(\tan\sec^{-1}u)^2 + 1 = (\sec\sec^{-1}u)^2;$

$$\sin\cos^{-1}u = \frac{1}{\csc\cos^{-1}u}.$$

By expressing the angle of the fundamental relations as an inverse function, we may develop relations between the inverse functions.

82. Given an angle, expressed as an inverse function of u, to find the value of any function of the angle in terms of u.

By the application of one or more of the fundamental relations, it is always possible to solve the stated problem.

Several illustrations are given below. The method employed can be readily applied to the other functions.

1. *To find the value of* $\tan \cos^{-1} u$ *in terms of* u.

If $\tan \cos^{-1} u$ is expressed in terms of the cosine of $\cos^{-1} u$, the problem is solved, since $\cos \cos^{-1} u = u$.

This may be done as follows:

$$\tan \cos^{-1} u = \frac{\sin \cos^{-1} u}{\cos \cos^{-1} u} = \frac{\pm \sqrt{1 - (\cos \cos^{-1} u)^2}}{u} = \frac{\pm \sqrt{1 - u^2}}{u}.$$

This result may be obtained geometrically. Since u is given, it is evident that $\cos^{-1} u$ represents, among others, two positive angles, α_1 and α_2, each less than 360°.

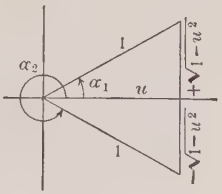
Let us assume u positive and let us construct these angles defined by $\cos^{-1} u$.

Then from the figure and the definition of the tangent,

$$\tan \alpha_1 = \frac{+\sqrt{1 - u^2}}{u}$$

and

$$\tan \alpha_2 = \frac{-\sqrt{1 - u_2}}{u}.$$



Since the tangent of any angle coterminal with α_1 is $\frac{+\sqrt{1 - u^2}}{u}$, and the tangent of any angle coterminal with α_2 is $\frac{-\sqrt{1 - u^2}}{u}$, and since $\cos^{-1} u$ represents all angles coterminal with either α_1 or α_2, we have

$$\tan \cos^{-1} u = \frac{\pm \sqrt{1 - u^2}}{u}.$$

2. *To find the value of* $\sec \cot^{-1} u$ *in terms of* u.

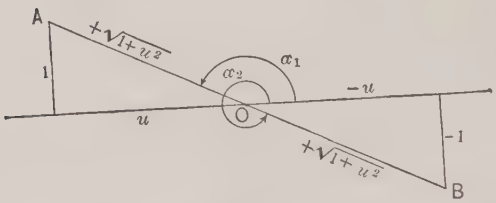
To solve this problem it is only necessary to express $\sec \cot^{-1} u$ in terms of $\cot \cot^{-1} u$.

Thus

$$\sec \cot^{-1} u = \pm\sqrt{1+(\tan \cot^{-1} u)^2} = \pm\sqrt{1+\frac{1}{(\cot \cot^{-1} u)^2}}$$

$$= \pm\sqrt{1+\frac{1}{u^2}} = \frac{\sqrt{u^2+1}}{u}.$$

This result may be obtained geometrically. Construct the angles given by $\cot^{-1} u$. Let us assume in this problem that u is negative and hence that $-u$ is positive. If the cotangent of an angle is negative the angle must terminate in either the second or fourth quadrant. Since OA and OB are the terminal lines of α_1 and α_2 respectively, and since the terminal line is always positive, we have



$$OA = OB = +\sqrt{1+u^2}.$$

Then $\sec \alpha_1 = \dfrac{+\sqrt{1+u^2}}{u}$ and $\sec \alpha_2 = \dfrac{+\sqrt{1+u^2}}{-u}$,

or by considerations similar to those in the previous example, we have

$$\sec \cot^{-1} u = \frac{+\sqrt{1+u^2}}{\pm u} = \frac{\pm\sqrt{1+u^2}}{u}.$$

3. *To find the value of* $\sin \cos^{-1} u$ *in terms of* u.

We have $\sin \cos^{-1} u = \pm\sqrt{1-(\cos \cos^{-1} u)^2} = \pm\sqrt{1-u^2}$.

4. *To find the value of* $\cot \operatorname{vers}^{-1} u$ *in terms of* u.

We have

$$\cot \operatorname{vers}^{-1} u = \frac{\cos \operatorname{vers}^{-1} u}{\pm\sqrt{1-(\cos \operatorname{vers}^{-1} u)^2}}$$

$$= \frac{1-\operatorname{vers} \operatorname{vers}^{-1} u}{\pm\sqrt{1-(1-\operatorname{vers} \operatorname{vers}^{-1} u)^2}} = \frac{1-u}{\pm\sqrt{1-(1-u)^2}}$$

$$= \frac{1-u}{\pm\sqrt{2u-u^2}}.$$

83. Some inverse functions expressed in terms of other inverse functions.

1. *To express* $\cos^{-1} u$ *in terms of an inverse tangent.*

From $\tan \cos^{-1} u = \dfrac{\pm\sqrt{1-u^2}}{u}$, (Art. 82, prob. 1) by taking the inverse tangent of each member (Art. 80), there results

$$\cos^{-1} u = \tan^{-1} \frac{\pm\sqrt{1-u^2}}{u}.$$

2. *To express the* $\cot^{-1} u$ *in terms of an inverse secant.*

From $\sec \cot^{-1} u = \dfrac{\pm\sqrt{1+u^2}}{u}$ (Art. 82, prob. 2) there results

$$\cot^{-1} u = \sec^{-1} \frac{\pm\sqrt{1+u^2}}{u}.$$

3. Similarly from $\sin \cos^{-1} u = \pm\sqrt{1-u^2}$ there results

$$\cos^{-1} u = \sin^{-1} (\pm\sqrt{1-u^2}).$$

4. Also from $\cot \operatorname{vers}^{-1} u = \dfrac{1-u}{\pm\sqrt{2u-u^2}}$ there results

$$\operatorname{vers}^{-1} u = \cot^{-1} \frac{1-u}{\pm\sqrt{2u-u^2}}.$$

By the method exemplified in Arts. 82 and 83 it is possible to express any inverse function in terms of any other inverse function.

In applying the above formulas care must be exercised in selecting the angles, since each inverse function represents an infinite number of angles and one member of the equation may represent angles not represented by the other. For example, in problem 1, if u be positive the $\cos^{-1} u$ represents angles terminating in the first and fourth quadrants; but $\tan^{-1} \dfrac{\pm\sqrt{1-u^2}}{u}$ represents angles terminating in the second and third quadrants as well as angles terminating in the first and fourth quadrants.

84. Some relations between inverse functions derived from the formulas for double angles, half angles, and the addition formulas.

The general method applicable to this class of problems will be illustrated by a few examples.

1. *To express* $\cos(2\sec^{-1}u)$ *in terms of* u.

$$\cos(2\sec^{-1}u) = 2(\cos\sec^{-1}u)^2 - 1 \quad \text{Art. 71, Eq. 4.}$$
$$= \frac{2}{(\sec\sec^{-1}u)^2} - 1 = \frac{2}{u^2} - 1.$$

From this relation it follows that,

$$2\sec^{-1}u = \cos^{-1}\left(\frac{2}{u^2} - 1\right).$$

2. *To express* $\tan(\frac{1}{2}\cos^{-1}u)$ *in terms of* u.

$$\tan(\tfrac{1}{2}\cos^{-1}u) = \pm\sqrt{\frac{1-\cos\cos^{-1}u}{1+\cos\cos^{-1}u}} = \pm\sqrt{\frac{1-u}{1+u}}.$$
Art. 72, Eq. 3.

From this relation it follows that,

$$\tfrac{1}{2}\cos^{-1}u = \tan^{-1}\left(\pm\sqrt{\frac{1-u}{1+u}}\right).$$

3. *To express* $\sin(\sin^{-1}u + \cos^{-1}v)$ *in terms of* u *and* v.

$$\sin(\sin^{-1}u + \cos^{-1}v) = \sin\sin^{-1}u \cdot \cos\cos^{-1}v + \cos\sin^{-1}u \cdot \sin\cos^{-1}v$$
Art. 62, Eq. 1.
$$= uv \pm \sqrt{1-u^2}\sqrt{1-v^2}.$$

From this relation it follows that,

$$\sin^{-1}u + \cos^{-1}v = \sin^{-1}(uv \pm \sqrt{1-u^2}\sqrt{1-v^2}).$$

85. EXAMPLES

Find the value of each of the following:

1. $\sin^{-1}\frac{1}{2}\sqrt{3}$.
2. $\cos^{-1}(-\frac{1}{2}\sqrt{3})$.
3. $\tan^{-1}1$.
4. $\tan\cot^{-1}4$.
5. $\sin\cot^{-1}4$.

Express the following in terms of u and v:

6. $\cos \cot^{-1} u$.

7. $\sec \cot^{-1} u$.

8. $\csc \cot^{-1} u$.

9. $\cos \sin^{-1} u$.

10. $\cos \tan^{-1} u$.

11. $\cos \sec^{-1} u$.

12. $\cos \csc^{-1} u$.

13. $\sin (2 \cos^{-1} u)$.

14. $\tan (2 \tan^{-1} u)$.

15. $\tan (2 \sec^{-1} u)$.

16. $\tan (2 \cos^{-1} u)$.

17. $\cos (2 \cos^{-1} u)$.

18. $\cos (2 \sin^{-1} u)$.

19. $\cos (2 \tan^{-1} u)$.

20. $\sin (\sin^{-1} u + \sin^{-1} v)$.

21. $\cos (\sin^{-1} u + \sin^{-1} v)$.

22. $\tan (\tan^{-1} u + \cot^{-1} v)$.

23. $\tan (\sec^{-1} u + \sec^{-1} v)$.

24. $\cos (\sec^{-1} u + \csc^{-1} v)$.

25. $\sin (\frac{1}{2} \cos^{-1} u)$.

26. $\cos (\frac{1}{2} \cos^{-1} u)$.

27. $\sin (\frac{1}{2} \sec^{-1} u)$.

28. $\sin (\frac{1}{2} \tan^{-1} u)$.

Find x in terms of a.

29. $\tan^{-1} x = \cot^{-1} a$.

30. $\sin^{-1} x = \tan^{-1} a$.

31. $\cos^{-1} x = 2 \sin^{-1} a$.

32. $\cos^{-1} x = \sin^{-1} a + \tan^{-1} a$.

33. $\sin^{-1} x = \frac{1}{2} \sec^{-1} a$.

Find x in terms of a and b.

34. $\tan^{-1} x = \sin^{-1} a + \sin^{-1} b$.

35. $\cos^{-1} x = \sec^{-1} a - \sec^{-1} b$.

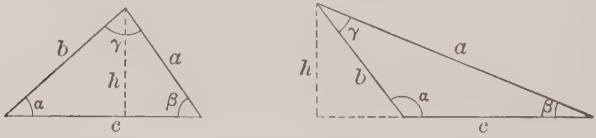
36. $\sin^{-1} x = 2 \cos^{-1} a + \frac{1}{2} \cos^{-1} b$.

CHAPTER VIII

OBLIQUE TRIANGLE

86. In the present chapter we develop the formulas by means of which a triangle may be completely solved when any three independent parts are given.

87. Law of sines. *In a plane triangle any two sides are to each other as the sines of the opposite angles.*



Let a, b, c be the sides of a triangle and α, β, γ the angles opposite these sides, respectively.

From the vertex of γ draw h perpendicular to the side c, or c produced.

Then, for each figure, $\sin\alpha = \frac{h}{b}$, Art. 21

and $\sin\beta = \frac{h}{a}$. Art. 29

Dividing the first equation by the second, we have

$$\frac{a}{b} = \frac{\sin\alpha}{\sin\beta}.$$

In a similar manner, dropping a perpendicular from the vertex of α, it is seen that

$$\frac{b}{c} = \frac{\sin\beta}{\sin\gamma}.$$

The last two equations may be written

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}.$$

88. Law of tangents. *The tangent of half the difference of two angles is to the tangent of half their sum, as the difference of the corresponding opposite sides is to their sum.*

From the law of sines we have

$$\frac{a}{b} = \frac{\sin \alpha}{\sin \beta}.$$

By division and composition, this becomes

$$\frac{a-b}{a+b} = \frac{\sin \alpha - \sin \beta}{\sin \alpha + \sin \beta},$$

which reduces to

$$\frac{a-b}{a+b} = \frac{2 \cos \tfrac{1}{2}(\alpha+\beta) \sin \tfrac{1}{2}(\alpha-\beta)}{2 \sin \tfrac{1}{2}(\alpha+\beta) \cos \tfrac{1}{2}(\alpha-\beta)} \quad \text{Art. 73}$$

$$= \cot \tfrac{1}{2}(\alpha+\beta) \tan \tfrac{1}{2}(\alpha-\beta)$$

or

$$\frac{\tan \tfrac{1}{2}(\alpha-\beta)}{\tan \tfrac{1}{2}(\alpha+\beta)} = \frac{a-b}{a+b}. \tag{1}$$

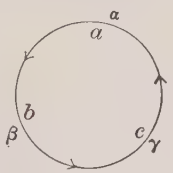
Similarly

$$\frac{\tan \tfrac{1}{2}(\beta-\gamma)}{\tan \tfrac{1}{2}(\beta+\gamma)} = \frac{b-c}{b+c}, \tag{2}$$

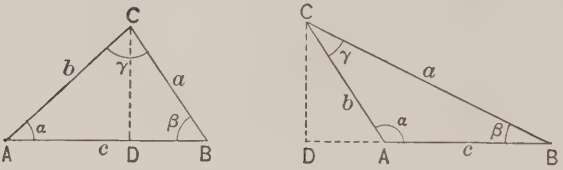
and

$$\frac{\tan \tfrac{1}{2}(\gamma-\alpha)}{\tan \tfrac{1}{2}(\gamma+\alpha)} = \frac{c-a}{c+a}. \tag{3}$$

89. Cyclic interchange of letters. Each formula pertaining to the oblique triangle gives rise to two other formulas of the same type by a cyclic interchange of letters. A cyclic interchange of letters may be accomplished by arranging the letters around the circumference of a circle as in the figure, and then replacing each letter by the next in order as indicated by the arrows. Thus by this cyclic interchange of letters formula (1) of Art. 88 gives rise to formula (2); likewise formula (2) gives rise to formula (3).



90. Law of cosines. *The square of any side of a plane triangle is equal to the sum of the squares of the other sides minus twice the product of those sides into the cosine of the included angle.*



For each figure

$$AB = AD + DB, \quad \text{or} \quad DB = AB - AD. \qquad \text{Art. 2}$$

But $AB = c$, $AD = b\cos\alpha$, hence $DB = c - b\cos\alpha$.

Also $DC = b\sin\alpha$.

From the right triangle CDB we have

$$\overline{BC}^2 = \overline{DC}^2 + \overline{DB}^2.$$

Substituting values, this equation becomes

$$\begin{aligned} a^2 &= (b\sin\alpha)^2 + (c - b\cos\alpha)^2 \\ &= b^2\sin^2\alpha + c^2 - 2bc\cos\alpha + b^2\cos^2\alpha \\ &= b^2(\sin^2\alpha + \cos^2\alpha) + c^2 - 2bc\cos\alpha. \end{aligned}$$

Hence $\boldsymbol{a^2 = b^2 + c^2 - 2bc\cos\alpha.}$

Similarly $\boldsymbol{b^2 = c^2 + a^2 - 2ca\cos\beta,}$

and $\boldsymbol{c^2 = a^2 + b^2 - 2ab\cos\gamma.}$

91. *To find the sine of half an angle of a plane triangle in terms of the sides of the triangle.*

From equation (1), Art. 72,

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}},$$

whence $$2\sin^2\frac{\alpha}{2} = 1 - \cos\alpha. \qquad (1)$$

From the cosine law

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}. \qquad (2)$$

From equations (1) and (2)

$$2\sin^2\frac{\alpha}{2} = 1 - \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{2bc - b^2 - c^2 + a^2}{2bc}$$

$$= \frac{a^2 - (b-c)^2}{2bc}$$

$$= \frac{(a-b+c)(a+b-c)}{2bc}. \qquad (3)$$

Let $$a+b+c = 2s. \qquad (4)$$

Subtracting $2a$, $2b$, and $2c$ from each member of (4) we have respectively

$$-a+b+c = 2(s-a)$$

$$a-b+c = 2(s-b)$$

$$a+b-c = 2(s-c).$$

Then equation (3) becomes

$$\sin\frac{\alpha}{2} = +\sqrt{\frac{(s-b)(s-c)}{bc}}.$$

Similarly $$\sin\frac{\beta}{2} = +\sqrt{\frac{(s-c)(s-a)}{ca}},$$

and $$\sin\frac{\gamma}{2} = +\sqrt{\frac{(s-a)(s-b)}{ab}}.$$

In these formulas the positive sign is given to the radical, since it is known that half an angle of any plane triangle is less than 90°. The same applies to the corresponding formulas for the cosine and the tangent of half an angle.

92. *To find the cosine of half an angle of a plane triangle in terms of the sides of the triangle.*

From equation (2), Art. 72, we have

$$2\cos^2\frac{\alpha}{2} = 1 + \cos\alpha.$$

Then
$$2\cos^2\frac{\alpha}{2} = 1 + \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{(b+c)^2 - a^2}{2bc}$$
$$= \frac{(a+b+c)(-a+b+c)}{2bc}$$

and
$$\cos^2\frac{\alpha}{2} = \frac{s(s-a)}{bc}.$$

Hence
$$\cos\frac{\alpha}{2} = +\sqrt{\frac{s(s-a)}{bc}}.$$

Similarly
$$\cos\frac{\beta}{2} = +\sqrt{\frac{s(s-b)}{ca}},$$

and
$$\cos\frac{\gamma}{2} = +\sqrt{\frac{s(s-c)}{ab}}.$$

93. *To find the tangent of half an angle of a plane triangle in terms of the sides of the triangle.*

Since
$$\tan\frac{\alpha}{2} = \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}},$$

$$\tan\frac{\alpha}{2} = +\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}. \qquad (1)$$

Similarly
$$\tan\frac{\beta}{2} = +\sqrt{\frac{(s-c)(s-a)}{s(s-b)}}, \qquad (2)$$

and
$$\tan\frac{\gamma}{2} = +\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}. \qquad (3)$$

Formula (1) may be written

$$\tan\frac{\alpha}{2}=\frac{1}{s-a}\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

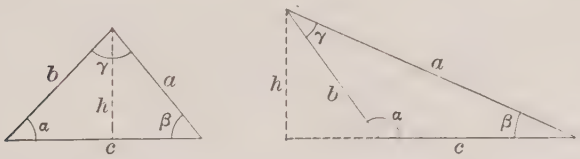
Letting
$$r=+\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}, \tag{4}$$

$$\tan\frac{\alpha}{2}=\frac{r}{s-a}. \tag{5}$$

Similarly
$$\tan\frac{\beta}{2}=\frac{r}{s-b}, \tag{6}$$

and
$$\tan\frac{\gamma}{2}=\frac{r}{s-c}. \tag{7}$$

94. *To find the area of a plane triangle in terms of two sides and the included angle.*



Let A be the area and h the altitude. Then in each figure

$$A=\tfrac{1}{2}ch,$$

and
$$h=b\sin\alpha.$$

Therefore
$$A=\tfrac{1}{2}bc\sin\alpha.$$

Similarly
$$A=\tfrac{1}{2}ca\sin\beta,$$

and
$$A=\tfrac{1}{2}ab\sin\gamma.$$

95. *To find the area of a plane triangle in terms of a side and two adjacent angles.*

From Art. 94,
$$A=\tfrac{1}{2}bc\sin\alpha.$$

From the sine law $b = \dfrac{c \sin \beta}{\sin \gamma}$,

$$A = \frac{c^2 \sin \alpha \sin \beta}{2 \sin \gamma}.$$

Then since $\alpha + \beta + \gamma = 180°$,

$$\boldsymbol{A = \frac{c^2 \sin \alpha \sin \beta}{2 \sin (\alpha + \beta)}}.$$

96. *To find the area of a plane triangle in terms of the three sides.*

From Art. 94.

$$A = \tfrac{1}{2} bc \sin \alpha.$$

Since $\sin \alpha = 2 \sin \dfrac{\alpha}{2} \cos \dfrac{\alpha}{2}$,

$$A = bc \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}.$$

Substituting the values of $\sin \dfrac{\alpha}{2}$ and $\cos \dfrac{\alpha}{2}$ as found in Arts. 91 and 92, we have, after reduction,

$$\boldsymbol{A = \sqrt{s(s-a)(s-b)(s-c)}}.$$

97. Formulas for solving an oblique triangle. The formulas developed in the present chapter are sufficient to solve a plane triangle when three independent parts are given.

The law of sines

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

is used when two of the given parts are an angle and the opposite side.

The law of tangents

$$\tan \tfrac{1}{2}(\alpha - \beta) = \frac{a-b}{a+b} \tan \tfrac{1}{2}(\alpha + \beta)$$

is used when two sides and the included angle are given.

If a, b, and γ are the given parts, $\frac{1}{2}(\alpha+\beta)$ is obtained from the relation $\alpha+\beta+\gamma=180°$. The formula then gives the value of $\frac{1}{2}(\alpha-\beta)$, which, united with the value of $\frac{1}{2}(\alpha+\beta)$, gives α and β.

The laws of half angles

$$\sin\frac{\alpha}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}},$$

$$\cos\frac{\alpha}{2}=\sqrt{\frac{s(s-a)}{bc}},$$

$$\tan\frac{\alpha}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}=\frac{r}{s-a},$$

are used when the three sides are given. The last formula is the most accurate, since the tangent varies more rapidly than either the sine or the cosine. The formula involving r is advantageous when *all* the angles are to be computed.

The law of cosines

$$a^2=b^2+c^2-2bc\cos\alpha$$

may be used to determine the third side when two sides and an angle are given. It may also be used to determine an angle when the three sides are given.

This formula is used with *natural functions*, not being adapted to logarithmic computation.

98. Check formulas. Any formula which was not used in the solution of the triangle may be used as a check formula.

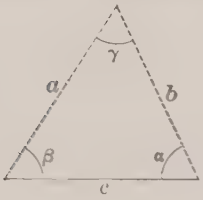
The relation $\alpha+\beta+\gamma=180°$ cannot be used as a check when a problem has been solved by the law of tangents, since the law of tangents involves this relation.

When two equations from the sine law have been used to find two elements of a triangle, the third equation from the sine law cannot be used as a check, since the first two equations involve the third.

99. Illustrative problems.

1. Given $c = 127.32$, $\alpha = 71° 58' 22''$, $\beta = 52° 19' 40''$; to find a, b, γ.

SOLUTION. *Construction and estimates.*



$a = 140$
$b = 120$
$\gamma = 60°$

Outline

$$a = \frac{c \sin \alpha}{\sin \gamma}$$

c	
α	
β	
$\alpha + \beta$	
γ	
log sin α	
log c	
colog sin γ	
log a	
a	

Check

$$\tan \tfrac{1}{2}(\alpha - \beta) = \frac{a - b}{a + b} \tan \tfrac{1}{2}(\alpha + \beta).$$

$a - b$	
$a + b$	
$\tfrac{1}{2}(a + b)$	
log $(a - b)$	
colog $(a + b)$	
log tan $\tfrac{1}{2}(\alpha + \beta)$	
log tan $\tfrac{1}{2}(\alpha - \beta)$	
$\tfrac{1}{2}(\alpha - \beta)$	
$\tfrac{1}{2}(\alpha + \beta)$	
α	
β	

$$b = \frac{c \sin \beta}{\sin \gamma}$$

log sin β	
log c	
colog sin γ	
log b	
b	

Or, more compactly,

log a	log sin α	
log b	log c	
	colog sin γ	
	log sin β	
	log a	
	log b	
	a	
	b	

Filling in the above outline, the completed work appears as follows:

$$a = \frac{c \sin \alpha}{\sin \gamma}$$

c	127.32
α	71° 58′ 22″
β	52° 19′ 40″
$\alpha + \beta$	124° 18′ 2″
γ	55° 41′ 58″
log sin α	9.97814−10
log c	2.10490
colog sin γ	0.08297
log a	2.16601
a	146.56

$$b = \frac{c \sin \beta}{\sin \gamma}$$

log sin β	9.89846−10
log c	2.10490
colog sin γ	0.08297
log b	2.08633
b	121.99

Check

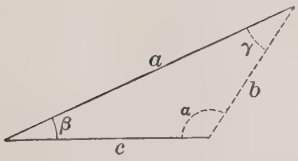
$$\tan \tfrac{1}{2}(\alpha - \beta) = \frac{a - b}{a + b} \tan \tfrac{1}{2}(\alpha + \beta)$$

$a - b$	24.57
$a + b$	268.55
$\frac{1}{2}(\alpha + \beta)$	62° 9′ 1″
log $(a - b)$	1.39041
colog $(a + b)$	7.57098
log tan $\frac{1}{2}(\alpha + \beta)$	0.27708
log tan $\frac{1}{2}(\alpha - \beta)$	9.23847−10
$\frac{1}{2}(\alpha - \beta)$	9° 49′ 28″
$\frac{1}{2}(\alpha + \beta)$	62° 9′ 1″
α	71° 58′ 29″
β	52° 19′ 33″

Or, in the more compact form,

log a { log b {	log sin α	9.97814−10
	log c	2.10490
	colog sin γ	0.08297
	log sin β	9.89846
	log a	2.16601
	log b	2.08633
	a	146.56
	b	121.99

2. Given $a = 1674.3$, $c = 1021.7$, $\beta = 28° 44′ 39″$; to find α, γ, b.



Estimates

$\alpha = 120°$

$\gamma = 30°$

$b = 900.$

$$\tan \tfrac{1}{2}(\alpha - \gamma) = \frac{a - c}{a + c} \tan \tfrac{1}{2}(\alpha + \gamma)$$

a	1674.3
c	1021.7
β	28° 44′ 39″
$a - c$	652.6
$a + c$	2696.0
$\alpha + \gamma$	151° 15′ 21″
$\frac{1}{2}(\alpha + \gamma)$	75° 37′ 40″
$\log (a - c)$	2.81465
$\operatorname{colog} (a + c)$	6.56928 − 10
$\log \tan \frac{1}{2}(\alpha + \gamma)$	0.59135
$\log \tan \frac{1}{2}(\alpha - \gamma)$	9.97528 − 10
$\frac{1}{2}(\alpha - \gamma)$	43° 22′ 12″
$\frac{1}{2}(\alpha + \gamma)$	75° 37′ 40″
α	118° 59′ 52″
γ	32° 15′ 28″

$$b = \frac{c \sin \beta}{\sin \gamma}$$

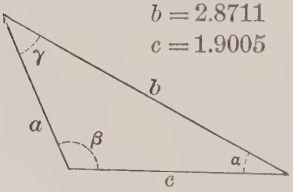
$\log c$	3.00932
$\log \sin \beta$	9.68205 − 10
$\operatorname{colog} \sin \gamma$	0.27268
$\log b$	2.96405
b	920.56

Check

$$b = \frac{a \sin \beta}{\sin \alpha}$$

$\log a$	3.22384
$\log \sin \beta$	9.68205 − 10
$\operatorname{colog} \sin \alpha$	0.05817
$\log b$	2.96406

3. Given $a = 1.4932$, $b = 2.8711$, $c = 1.9005$ to find α, β, γ.



Estimates

$\alpha = 25°$

$\beta = 120°$

$\gamma = 35°$

$$\tan \frac{\alpha}{2} = \frac{r}{s - a}$$

$$\tan \frac{\beta}{2} = \frac{r}{s - b}$$

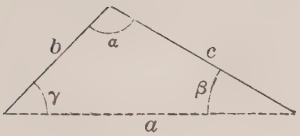
$$\tan \frac{\gamma}{2} = \frac{r}{s - c}$$

a	1.4932
b	2.8711
c	1.9005
$2s$	6.2648
s	3.1324
$s - a$	1.6392
$s - b$	0.2613
$s - c$	1.2319
s	3.1324

$$r = \sqrt{\frac{(s - a)(s - b)(s - c)}{s}}$$

$\log (s - a)$	0.21463
$\log (s - b)$	9.41714 − 10
$\log (s - c)$	0.09058
$\operatorname{colog} s$	9.50412 − 10
$\log r^2$	19.22647 − 20
$\log r$	9.61324 − 10
$\log \tan \frac{\alpha}{2}$	9.39861 − 10
$\log \tan \frac{\beta}{2}$	0.19610
$\log \tan \frac{\gamma}{2}$	9.52266 − 10
$\frac{\alpha}{2}$	14° 3′ 26″
$\frac{\beta}{2}$	57° 31′ 2″
$\frac{\gamma}{2}$	18° 25′ 34″
α	28° 6′ 52″
β	115° 2′ 4″
γ	36° 51′ 8″
Check: $\alpha + \beta + \gamma$	180° 00′ 4″

4. Given $b = .0060041$, $c = .0093284$, $\gamma = 44° 47' 58''$; to find β, α, a.



Estimates

$\beta = 30°$
$\alpha = 105°$
$a = .012$.

$$\sin\beta = \frac{b\sin\gamma}{c}$$

b	.0060041
c	.0093284
γ	44° 47′ 58″
$\log b$	7.77845 − 10
$\operatorname{colog} c$	2.03019
$\log\sin\gamma$	9.84796 − 10
$\log\sin\beta$	9.65660 − 10
β	26° 58′ 12″
$\beta+\gamma$	71° 46′ 10″
α	108° 13′ 50″

$$a = \frac{c\sin\alpha}{\sin\gamma}$$

$\log c$	7.96981 − 10
$\operatorname{colog}\sin\gamma$	0.15204
$\log\sin\alpha$	9.97764 − 10
$\log a$	8.09949 − 10
a	0.012574

Check

$$\tan\tfrac{1}{2}(\alpha-\gamma) = \frac{a-c}{a+c}\tan\tfrac{1}{2}(\alpha+\gamma)$$

a	.012574
c	.0093284
$a-c$	.0032456
$a+c$	.021902
$\alpha+\gamma$	153° 1′ 48″
$\frac{1}{2}(\alpha+\gamma)$	76° 30′ 54″
$\log(a-c)$	7.51129 − 10
$\operatorname{colog}(a+c)$	1.65952
$\log\tan\frac{1}{2}(\alpha+\gamma)$	0.62015
$\log\tan\frac{1}{2}(\alpha-\gamma)$	9.79096 − 10
$\frac{1}{2}(\alpha-\gamma)$	31° 42′ 51
$\frac{1}{2}(\alpha+\gamma)$	76° 30′ 54″
α	108° 13′ 45″
γ	44° 48′ 3″

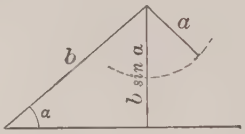
100. The ambiguous case. When two sides and an angle opposite one of them are given, the triangle may admit of no solution, of one solution, or of two solutions.

Let a, b, α be given. The formula

$$\sin\beta = \frac{b\sin\alpha}{a}$$

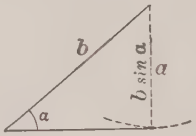
determines β.

If the calculated $\sin \beta$ is greater than 1, there can be no solution.



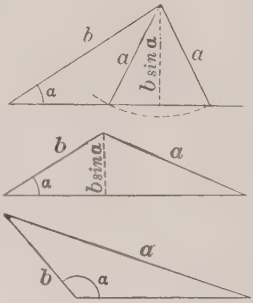
$a < b \sin \alpha$

If the calculated $\sin \beta$ equals 1, $\beta = 90°$ and there is one solution.



$a = b \sin \alpha$

If the calculated $\sin \beta$ is less than 1, two supplementary values of β are determined, giving two solutions unless the larger value of β plus α is equal to or greater than 180°.

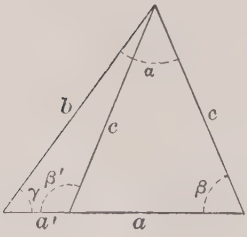


$a > b \sin \alpha$
$a < b$

$a \overline{>} b$

$a > b$

Given $b = 420$, $c = 389.73$, $\gamma = 53° 47' 20''$; to find β, α, a.



Estimates	*Two solutions*
$\beta = 65°$	$\beta' = 115°$
$\alpha = 60°$	$\alpha' = 10°$
$a = 390.$	$a' = 90.$

$$\sin \beta = \frac{b \sin \gamma}{c}$$

b	420
c	389.73
γ	53° 47′ 20″
$\log b$	2.62325
$\operatorname{colog} c$	7.40924 − 10
$\log \sin \gamma$	9.90679 − 10
$\log \sin \beta$	9.93928 − 10
β	60° 24′ 9″
$180° - \beta = \beta'$	119° 35′ 51″
$\beta + \gamma$	114° 11′ 29″
$\beta' + \gamma$	173° 22′ 11″
α	65° 48′ 31″
α'	6° 37′ 49″

Check of 1st solution

$$\tan \tfrac{1}{2}(\alpha - \gamma) = \frac{a - c}{a + c} \tan \tfrac{1}{2}(\alpha + \gamma)$$

a	440.61
c	389.73
$a - c$	50.88
$a + c$	830.34
$\alpha + \gamma$	119° 35′ 51′
$\frac{1}{2}(\alpha + \gamma)$	59° 47′ 50″
$\log (a - c)$	1.70655
$\operatorname{colog} (a + c)$	7.08074 − 10
$\log \tan \frac{1}{2}(\alpha + \gamma)$	0.23502
$\log \tan \frac{1}{2}(\alpha - \gamma)$	9.02231 − 10
$\frac{1}{2}(\alpha - \gamma)$	6° 0′ 34″
$\frac{1}{2}(\alpha + \gamma)$	59° 47′ 50″
α	65° 48′ 24″
γ	53° 47′ 16″

$$a = \frac{c \sin \alpha}{\sin \gamma}$$

$$a' = \frac{c \sin \alpha'}{\sin \gamma}$$

$\log \sin \alpha$	9.96008 − 10
$\log c$	2.59076
$\operatorname{colog} \sin \gamma$	0.09321
$\log \sin \alpha'$	9.06244 − 10
$\log a$	2.64405
$\log a'$	1.74641
a	440.61
a'	55.771

Check of 2d solution

$$\tan \tfrac{1}{2}(\gamma - \alpha') = \frac{c - a'}{c + a'} \tan \tfrac{1}{2}(\gamma + \alpha')$$

c	389.73
a'	55.771
$c - a'$	333.96
$c + a'$	445.50
$\gamma + \alpha'$	60° 25′ 9″
$\frac{1}{2}(\gamma + \alpha')$	30° 12′ 34″
$\log (c - a')$	2.52370
$\operatorname{colog} (c + a')$	7.35115 −10
$\log \tan \frac{1}{2}(\gamma + \alpha')$	9.76509 − 10
$\log \tan \frac{1}{2}(\gamma - \alpha')$	9.63994 − 10
$\frac{1}{2}(\gamma - \alpha')$	23° 34′ 45″
$\frac{1}{2}(\gamma + \alpha')$	30° 12′ 34″
γ	53° 47′ 19″
α'	6° 37′ 49″

FIRST SOLUTION

$\beta = 60° 24′ 9″$
$\alpha = 65° 48′ 31″$
$a = 440.61$

SECOND SOLUTION

$\beta' = 119° 35′ 51″$
$\alpha' = 6° 37′ 49″$
$a' = 55.771$

101. EXAMPLES

Solve the following triangles, using a three-place table or a slide-rule.

1. $a = 26$, $\alpha = 53°$, $\beta = 49°$

2. $a = 48$, $\beta = 61°$, $\gamma = 69°$

3. $\alpha = 73°$, $\beta = 59°$, $b = 55$

4. $\alpha = 69°$, $a = 80$, $b = 64$

5. $\alpha = 54° 40'$, $b = 122$, $c = 110$

6. $a = 163$, $b = 241$, $\gamma = 34° 20'$

7. $a = 42$, $b = 28$, $\gamma = 72°$

8. $b = 115$, $c = 96$, $\alpha = 110°$

9. $a = 51$, $b = 63$, $c = 70$

10. $a = 8.03$, $b = 6.42$, $c = 7.15$

11. $\alpha = 20°$, $a = 15$, $b = 25$

12. $\gamma = 43° 50'$, $a = .34$, $c = .30$

Solve the following triangles:

13. $b = 63.67$, $\beta = 100° 10'$, $\alpha = 40° 0' 10''$

14. $b = 20.007$, $\alpha = 40° 27' 30''$, $\gamma = 42° 30' 15''$

15. $a = 238.61$, $b = 216.77$, $c = 98.435$

16. $a = 8.0038$, $b = 4.6259$, $c = 4.3167$

17. $b = .76328$, $c = 2.4359$, $\gamma = 120° 46' 18''$

18. $b = 85.249$, $c = 105.63$, $\alpha = 50° 40' 24''$

19. $a = 1.4562$
$c = .45296$
$\beta = 74° 19' 38''$

20. $a = 83.831$
$b = 56.479$
$c = 74.025$

21. $a = 2.1469$
$b = 3.2824$
$c = 4.0026$

22. $c = 7.2693$
$a = .54871$
$\alpha = 5° 41' 30''$

23. $b = .06532$
$c = .01846$
$\gamma = 8° 0' 20''$

24. $b = 10.246$
$c = 18.075$
$\beta = 33° 30' 5''$

25. $a = 764.38$
$\alpha = 143° 18' 31''$
$\beta = 13° 34' 26''$

26. $a = 962.27$
$b = 637.34$
$c = 655.80$

Find the areas of the following triangles:

27. $a = 15$
$b = 20$
$c = 25$

28. $a = 172$
$b = 103$
$c = 141$

29. $a = 20.46$
$b = 19.72$
$c = 15.04$

30. $a = 18.3$
$b = 22.4$
$\gamma = 32°$

31. $b = 3.46$
$c = 4.09$
$\alpha = 56° 10'$

32. $c = 435.3$
$a = 289.6$
$\beta = 31° 7'$

33. $a = 48$
$\beta = 26°$
$\gamma = 43°$

34. $b = 10.34$
$\alpha = 83° 22'$
$\gamma = 60° 40'$

35. The horizontal distance from a point on top of a tower to a distant flagpole is 468 ft. The angle of elevation of the top of the flagpole is $5° 8' 30''$. The angle of depression of the foot of the pole is $15° 36'$. Find the height of the flagpole.

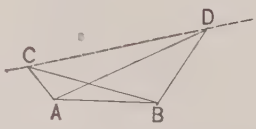
36. A tower is situated on a hill which inclines at an angle of 23° 19′ 10″ to the horizontal. The angle of elevation of the top of the tower, from a point on the hillside, was measured and found to be 43° 39′ 50″. At a point 75.5 ft. farther down, the angle of elevation was found to be 39° 23′ 20″. How high is the tower?

37. From a point 5 miles from one end of a lake and 3 miles from the other end, the lake subtends an angle of 47° 34′ 30″. Find the length of the lake.

38. Find the altitudes of a triangle whose sides are 125.4, 230.6, and 179.8.

39. A flagpole stands on the summit of a hill. The hill inclines 32° 18′ 20″ to the horizontal. At a point 25.5 ft. from the base of the flagpole, measured along the incline, the angle subtended by the flagpole was found to be 41° 24′. Find the height of the flagpole.

40. Two observers, A and B, stationed 4000 ft. apart, at the same instant observe the angles BAC and CBA to an automobile traveling on a straight road. Three minutes later they measure the angles DAB and ABD to the second position of the automobile.



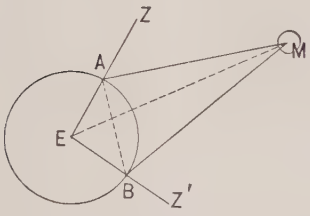
$$\text{If } \angle BAC = 136° 28',$$
$$\angle CBA = 32° 8',$$
$$\angle DAB = 40° 12',$$
$$\angle ABD = 118° 44',$$

what is the rate of the automobile?

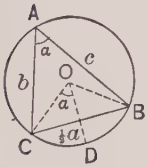
41. A man wishes to measure the length of a lake from his position on a hill top 185 ft. above the level of the lake. He finds the angles of depression of the ends of the lake to be 6° 18′ and 2° 30′. The angle subtended by the lake, formed

by the two lines of sight, is 66° 27′. Find the length of the lake.

42. The angles of elevation of a cloud, directly above a straight road, from two points of the road on opposite sides of the cloud, are 78° 15′ 20″ and 59° 47′ 40″. Find the height of the cloud, the distance between the two points of observation being 5000 ft.



43. Two observers at A and B, whose longitudes are the same, simultaneously observe the moon and find the angles ZAM and $Z'BM$ to be 35° 2′ 20″ and 51° 17′ 10″ respectively. The latitude of A (Greenwich) is N. 51° 17′ 15″ and the latitude of B (Cape of Good Hope) is S. 33° 45′ 16″. The moon is in the plane determined by A, E, and B. Find EM, the distance from the center of the earth to the moon, the radius of the earth being 3960 miles.

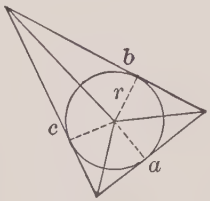


44. Show that twice the radius, $2R$, of a circle circumscribing a triangle is given by the equations

$$2R = \frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}.$$

Suggestion. $\angle BAC = \angle DOC$.

45. Find the radius of a circle inscribed in a triangle whose sides are given.



Solution. Representing the area of the triangle by A, we have

$$A = \tfrac{1}{2}ar + \tfrac{1}{2}br + \tfrac{1}{2}cr = rs.$$

But $A = \sqrt{s(s-a)(s-b)(s-c)}$. Art. 96.

Therefore $r = \sqrt{\dfrac{(s-a)(s-b)(s-c)}{s}}$. See Art. 93.

MISCELLANEOUS EXERCISES

102. **1.** An angle of 4 radians, having its vertex at the center of a circle, intercepts an arc of 7 inches; find the radius of the circle.

2. Express the following in degrees: $\frac{\pi}{3}$ radians, $\frac{2\pi}{5}$ radians, $\frac{3\pi}{4}$ radians.

3. Express the following in radian measure: 40°, 55°, 38°, 52° 16′.

4. Reduce $\frac{3}{4}$ radians to degrees.

5. Find the number of degrees in a central angle which intercepts an arc of 5 feet in a circle whose radius is 8 feet.

6. An arc of 12 inches subtends a central angle of 50°; find the radius of the circle.

7. The number of minutes in an angle is $7\frac{1}{2}$ times the number of degrees in its supplement; find the number of radians in the angle.

8. An angle exceeds another by $\frac{\pi}{8}$ radians, and their sum is 160°; express each angle in radian measure.

9. The angles of a triangle are to each other as 2 : 3 : 4; express each angle in radian measure.

10. The angles of a triangle are in arithmetical progression, and the mean angle is twice the smallest; express each angle in radian measure.

11. The circumference of a circle is divided into 7 parts in arithmetical progression, the greatest part being 10 times the least; express in radians the angle which each arc subtends at the center.

12. An arc of 40° on a circle whose radius is 6 inches is equal in length to an arc of 25° on another circle; what is the radius of the latter circle?

Prove each of the following:

13. $\sin\alpha\cos\alpha = \sin^3\alpha\cos\alpha + \cos^3\alpha\sin\alpha.$

14. $\cot\alpha\csc\alpha = \dfrac{1}{\sec\alpha - \cos\alpha}.$

15. $\dfrac{1+\tan^2\alpha}{1+\cot^2\alpha} = \dfrac{\sin^2\alpha}{\cos^2\alpha}.$

16. $\sin\beta + \dfrac{\sin\beta}{\cot\beta - 1} = \dfrac{\cos\beta}{1-\tan\beta} - \cos\beta.$

17. $\sin^6\alpha + \cos^6\alpha = 1 - 3\sin^2\alpha\cos^2\alpha.$

18. $\cot\alpha - \tan\alpha = \csc\alpha\sec\alpha\,(1 - 2\sin^2\alpha).$

19. $\dfrac{1-\tan\beta}{1+\tan\beta} = \dfrac{\cot\beta - 1}{\cot\beta + 1}.$

20. $\sec^4\alpha - \tan^4\alpha = 2\sec^2\alpha - 1.$

21. $\tan 15° = 2 - \sqrt{3}.$

22. $\sin 4x = 4(\cos^3 x\sin x - \sin^3 x\cos x).$

23. $\cos 4x = 4\cos^4 x + 4\sin^4 x - 3.$

24. $\sin 5x = 16\sin^5 x - 20\sin^3 x + 5\sin x.$

25. $\cos 5x = 16\cos^5 x - 20\cos^3 x + 5\cos x.$

26. $\sin(\alpha+\beta+\gamma) = \sin\alpha\cos\beta\cos\gamma + \cos\alpha\sin\beta\cos\gamma + \cos\alpha\cos\beta\sin\gamma - \sin\alpha\sin\beta\sin\gamma.$

27. $\tan(\alpha+\beta+\gamma) = \dfrac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\beta\tan\gamma-\tan\gamma\tan\alpha-\tan\alpha\tan\beta}.$

28. $2 \sin(\alpha - \beta) \cos \alpha = \sin(2\alpha - \beta) - \sin \beta.$

29. $\cos 2\alpha = \dfrac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}.$

30. $\sin\left(\dfrac{\pi}{4} + \theta\right) = \dfrac{\sin \theta + \cos \theta}{\sqrt{2}}.$

31. $\cos\left(\dfrac{\pi}{4} - \theta\right) = \dfrac{\sin \theta + \cos \theta}{\sqrt{2}}.$

32. $\tan\left(\dfrac{\pi}{4} + \theta\right) = \dfrac{1 + \tan \theta}{1 - \tan \theta}.$

33. $\tan \alpha + \tan \beta = \dfrac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta}.$

34. $\cot \alpha + \cot \beta = \dfrac{\sin(\alpha + \beta)}{\sin \alpha \sin \beta}.$

35. $\cos(\alpha + \beta) \cos(\alpha - \beta) = \cos^2 \alpha - \sin^2 \beta.$

36. $\cos \theta = \dfrac{1 - \tan^2 \frac{1}{2}\theta}{1 + \tan^2 \frac{1}{2}\theta}.$

37. $\sec 2\theta = \dfrac{\cot \theta + \tan \theta}{\cot \theta - \tan \theta}.$

38. $\tan \theta + \cot \theta = 2 \operatorname{cosec} 2\theta.$

39. $\dfrac{\cos \alpha + \sin \alpha}{\cos \alpha - \sin \alpha} = \tan 2\alpha + \sec 2\alpha.$

40. $\dfrac{\cos(45° + \theta)}{\cos(45° - \theta)} = \sec 2\theta - \tan 2\theta.$

41. $\sin(\alpha + \beta) \cos(\alpha - \beta) + \cos(\alpha + \beta) \sin(\alpha - \beta) = \sin 2\alpha.$

42. $\dfrac{\sin \alpha - \sin \beta}{\cos \alpha + \cos \beta} = \tan\left(\dfrac{\alpha - \beta}{2}\right).$

43. $\tan 2\alpha - \tan\alpha = \tan\alpha \sec 2\alpha.$

44. $\sin\alpha = \dfrac{2\cos^2\frac{1}{2}\alpha}{\cot\frac{1}{2}\alpha}.$

45. $\cos 3\alpha\cos\alpha + \sin 3\alpha\sin\alpha = \cos 2\alpha.$

46. $\left[\sin\left(\dfrac{\alpha+\beta}{2}\right) + \cos\left(\dfrac{\alpha+\beta}{2}\right)\right]$
$\left[\sin\left(\dfrac{\alpha-\beta}{2}\right) + \cos\left(\dfrac{\alpha-\beta}{2}\right)\right] = \sin\alpha + \cos\beta.$

47. $4\sin\theta\sin(60°-\theta)\sin(60°+\theta) = \sin 3\theta.$

48. $\tan^2\frac{1}{2}\theta = \dfrac{2\sin\theta - \sin 2\theta}{2\sin\theta + \sin 2\theta}.$

Solve each of the following equations for all values of the unknown quantity less than 360°.

49. $\tan\theta = \sin\theta.$

50. $(3 - 4\cos^2\alpha)\cos 2\alpha = 0.$

51. $\sin\alpha + \cos\alpha\cot\alpha = 2.$

52. $\tan(45° + \theta) = 3\tan(45° - \theta).$

53. $5\sin\theta = \tan\theta.$

54. $1 + \sin^2\alpha = 3\sin\alpha\cos\alpha.$

55. $2\cos x + \sec x = 3.$

56. $\tan^4 y - 4\tan^2 y + 3 = 0.$

57. $\sec\beta + \tan\beta = 2.$

58. $2\sin\alpha + 5\cos\alpha = 2.$

59. $\sin 5x + \sin 3x = 0.$

60. $\cos 7x - \cos x = 0.$

61. $\tan 6\theta = 1.$

62. $\tan 2\theta\tan\theta = 1.$

63. $\sin 4\theta + \sin 2\theta + \cos\theta = 0.$

64. $\sin 2\theta - 2\cos\theta + 2\sin\theta - 2 = 0.$

65. $\sin^2\theta + \sin 3\theta = 2\sin\theta\cos 2\theta - \frac{1}{4}.$

66. $\tan 2\theta\cot\theta - \tan 2\theta + \cot\theta - 1 = 0.$

67. $\cos 7\theta + \cos 5\theta + \cos 3\theta = 0.$

68. $\sec^2\theta\csc^2\theta + 4 = 4\csc^2\theta + \sec^2\theta.$

69. Given $\tan\beta = u$, find $\sin 2\beta$.

70. Given $\tan\theta = \csc 2\theta$, find $\cos\theta$.

71. Given $\cos x = \frac{1}{2}$, find $\cos\frac{1}{2}x$.

72. Given $\cos x = \frac{3}{7}$, find $\tan\frac{1}{2}x$ and $\tan 2x$.

73. Given $\tan 2x = m$, find $\tan x$.

74. If $\alpha + \beta + \gamma = 180°$, show that

$$\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha\tan\beta\tan\gamma.$$

75. If $\alpha + \beta + \gamma = 180°$, show that

$$\cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}.$$

76. If $\alpha + \beta + \gamma = 180°$, show that

$$\sin\alpha + \sin\beta + \sin\gamma = 4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}.$$

Prove the following:

77. $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{3}{5} = \frac{\pi}{4}.$

78. $\tan^{-1}\frac{1}{6} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{14}{27} = \frac{\pi}{4}.$

79. $\tan^{-1}k + \tan^{-1}l = \tan^{-1}\frac{k+l}{1-kl}.$

80. $3\tan^{-1}u = \tan^{-1}\frac{3u - u^3}{1 - 3u^2}.$

81. $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}.$

82. $\sin^{-1}\sqrt{\dfrac{m}{m+n}} = \tan^{-1}\sqrt{\dfrac{m}{n}}$.

83. $\cos^{-1}\dfrac{m}{n} = \operatorname{cosec}^{-1}\sqrt{\dfrac{n}{n-m}}$.

84. $\cot^{-1}\dfrac{k^2-2}{2\sqrt{k^2-1}} = 2\csc^{-1}k$.

Solve the following for y:

85. $2\sin^{-1}\dfrac{1}{2} + \cot^{-1}y = \dfrac{\pi}{2}$.

86. $\sin^{-1}y + \sin^{-1}2y = \dfrac{\pi}{2}$.

87. $\dfrac{\pi}{2} - \cot^{-1}3y = 2\tan^{-1}y$.

88. $\tan^{-1}y = \sin^{-1}a + \cos^{-1}b$.

89. Prove that in a plane triangle, right-angled at γ,

$$\sin 2\alpha = \frac{2ab}{b^2+a^2}; \quad \cos 2\alpha = \frac{b^2-a^2}{b^2+a^2}.$$

90. In a right triangle, c being the hypothenuse, prove that

$$\sin^2\frac{1}{2}\alpha = \frac{c-b}{2c}; \quad \cos^2\frac{1}{2}\alpha = \frac{c+b}{2c}.$$

91. In an isosceles triangle in which $\alpha = b$, prove that

$$\cos\alpha = \frac{c}{2a}; \quad \operatorname{vers}\gamma = \frac{c^2}{2a^2}.$$

92. In a triangle in which $\gamma = 60°$, prove that

$$\cos(60° - \beta) = \frac{a+b}{2c}.$$

93. Show that the area of a regular polygon, inscribed in a circle whose radius is r, is $\dfrac{nr^2}{2}\sin\dfrac{2\pi}{n}$.

94. Show that the area of a regular polygon, circumscribed about a circle whose radius is r, is $nr^2 \tan \frac{\pi}{n}$.

95. Show that the area of a regular polygon of n sides is $\frac{na^2}{4} \cot \frac{\pi}{n}$, a being the length of a side.

96. Prove that the areas of an equilateral triangle and of a regular hexagon, of equal perimeters, are to each other as $2 : 3$.

97. A flagpole 125 ft. high, standing on a horizontal plane, casts a shadow 250 ft. long; find the altitude of the sun.

98. From the top of a rock that rises vertically 325.6 ft. out of the water, the angle of depression of a boat was found to be 24° 35′; find the distance of the boat from the rock.

99. A balloon is directly above a station A. From a second station B, in the same horizontal plane with A, the angle of elevation of the balloon is 60° 30′. If AB = 300 ft., find the height of the balloon.

100. From a tower 64 ft. high the angles of depression of two objects in the same horizontal line with the base of the tower and on the same side of the tower, are measured and found to be 28° 14′ and 42° 47′ respectively; find the distance between the objects.

101. Find the area of a triangular field whose sides are 48 rods, 62 rods, and 74 rods.

102. The earth subtends an angle of $17\frac{1}{2}''$ at the sun; find the distance of the sun from the earth, the radius of the earth being 3960 mi.

103. A vertical tower makes an angle of 113° 12′ with the inclined plane on which it stands; and at a distance of 89 ft. 8 in. from its base, measured down the plane, the angle subtended by the tower is 23° 26′. Find the height of the tower.

104. In a circle of radius 8, find the area of a sector with an arc of 50°.

105. In a circle of radius 12, find the area of a segment with an arc of 132°.

106. In a circle of radius r, find the area of a sector with an arc of 1 radian.

107. The area of a square inscribed in a circle is 200 sq. ft.; find the area of an equilateral triangle inscribed in the same circle.

108. Find the area of a triangle of which two sides are $6+\sqrt{5}$ and $6-\sqrt{5}$, the included angle being 32° 12′.

109. At what latitude is the radius of the circle of latitude equal to $\frac{1}{3}$ the radius of the earth?

110. From the top of a lighthouse 85 feet high, standing on a rock, the angle of depression of a ship was 3° 38′, and at the bottom of the lighthouse the angle of depression was 2° 43′; find the horizontal distance of the vessel and the height of the rock.

111. At a point directly south of a flagpole, in the horizontal plane of its base, I observed its elevation, 45°; then going east 200 ft. its elevation was 35°. Find the height of the flagpole.

112. A castle and a monument stand on the same horizontal plane. The angles of depression of the top and the bottom of the monument viewed from the top of the castle are 40° 32′ 18″ and 80° 17′ 46″, and the height of the castle is $104\frac{3}{4}$ feet. Find the height of the monument.

113. At the distance a from the foot of a tower the angle of elevation α of the top of the tower is the complement of the angle of elevation of a flagstaff on top of the tower; show that the length of the staff is $2\,a\cot 2\,\alpha$.

114. A flagstaff a feet high is on a tower $3a$ feet high; the observer's eye is on a level with the top of the staff, and the staff and tower subtend equal angles. How far is the observer from the top of the staff?

115. Two towers on a horizontal plane are 120 feet apart. A person standing successively at their bases observes that the angular elevation of one is double that of the other; but, when he is halfway between them, the elevations are complementary. Find the heights of the towers.

116. An observer sailing north sees two lighthouses 8 miles apart, in a line due west; after an hour's sailing one lighthouse bears S. W., and the other S. S. W. Find the ship's rate.

117. From the top of a house 42 ft. high, the angle of elevation of the top of a pole is 14° 26′ 9″; at the bottom of the house it is 23° 21′ 33″; find the height of the pole.

118. From the top of a hill I observe that the angles of depression of two successive milestones in the horizontal plain below, in a straight line before me, are 14° 20′ 24″ and 5° 31′ 14″. Find the height of the hill.

119. Along the bank of a river is measured a line 500 ft. in length; the angles between this line and the lines of sight from its extremities to an object on the opposite bank are 53° 1′ 7″ and 79° 44′ 55″. Find the breadth of the river.

120. A cape bears N. by E. ($\frac{1}{8}$ of 90° E. of N.), as seen from a ship. The ship sails N. W. 30 miles and then the cape bears E. How far is it from the second point of observation?

121. Find the height of a precipice, its angles of elevation at two stations in a horizontal line with its base being 39° 30′ and 34° 15′ and the distance between the stations being 145 ft.

122. From the summit of a hill, 360 ft. above a plain, the angles of depression of the top and bottom of a tower standing on the plain were 41° and 54°; required the height of the tower.

123. At one side of a canal is a flagstaff 21 feet high fixed on the top of a wall 15 feet high; on the other side of the canal, at a point on the ground directly opposite, the flagstaff and the wall subtend equal angles. Find the width of the canal.

124. From the top C of a cliff 600 feet high, the angle of elevation of a balloon B was observed to be 47° 22′, and the angle of depression of its shadow S upon the sea was 61° 10′; find the height of the balloon, the altitude of the sun being 65° 31′ and B, S, C being in the same vertical plane and the sun being behind the observer.

125. At each extremity of a base $AB = 758$ yards, the angles between the other extremity and two objects C and D were observed, viz. $CAB = 103°\ 50'\ 41''$, $DAB = 53°\ 17'\ 24''$, $DBA = 85°\ 47'\ 30''$, and $CBA = 46°\ 13'\ 27''$; find CD.

126. Show that if r be the radius of the earth, h the height of the observer above the sea, and d the angle of depression of the horizon; then $\tan d = \dfrac{\sqrt{(2r+h)h}}{r}$.

127. A privateer lies 12.75 miles S. W. of a harbor, and a merchantman leaves the harbor in a direction E. by S., at the rate of 10 miles an hour; on what course and at what rate must the privateer sail in order to overtake the merchantman in $1\frac{1}{2}$ hours?

128. From the top of a hill the angles of depression of two objects in the plain at its base were observed to be 45° and 30°, and the horizontal angle between them is also 30°; find the height of the hill in terms of the distance a between the objects.

129. The topmast, 120 feet above the water line of a man-of-war coming into port at the rate of 10 miles an hour, was first seen on the horizon at 8.45 A.M. by a person swimming near the water's edge; and at 10.06 A.M. she cast anchor. Find an approximate value for the radius of the earth.

130. A railway curve which is a circular quadrant has telegraph poles at its extremities and at equal distance along the arc, the whole number of poles being 10. A person in one of the extreme radii produced and at a distance of 300 feet from its extremity, sees the third and sixth poles in line. Find the radius of the curve.

CHAPTER IX

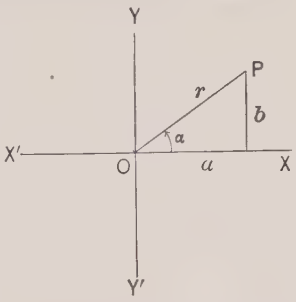
DE MOIVRE'S THEOREM WITH APPLICATIONS

103. In the present chapter De Moivre's theorem is introduced with some of its applications, including the demonstration of the fundamental series by means of which we may calculate the trigonometric tables.

A few preliminary considerations pertaining to the complex number furnish the necessary basis.

104. Geometric representation of a complex number. In algebra it is shown that every complex number can be reduced to the form $a + b\sqrt{-1}$ or $a + ib$ where a and b are any real numbers, and $i = \sqrt{-1}$.

Having given a complex number $a + ib$, we can construct a point P whose coordinates are a and b. This point may be considered the geometric representation of the complex number. It is thus seen that to every complex number there corresponds a point in the plane.



Conversely, to every point in the plane there corresponds a complex number, since the coordinates of the point represent the two elements, a and b, of the complex number.

The line OP, instead of the point P, may be considered the geometric representation of the complex number.

From the figure it is evident that

$$a = r \cos \alpha,$$

and $$b = r \sin \alpha.$$

Therefore $$a + ib = r \cos \alpha + ir \sin \alpha.$$

Hence any complex number, $a + ib$, can be reduced to the form

$$r(\cos \alpha + i \sin \alpha). \qquad (1)$$

The form (1) includes all real and all pure imaginary numbers as special cases. Letting $\alpha = 0°$ or $180°$, (1) reduces to r or $-r$, which represents any real number since r represents any real positive number. Letting $\alpha = 90°$ or $270°$, (1) reduces to $\pm ri$, any pure imaginary number.

The angle XOP or α is the **argument** of the complex number $a + ib$. It is determined by the equation

$$\tan \alpha = \frac{b}{a}.$$

The distance OP or r is the **modulus** of the complex number $a + ib$. It is determined by the equation

$$r = \sqrt{a^2 + b^2}.$$

105. *To show that*

$$[r(\cos \alpha + i \sin \alpha)]^n = r^n(\cos n\alpha + i \sin n\alpha),$$

n being a positive integer.

Squaring the quantity $r(\cos \alpha + i \sin \alpha)$, we have

$$[r(\cos \alpha + i \sin \alpha)]^2 = r^2(\cos^2 \alpha - \sin^2 \alpha + 2i \sin \alpha \cos \alpha)$$
$$= r^2(\cos 2\alpha + i \sin 2\alpha) \text{ by Art. 71.} \qquad (1)$$

Multiplying each member of (1) by $r(\cos \alpha + i \sin \alpha)$ we have

$$[r(\cos \alpha + i \sin \alpha)]^3 = r^3[(\cos 2\alpha \cos \alpha - \sin 2\alpha \sin \alpha)$$
$$+ i(\sin 2\alpha \cos \alpha + \sin \alpha \cos 2\alpha)]$$
$$= r^3(\cos 3\alpha + i \sin 3\alpha) \text{ by Art. 68.} \qquad (2)$$

From (2) we have, similarly,

$$[r(\cos \alpha + i \sin \alpha)]^4 = r^4(\cos 4\alpha + i \sin 4\alpha). \qquad (3)$$

Equations (1), (2), and (3) have the form

$$[r(\cos \alpha + i \sin \alpha)]^n = r^n(\cos n\alpha + i \sin n\alpha). \qquad (4)$$

Multiplying both members of (4) by $r(\cos\alpha + i\sin\alpha)$, we have

$$[r(\cos\alpha + i\sin\alpha)]^{n+1} = r^{n+1}[\cos(n+1)\alpha + i\sin(n+1)\alpha]. \quad (5)$$

Hence, assuming the law expressed in (4) to be true, equation (5) shows that it is true when n is increased by unity. But the law is true for $n=4$, by equation (3); hence it is true for $n=5$. Being true for $n=5$, it must also be true for $n=6$, etc. Hence equation (4) is true for all positive integral values of n.

It can be shown that equation (4) is still true when n is a negative integer, or a fraction.

From equation (4) it is seen that the nth power of a complex number is a complex number having an argument n times the argument of the given number and a modulus equal to the nth power of the given modulus.

Letting $r=1$, equations (1), (2), (3), and (4) become

$$(\cos\alpha + i\sin\alpha)^2 = \cos 2\alpha + i\sin 2\alpha \quad (5)$$
$$(\cos\alpha + i\sin\alpha)^3 = \cos 3\alpha + i\sin 3\alpha \quad (6)$$
$$(\cos\alpha + i\sin\alpha)^4 = \cos 4\alpha + i\sin 4\alpha \quad (7)$$
$$(\cos\alpha + i\sin\alpha)^n = \cos n\alpha + i\sin n\alpha. \quad (8)$$

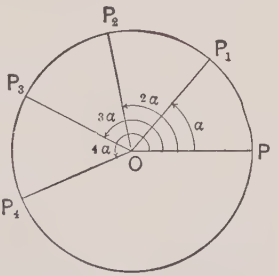
The last equation is known as De Moivre's theorem.

Prob. Show De Moivre's theorem is true when $n = -3$.

106. Geometric Interpretation. Since each of the complex numbers

$$\cos\alpha + i\sin\alpha$$
$$\cos 2\alpha + i\sin 2\alpha$$
$$\cos 3\alpha + i\sin 3\alpha$$
$$\cos 4\alpha + i\sin 4\alpha$$

has a modulus equal to unity, the lines representing these numbers terminate in points lying on the circumference of a circle whose radius is unity. The arguments of any two consecutive integral powers of $\cos\alpha + i\sin\alpha$ differ by α, hence the lines representing any two consecutive powers differ in direction by α.



107. Applications of De Moivre's theorem. De Moivre's theorem may be used to find the various roots of unity, to extract any root of a complex number, to obtain the sine and cosine of any multiple of an angle, and to expand the sine and cosine of an angle in a series of powers of the angle.

108. *To find the cube roots of unity.*

If the cube roots of unity are real numbers, or complex numbers, we may assume, by Art. 104, that

$$\sqrt[3]{1} = r(\cos\alpha + i\sin\alpha). \tag{1}$$

Then, cubing, $1 = r^3(\cos 3\alpha + i\sin 3\alpha)$. (2)

Also $1 = r^3\cos(3\alpha - 2n\pi) + ir^3\sin(3\alpha - 2n\pi)$, (3)

n being an integer.

Equating the real and imaginary parts of equation (3), we have

$$r^3\cos(3\alpha - 2n\pi) = 1$$

and $$r^3\sin(3\alpha - 2n\pi) = 0.$$

These equations of condition are satisfied when

$$3\alpha - 2n\pi = 0 \text{ and } r = 1,$$

whence $$\alpha = \frac{2n\pi}{3}.$$

When $n = 0,\ 1,\ 2,\ 3,\ 4$, etc.

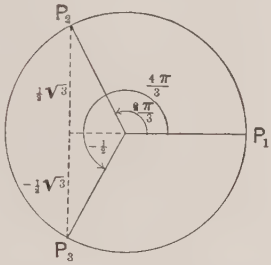
$$\alpha = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{6\pi}{3}, \frac{8\pi}{3}, \text{ etc., respectively.} \tag{4}$$

Every angle in this series is coterminal with either 0, $\frac{2\pi}{3}$, or $\frac{4\pi}{3}$; hence all the values of $\sin\alpha$ and $\cos\alpha$ are obtained by using only the first three values of α. Substituting these values of α in equation (1), and remembering that $r = 1$, we have

$\sqrt[3]{1} = 1$ for $n = 0$,

$\sqrt[3]{1} = -\frac{1}{2} + i\,\frac{1}{2}\sqrt{3}$ for $n = 1$,

$\sqrt[3]{1} = -\frac{1}{2} - i\,\frac{1}{2}\sqrt{3}$ for $n = 2$.



The three cube roots of unity are represented geometrically by P_1, P_2, and P_3.

We can now write the three cube roots of any real number a, for letting a_1, a_2, and a_3 be the cube roots of a, we have

$$a_1 = \sqrt[3]{a} \cdot 1,\quad a_2 = \sqrt[3]{a}\,(-\tfrac{1}{2} + i\,\tfrac{1}{2}\sqrt{3}),\quad a_3 = \sqrt[3]{a}\,(-\tfrac{1}{2} - i\,\tfrac{1}{2}\sqrt{3}),$$

where $\sqrt[3]{a}$ is the arithmetical cube root.

PROBLEM. Show that $(-\frac{1}{2} + i\,\frac{1}{2}\sqrt{3})^3 = 1$, thus justifying the assumption made in Eq. 1.

109. *To find the fifth roots of unity.*

Let $$\sqrt[5]{1} = r(\cos\alpha + i\,\sin\alpha). \tag{1}$$

Then, raising each member to the fifth power,

$$1 = r^5(\cos 5\,\alpha + i\,\sin 5\,\alpha). \tag{2}$$

Also $$1 = r^5\cos(5\,\alpha - 2\,n\pi) + ir^5\sin(5\,\alpha - 2\,n\pi), \tag{3}$$

n being an integer.

Equating the real and imaginary parts of equation (3), we have

$$r^5\cos(5\,\alpha - 2\,n\pi) = 1,$$

and

$$r^5\sin(5\,\alpha - 2\,n\pi) = 0.$$

These equations of condition are satisfied when

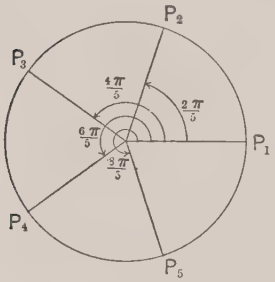
$$5\,\alpha - 2\,n\pi = 0, \text{ and } r = 1.$$

Therefore $$\alpha = \frac{2\,n\pi}{5}.$$

When $n = 0,\ 1,\ 2,\ 3,\ 4,$ etc.

$$\alpha = 0,\ \frac{2\,\pi}{5},\ \frac{4\,\pi}{5},\ \frac{6\,\pi}{5},\ \frac{8\,\pi}{5},\ \text{etc., respectively.} \tag{4}$$

Substituting the values from (4) in equation (1), and remembering that $r = 1$, we have



$$\sqrt[5]{1} = 1 \qquad \text{for } n = 0,$$

$$\sqrt[5]{1} = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5} \quad \text{for } n = 1,$$

$$\sqrt[5]{1} = \cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5} \quad \text{for } n = 2,$$

$$\sqrt[5]{1} = \cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5} \quad \text{for } n = 3,$$

$$\sqrt[5]{1} = \cos\frac{8\pi}{5} + i\sin\frac{8\pi}{5} \quad \text{for } n = 4.$$

110. *To extract the square root of* $a + ib$.

Let $\sqrt{a + ib} = r(\cos\alpha + i\sin\alpha)$. (1)

Squaring, $a + ib = r^2(\cos 2\alpha + i\sin 2\alpha)$. (2)

Also $a + ib = r^2[\cos(2\alpha - 2n\pi) + i\sin(2\alpha - 2n\pi)]$, (3)

where n is any integer.

Equating the real and imaginary parts,

$$r^2\cos(2\alpha - 2n\pi) = a, \tag{4}$$

$$r^2\sin(2\alpha - 2n\pi) = b. \tag{5}$$

Squaring and adding, we have

$$r^4 = a^2 + b^2, \tag{6}$$

or $$r^2 = \sqrt{a^2 + b^2}, \tag{7}$$

and $$r = \sqrt[4]{a^2 + b^2}. \tag{8}$$

Equation (8) gives the value of r in terms of the known numbers a and b.

From equations (4) and (7)

$$\cos(2\alpha - 2n\pi) = \frac{a}{r^2} = \frac{a}{\sqrt{a^2 + b^2}},$$

whence $$2\alpha - 2n\pi = \cos^{-1}\frac{a}{\sqrt{a^2 + b^2}},$$

the quadrant of $2\alpha - 2n\pi$ being determined by (4) and (5).

Then $$\alpha = \tfrac{1}{2}\cos^{-1}\frac{a}{\sqrt{a^2+b^2}} \qquad \text{for } n=0, \tag{9}$$

and $$\alpha = \pi + \tfrac{1}{2}\cos^{-1}\frac{a}{\sqrt{a^2+b^2}} \qquad \text{for } n=1. \tag{10}$$

The values of α for $n = 2, 3, 4$, etc., are coterminal with the values of α in equations (9) or (10). Hence there are only two values for $\sin\alpha$ and $\cos\alpha$; namely those for which $n = 0$ and $n = 1$.

Substituting these values of α in equation (1), we have, finally,

$$\sqrt{a+ib} = \sqrt[4]{a^2+b^2}\left[\cos\left(\tfrac{1}{2}\cos^{-1}\frac{a}{\sqrt{a^2+b^2}}\right) + i\sin\left(\tfrac{1}{2}\cos^{-1}\frac{a}{\sqrt{a^2+b^2}}\right)\right], \tag{11}$$

and $$\sqrt{a+ib} = \sqrt[4]{a^2+b^2}\left[\cos\left(\pi+\tfrac{1}{2}\cos^{-1}\frac{a}{\sqrt{a^2+b^2}}\right) + i\sin\left(\pi+\tfrac{1}{2}\cos^{-1}\frac{a}{\sqrt{a^2+b^2}}\right)\right]. \tag{12}$$

These values of $\sqrt{a+ib}$, while more complicated than $\sqrt{a+ib}$ itself, are nevertheless in the standard form of complex numbers,

$$r(\cos\alpha + i\sin\alpha).$$

111. *To extract the kth root of $a + ib$.*

Let $$\sqrt[k]{a+ib} = r(\cos\alpha + i\sin\alpha). \tag{1}$$

Then $$a+ib = r^k(\cos k\alpha + i\sin k\alpha) \tag{2}$$

$$= r^k[\cos(k\alpha - 2n\pi) + i\sin(k\alpha - 2n\pi)], \tag{3}$$

where n is an integer.

Equating the real and imaginary parts of equation (3), we have

$$r^k\cos(k\alpha - 2n\pi) = a, \tag{4}$$

and $$r^k\sin(k\alpha - 2n\pi) = b. \tag{5}$$

Squaring and adding equations (4) and (5), we have

$$r^{2k} = a^2 + b^2. \tag{6}$$

Then
$$r^k = \sqrt{a^2 + b^2}, \tag{7}$$

and
$$r = \sqrt[2k]{a^2 + b^2}. \tag{8}$$

From equations (4) and (7), we have

$$\cos(k\alpha - 2n\pi) = \frac{a}{r^k} = \frac{a}{\sqrt{a^2 + b^2}},$$

or
$$k\alpha - 2n\pi = \cos^{-1}\frac{a}{\sqrt{a^2 + b^2}},$$

the quadrant of $k\alpha - 2n\pi$ being determined by the signs of (4) and (5).

Therefore
$$\alpha = \frac{2n\pi}{k} + \frac{1}{k}\cos^{-1}\frac{a}{\sqrt{a^2 + b^2}}, \tag{9}$$

whence
$$\alpha = \frac{1}{k}\cos^{-1}\frac{a}{\sqrt{a^2 + b^2}} \text{ for } n = 0, \tag{10}$$

$$\alpha = \frac{2\pi}{k} + \frac{1}{k}\cos^{-1}\frac{a}{\sqrt{a^2 + b^2}} \text{ for } n = 1, \tag{11}$$

$$\alpha = \frac{4\pi}{k} + \frac{1}{k}\cos^{-1}\frac{a}{\sqrt{a^2 + b^2}} \text{ for } n = 2. \tag{12}$$

. . . . .

Substituting the values for r and α as found in equations (8), (10), (11), (12), etc., in equation (1), we have the several kth roots of $a + ib$.

When k is an integer there are k roots. Consecutive values of α, as given by (9), differ by $\frac{2\pi}{k}$, hence all the values of α after the kth are coterminal with one of the first k values.

112. *To express* $\sin n\alpha$ *and* $\cos n\alpha$ *in terms of* $\sin\alpha$ *and* $\cos\alpha$.

We have

$$\cos n\alpha + i \sin n\alpha = (\cos\alpha + i \sin\alpha)^n.$$

Expanding the second member by the binomial theorem and equating the real and imaginary parts, the problem is solved.

Thus, for $\sin 4\alpha$ and $\cos 4\alpha$ we have

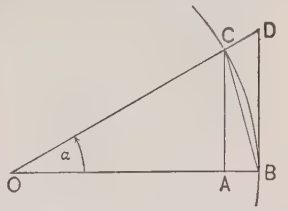
$\cos 4\alpha + i \sin 4\alpha = (\cos\alpha + i \sin\alpha)^4 = \cos^4\alpha,$

$+ 4i \cos^3\alpha \sin\alpha - 6\cos^2\alpha \sin^2\alpha - 4i \cos\alpha \sin^3\alpha + \sin^4\alpha.$

Therefore $\sin 4\alpha = 4\cos^3\alpha \sin\alpha - 4\cos\alpha \sin^3\alpha.$

and $\cos 4\alpha = \cos^4\alpha - 6\cos^2\alpha \sin^2\alpha + \sin^4\alpha.$

113. Comparison of the values of sin α, α, and tan α, α being an acute angle. Let α be any acute angle expressed in radians. With the vertex O as a center and any radius OB, describe the arc BC. Draw AC and BD perpendicular to OB, and join B with C.



The area of the triangle OBC is less than the area of the sector OBC, and the sector OBC is less than the triangle OBD.

But since

$AC = OC \sin\alpha$, $BD = OB \tan\alpha$, and arc $BC = OB \cdot \alpha$, Art. 17,

the area of the triangle OBC is equal to $\frac{1}{2} \cdot OB \cdot OC \sin\alpha$,

the area of the sector OBC is equal to $\frac{1}{2} \cdot OB \cdot OB \cdot \alpha$,

and the area of the triangle OBD is equal to $\frac{1}{2} \cdot OB \cdot OB \tan\alpha$.

Hence

$$\tfrac{1}{2} \cdot OB \cdot OC \sin\alpha < \tfrac{1}{2} \cdot OB \cdot OB \cdot \alpha < \tfrac{1}{2} \cdot OB \cdot OB \tan\alpha,$$

or $$\sin\alpha < \alpha < \tan\alpha.$$

114. Value of $\frac{\sin\alpha}{\alpha}$ for small values of α. In Art. 40 we saw that $\sin\alpha$ approaches 0 as α approaches 0. The value of $\frac{\sin\alpha}{\alpha}$ therefore approaches $\frac{0}{0}$ as α approaches 0.

But from the previous article

$$\sin\alpha < \alpha < \tan\alpha.$$

Dividing by $\sin\alpha$, we have

$$1 < \frac{\alpha}{\sin\alpha} < \frac{1}{\cos\alpha}$$

or

$$1 > \frac{\sin\alpha}{\alpha} > \cos\alpha.$$

Since $\frac{\sin\alpha}{\alpha}$ lies between 1 and $\cos\alpha$, $\frac{\sin\alpha}{\alpha}$ must approach 1 as α approaches 0, since $\cos\alpha$ approaches 1.

Then for very small angles $\sin\alpha$ may be replaced by α, expressed in radians. The error thus introduced is so small that it may be neglected in many problems. Thus, to five decimal places,

$\sin 1^\circ = 0.01745$	$1^\circ = 0.01745$ radians
$\sin 2^\circ = 0.03490$	$2^\circ = 0.03491$ radians
$\sin 3^\circ = 0.05234$	$3^\circ = 0.05236$ radians
$\sin 4^\circ = 0.06976$	$4^\circ = 0.06981$ radians

115. *To develop* $\sin\alpha$ *and* $\cos\alpha$ *in terms of* α.

By De Moivre's theorem,

$$\cos n\theta + i\sin n\theta = (\cos\theta + i\sin\theta)^n.$$

On expanding the second member by the binomial theorem, we have

$$\cos n\theta + i\sin n\theta = \cos^n\theta + in\cos^{n-1}\theta\sin\theta$$
$$-\frac{n(n-1)}{\lfloor 2}\cos^{n-2}\theta\sin^2\theta - i\frac{n(n-1)(n-2)}{\lfloor 3}\cos^{n-3}\theta\sin^3\theta$$
$$+\frac{n(n-1)(n-2)(n-3)}{\lfloor 4}\cos^{n-4}\theta\sin^4\theta + \cdots. \qquad (1)$$

Equating the imaginary parts, we have

$$\sin n\theta = n\cos^{n-1}\theta\sin\theta - \frac{n(n-1)(n-2)}{\lfloor 3}\cos^{n-3}\theta\sin^3\theta$$
$$+\frac{n(n-1)(n-2)(n-3)(n-4)}{\lfloor 5}\cos^{n-5}\theta\sin^5\theta - \cdots. \qquad (2)$$

Let $n\theta = \alpha$. Then equation (2) may be written

$$\sin\alpha = \frac{\alpha}{\theta}\cos^{n-1}\theta\sin\theta - \frac{\frac{\alpha}{\theta}\left(\frac{\alpha}{\theta}-1\right)\left(\frac{\alpha}{\theta}-2\right)}{\lfloor 3}\cos^{n-3}\theta\sin^3\theta$$
$$+\frac{\frac{\alpha}{\theta}\left(\frac{\alpha}{\theta}-1\right)\left(\frac{\alpha}{\theta}-2\right)\left(\frac{\alpha}{\theta}-3\right)\left(\frac{\alpha}{\theta}-4\right)}{\lfloor 5}\cos^{n-5}\theta\sin^5\theta - \cdots, \quad (3)$$

$$\text{or } \sin\alpha = \alpha\cos^{n-1}\theta\left(\frac{\sin\theta}{\theta}\right) - \frac{\alpha(\alpha-\theta)(\alpha-2\theta)}{\lfloor 3}\cos^{n-3}\theta\left(\frac{\sin^3\theta}{\theta^3}\right)$$
$$+\frac{\alpha(\alpha-\theta)(\alpha-2\theta)(\alpha-3\theta)(\alpha-4\theta)}{\lfloor 5}\cos^{n-5}\theta\left(\frac{\sin^5\theta}{\theta^5}\right) - \cdots. \quad (4)$$

Let α remain constant while n increases indefinitely. Then θ necessarily decreases indefinitely, since $n\theta = \alpha$, a constant. By Art. 114, when θ approaches 0, $\frac{\sin\theta}{\theta}$ approaches 1, and $\cos\theta$ approaches 1. Making these substitutions in equation (4), we have

$$\sin\alpha = \alpha - \frac{\alpha^3}{\lfloor 3} + \frac{\alpha^5}{\lfloor 5} - \frac{\alpha^7}{\lfloor 7} + \cdots.$$

Equating the real parts of equation (1), we have

$$\cos n\theta = \cos^n\theta - \frac{n(n-1)}{\lfloor 2}\cos^{n-2}\theta\sin^2\theta$$
$$+\frac{n(n-1)(n-2)(n-3)}{\lfloor 4}\cos^{n-4}\theta\sin^4\theta - \cdots. \quad (5)$$

By the same process as above, equation (5) becomes

$$\cos\alpha = 1 - \frac{\alpha^2}{\lfloor 2} + \frac{\alpha^4}{\lfloor 4} - \frac{\alpha^6}{\lfloor 6} + \cdots.$$

The series for $\sin\alpha$ and $\cos\alpha$ are convergent for all finite values of α.* They enable us to compute the sine and cosine of any angle. It is then possible to construct a table of natural functions, from which the logarithmic functions may be obtained. In using these series α must, of course, be expressed in radian measure.

* See any College Algebra on the convergency of series.

116. EXAMPLES

1. Find the four fourth roots of unity by De Moivre's theorem.

2. Find the six sixth roots of unity by De Moivre's theorem.

3. Find the square root of $5 - 3\,i$.

SOLUTION. — Let $\sqrt{5 - 3\,i} = r\,(\cos\alpha + i\sin\alpha)$. (1)

Then $5 - 3\,i = r^2\,(\cos 2\,\alpha + i\sin 2\,\alpha)$

$= r^2[\cos(2\,\alpha - 2\,n\pi) + i\sin(2\,\alpha - 2\,n\pi)]$.

Equating the real and the imaginary parts,

$$r^2\cos(2\,\alpha - 2\,n\pi) = 5, \tag{2}$$

and $$r^2\sin(2\,\alpha - 2\,n\pi) = -3. \tag{3}$$

Squaring and adding (2) and (3), we have

$$r^4 = 34,\ \therefore\ r^2 = \sqrt{34},\ \text{and}\ r = \sqrt[4]{34}. \tag{4}$$

Then $$\cos(2\,\alpha - 2\,n\pi) = \frac{5}{\sqrt{34}},$$

and $$2\,\alpha - 2\,n\pi = 329°\ 2', \tag{5}$$

the quadrant being determined by (2) and (3).

When $$n = 0,\ \alpha = 164°\ 31'; \tag{6}$$

$$n = 1,\ \alpha = 344°\ 31'. \tag{7}$$

Substituting from (4) and (6) in (1), we have

$$\sqrt{5 - 3\,i} = -2.3271 + .6446\,i.$$

Substituting from (4) and (7) in (1), we have

$$\sqrt{5 - 3\,i} = 2.3271 - .6446\,i.$$

4. Find the square root of $3 + 4\,i$.

5. Find the square root of $-3 - 4\,i$.

6. Find the square root of $1 + 2\,i$.

7. Find the square root of i.

8. Find the square root of $-i$.

9. Find the cube root of $2 - 3\,i$.

SOLUTION. — Let $\sqrt[3]{2 - 3\,i} = r\,(\cos\alpha + i\sin\alpha)$. (1)

Then $2 - 3\,i = r^3\,(\cos 3\,\alpha + i\sin 3\,\alpha)$

$= r^3\,[\cos(3\,\alpha - 2\,n\pi) + i\sin\,(3\,\alpha - 2\,n\pi)]$.

Equating the real and imaginary parts,

$$r^3 \cos(3\alpha - 2n\pi) = 2; \tag{2}$$

and

$$r^3 \sin(3\alpha - 2n\pi) = -3. \tag{3}$$

Squaring and adding (2) and (3), we have

$$r^6 = 13, \ \therefore \ r^3 = \sqrt{13}, \text{ and } r = \sqrt[6]{13}. \tag{4}$$

Then

$$\cos(3\alpha - 2n\pi) = \frac{2}{\sqrt{13}},$$

and

$$3\alpha - 2n\pi = 303^\circ 41'; \tag{5}$$

the quadrant being determined by (2) and (3).

When

$$n = 0, \ \alpha = 101^\circ 14'; \tag{6}$$

$$n = 1, \ \alpha = 221^\circ 14'; \tag{7}$$

$$n = 2, \ \alpha = 341^\circ 14'. \tag{8}$$

Substituting from (4) and (6) in (1), we have

$$\sqrt[3]{2 - 3i} = -.2987 + 1.5041\,i.$$

Substituting from (4) and (7) in (1), we have

$$\sqrt[3]{2 - 3i} = -1.1530 - 1.0107\,i.$$

Substituting from (4) and (8) in (1), we have

$$\sqrt[3]{2 - 3i} = 1.4518 - .4933\,i.$$

We have thus found the three cube roots of $2 - 3i$.

10. Find the cube root of $1 + i$.

11. Find the cube root of $-1 + i$.

12. Find the cube root of $2 + 3i$.

13. Find the values of $\sin 3x$ and $\cos 3x$ in terms of $\sin x$ and $\cos x$.

14. Find the values of $\sin 5x$ and $\cos 5x$ in terms of $\sin x$ and $\cos x$.

15. Prove by De Moivre's theorem that

$$\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}, \text{ also } \cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}.$$

SUGGESTION. $\cos\alpha + i\sin\alpha = \left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\right)^2$.

16. Show that $\cos\alpha = \cos^3\frac{\alpha}{3} - 3\cos\frac{\alpha}{3}\sin^2\frac{\alpha}{3}$,

$$\sin\alpha = 3\cos^2\frac{\alpha}{3}\sin\frac{\alpha}{3} - \sin^3\frac{\alpha}{3}.$$

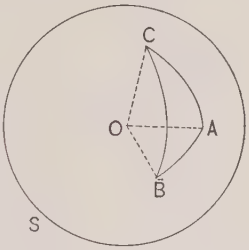
SPHERICAL TRIGONOMETRY

CHAPTER X

FUNDAMENTAL FORMULAS

117. The spherical triangle.* Spherical trigonometry treats of the relations between the various parts of a spherical triangle and of the methods of solving the spherical triangle.

The sides of a spherical triangle are always arcs of great circles.



Having given a spherical triangle ABC, situated upon a sphere S, a triedral angle O–ABC may be formed by passing planes through O, the center of the sphere, and through the sides of the triangle.

It is known from geometry that the arc AB and the angle AOB contain the same number of degrees, and that the angle CAB and the diedral angle C–AO–B contain the same number of degrees.

The sides and angles of a spherical triangle may have any values between 0° and 360°.

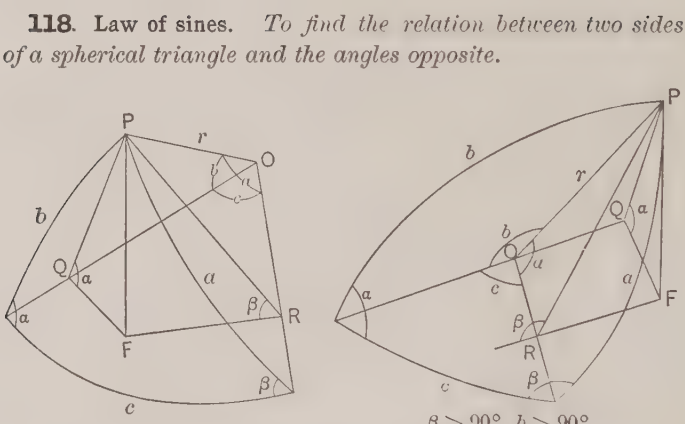
A triangle having one or more of its parts greater than 180° is called a **general spherical triangle.**

A triangle having each of its parts less than 180° is called **a spherical triangle.**

We shall consider only those triangles whose parts are each less than 180°.

* For a course on the right spherical triangle read Arts. 117 and 126 and from Art. 128 to end of Chapter XI.

118. Law of sines. *To find the relation between two sides of a spherical triangle and the angles opposite.*



$\beta > 90°$, $b > 90°$.

Given a spherical triangle and its accompanying triedral angle; through the vertex P pass planes perpendicular to OR and OQ, intersecting in PF a line perpendicular to the plane ORQ.

Then $$\sin\alpha = \frac{FP}{QP} \text{ and } \sin\beta = \frac{FP}{RP},$$

therefore $$\frac{\sin\alpha}{\sin\beta} = \frac{RP}{QP}.$$

Also $$\sin\alpha = \frac{RP}{OP} \text{ and } \sin b = \frac{QP}{OP},$$

therefore $$\frac{\sin\alpha}{\sin b} = \frac{RP}{QP}.$$

Hence $$\frac{\sin\alpha}{\sin\beta} = \frac{\sin a}{\sin b}.$$

Likewise $$\frac{\sin\beta}{\sin\gamma} = \frac{\sin b}{\sin c}.$$

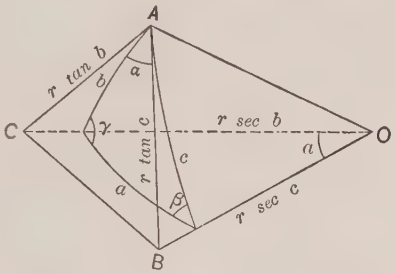
Uniting these equations, we have

$$\frac{\sin a}{\sin\alpha} = \frac{\sin b}{\sin\beta} = \frac{\sin c}{\sin\gamma}. \tag{1}$$

This demonstration applies to similar figures drawn for all possible cases,* hence the theorem is always true.

119. Law of cosines. *To find the relation between the three sides and an angle.*

Given a spherical triangle and its accompanying triedral angle; pass a plane through the vertex A perpendicular to OA, intersecting the planes of the triedral angle in the lines AB, AC, and BC.



$b < 90°$, $c < 90°$; $a < 180°$, $\alpha < 180°$.

Then $AB = r \tan c$, $OB = r \sec c$, $AC = r \tan b$, $OC = r \sec b$. From the triangle OBC, by Art. 90,

$$\overline{BC}^2 = (r \sec b)^2 + (r \sec c)^2 - 2\,(r \sec b)\,(r \sec c)\cos a. \quad (1)$$

Likewise from the triangle ABC,

$$\overline{BC}^2 = (r \tan b)^2 + (r \tan c)^2 - 2\,(r \tan b)\,(r \tan c)\cos \alpha. \quad (2)$$

Subtracting (2) from (1), we have

$$0 = r^2(\sec^2 b - \tan^2 b) + r^2(\sec^2 c - \tan^2 c) - 2\,r^2 \sec b \sec c \cos a + 2\,r^2 \tan b \tan c \cos \alpha,$$

which reduces to

$$0 = 1 - \sec b \sec c \cos a + \tan b \tan c \cos \alpha,$$

or $$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha. \quad (3)$$

Also $$\cos b = \cos c \cos a + \sin c \sin a \cos \beta, \quad (4)$$

and $$\cos c = \cos a \cos b + \sin a \sin b \cos \gamma. \quad (5)$$

* The following seven cases can arise:

(1) 3 sides $< 90°$, 3 angles $< 90°$
(2) 3 sides $< 90°$, 2 angles $< 90°$
(3) 2 sides $< 90°$, 2 angles $< 90°$
(4) 1 side $< 90°$, 2 angles $< 90°$
(5) 1 side $< 90°$, 1 angle $< 90°$
(6) 1 side $< 90°$, 0 angle $< 90°$
(7) 0 side $< 90°$, 0 angle $< 90°$.

It is to be understood that all parts not mentioned are greater than 90°.

120. *To extend the law of cosines.*

In the derivation of the formula

$$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha,$$

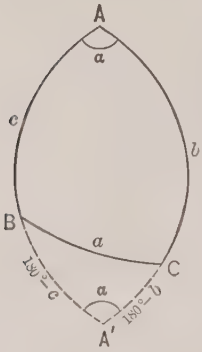
b and c were less than 90°, while a and α were less than 180°.

To show that the formula is true in general, it is necessary to consider two additional cases:

1st. Both b and c greater than 90°.

2d. Either b or c greater than 90°.

Since β and γ do not enter the formula, they may have any value consistent with the above conditions.



First. Given the triangle ABC in which $b > 90°$ and $c > 90°$.

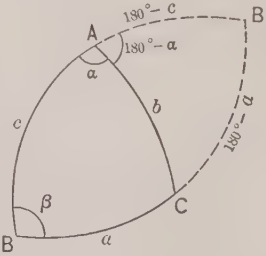
Extend the sides b and c of the triangle ABC, forming the lune whose angle is α. Then in the triangle $A'BC$, the sides $A'B$ and $A'C$ are each less than 90°, hence by Art. 119

$$\cos a = \cos (180° - b) \cos (180° - c) + \sin (180° - b) \sin (180° - c) \cos \alpha,$$

or $$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha.$$

Hence the law of cosines holds when both b and c are greater than 90°.

Second. Given the triangle ABC, in which $b < 90°$ and $c > 90°$.



Extend the sides a and c of the triangle ABC, forming the lune whose angle is β. Then in the triangle $AB'C$, the sides AB' and AC are each less than 90°, and the angle $B'AC$ is equal to $180° - \alpha$.

Then, by Art. 119,

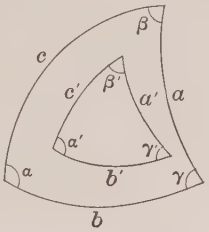
$$\cos (180° - a) = \cos b \cos (180° - c) + \sin b \sin (180° - c) \cos (180° - \alpha),$$

or $$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha.$$

Hence the law of cosines holds when either b or c is greater than 90°. The law of cosines is therefore true in general.

121. *To find the relation between one side and the three angles.*

Let a, b, and c be the sides of any spherical triangle, and a', b', c' the sides of its polar triangle.



Applying the law of cosines to the polar triangle, we have

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos \alpha'.$$

But

$$a' = 180° - \alpha, \quad b' = 180° - \beta, \text{ etc.}$$

Therefore

$$\cos(180° - \alpha) = \cos(180° - \beta)\cos(180° - \gamma) + \sin(180° - \beta)\sin(180° - \gamma)\cos(180° - a),$$

or
$$\cos \alpha = -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cos a. \quad (1)$$

Also
$$\cos \beta = -\cos \gamma \cos \alpha + \sin \gamma \sin \alpha \cos b, \quad (2)$$

and
$$\cos \gamma = -\cos \alpha \cos \beta + \sin \alpha \sin \beta \cos c. \quad (3)$$

122. **The sine-cosine law.** *To find the relation between three sides and two angles.*

We have

$$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha, \quad (1)$$

and
$$\cos b = \cos c \cos a + \sin c \sin a \cos \beta. \quad (2)$$

Eliminating $\cos a$ by substitution,

$$\cos b = \cos b \cos^2 c + \sin b \sin c \cos c \cos \alpha + \sin c \sin a \cos \beta.$$

Transposing and factoring,

$$\cos b(1 - \cos^2 c) = \sin b \sin c \cos c \cos \alpha + \sin a \sin c \cos \beta.$$

Replacing $(1 - \cos^2 c)$ by $\sin^2 c$, and dividing by $\sin c$, we have

$$\cos b \sin c = \sin b \cos c \cos \alpha + \sin a \cos \beta.$$

Rearranging terms,

$$\sin a \cos \beta = \cos b \sin c - \sin b \cos c \cos \alpha. \quad (3)$$

Also $\sin b \cos \gamma = \cos c \sin a - \sin c \cos a \cos \beta,$ (4)

and $\sin c \cos \alpha = \cos a \sin b - \sin a \cos b \cos \gamma.$ (5)

Interchanging β and γ and consequently b and c, we have from (3)

$$\sin a \cos \gamma = \cos c \sin b - \sin c \cos b \cos \alpha. \quad (6)$$

Similarly from (4)

$$\sin b \cos \alpha = \cos a \sin c - \sin a \cos c \cos \beta, \quad (7)$$

and from (5)

$$\sin c \cos \beta = \cos b \sin a - \sin b \cos a \cos \gamma. \quad (8)$$

123. *To find the relation between two sides and the three angles.*

Applying the sine-cosine law to the polar of the given triangle, we have

$$\sin a' \cos \beta' = \cos b' \sin c' - \sin b' \cos c' \cos \alpha'.$$

But $a' = 180° - \alpha$, $\beta' = 180° - b$, etc.

Then

$$\sin (180° - \alpha) \cos (180° - b) = \cos (180° - \beta) \sin (180° - \gamma) - \sin (180° - \beta) \cos (180° - \gamma) \cos (180° - a).$$

Therefore $\sin \alpha \cos b = \cos \beta \sin \gamma + \sin \beta \cos \gamma \cos a.$ (1)

Also $\sin \beta \cos c = \cos \gamma \sin \alpha + \sin \gamma \cos \alpha \cos b,$ (2)

$\sin \gamma \cos a = \cos \alpha \sin \beta + \sin \alpha \cos \beta \cos c,$ (3)

$\sin \alpha \cos c = \cos \gamma \sin \beta + \sin \gamma \cos \beta \cos a,$ (4)

$\sin \beta \cos a = \cos \alpha \sin \gamma + \sin \alpha \cos \gamma \cos b,$ (5)

$\sin \gamma \cos b = \cos \beta \sin \alpha + \sin \beta \cos \alpha \cos c.$ (6)

124. *To find the relation between two sides and two angles, one of the angles being included between the given sides.*

From Art. 122

$$\sin a \cos \beta = \cos b \sin c - \sin b \cos c \cos \alpha.$$

Dividing this equation by

$$\sin a \sin \beta = \sin b \sin \alpha, \qquad \text{Art. 118}$$

member by member, we have

$$\cot \beta = \frac{\cot b \sin c - \cos c \cos \alpha}{\sin \alpha}.$$

Therefore $\sin \alpha \cot \beta = \cot b \sin c - \cos c \cos \alpha.$ (1)

Similarly $\sin \beta \cot \gamma = \cot c \sin a - \cos a \cos \beta,$ (2)

and $\sin \gamma \cot \alpha = \cot a \sin b - \cos b \cos \gamma.$ (3)

Interchanging α and β and consequently a and b, we have from (1)

$$\sin \beta \cot \alpha = \cot a \sin c - \cos c \cos \beta.$$ (4)

Similarly from (2)

$$\sin \gamma \cot \beta = \cot b \sin a - \cos a \cos \gamma,$$ (5)

and from (3)

$$\sin \alpha \cot \gamma = \cot c \sin b - \cos b \cos \alpha.$$ (6)

125. Formulas independent of the radius of the sphere. It will be noticed that r, the radius of the sphere, does not enter any of the formulas thus far developed; hence they are independent of the radius of the sphere, and may be applied, without modification, to triangles on any sphere. The fact is serviceable in problems of Astronomy and Geodesy where the formulas are applied to triangles situated upon the celestial and terrestrial spheres.

CHAPTER XI

SPHERICAL RIGHT TRIANGLE

126. The spherical right triangle is a spherical triangle one of whose angles is a right angle. The other parts may have any values between 0° and 180°.

In the work that follows the angle γ will be taken as the right angle.

127. Formulas for the solution of right triangles. The formulas for the solution of any spherical right triangle are obtained from the general formulas of Chapter X by letting $\gamma = 90°$. We thus have

from eq. (1) Art. 118	$\sin a = \sin c \sin \alpha$
from eq. (1) Art. 118	$\sin b = \sin c \sin \beta$
from eq. (5) Art. 119	$\cos c = \cos a \cos b$
from eq. (1) Art. 121	$\cos \alpha = \sin \beta \cos a$
from eq. (2) Art. 121	$\cos \beta = \sin \alpha \cos b$
from eq. (3) Art. 121	$\cos c = \cot \alpha \cot \beta$
from eq. (2) Art. 124	$\cos \beta = \tan a \cot c$
from eq. (3) Art. 124	$\sin b = \cot \alpha \tan a$
from eq. (5) Art. 124	$\sin a = \cot \beta \tan b$
from eq. (6) Art. 124	$\cos \alpha = \tan b \cot c.$

128. Direct geometric derivation of formulas. Let a and b be the sides of a given spherical right triangle, α and β the angles opposite, and c its hypotenuse.

Let O–ABC be its accompanying triedral angle.

Through the vertex B pass a plane perpendicular to OA, intersecting the planes of the triedral angle in AB, BC, and CA.

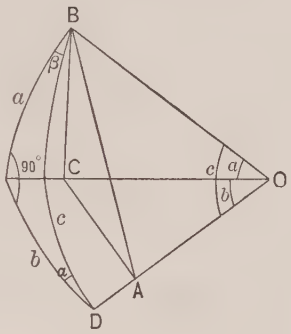
Then $\angle BAC=\alpha$, $\angle BOC=a$, $\angle AOC=b$, and $\angle AOB=c$. Also $\angle BCA$, $\angle BCO$, $\angle CAO$, $\angle BAO$ are each a right angle.

From the triangle ABC we have

$$\sin\alpha=\frac{CB}{AB}=\frac{\frac{CB}{OB}}{\frac{AB}{OB}}=\frac{\sin a}{\sin c}, \quad (1)$$

$$\cos\alpha=\frac{AC}{AB}=\frac{\frac{AC}{OA}}{\frac{AB}{OA}}=\frac{\tan b}{\tan c}, \quad (2)$$

$$\tan\alpha=\frac{CB}{AC}=\frac{\frac{CB}{OC}}{\frac{AC}{OC}}=\frac{\tan a}{\sin b}. \quad (3)$$



By interchanging α and β and consequently a and b, or by passing a plane through $D\perp$ to OB and proceeding as above, we have

$$\sin\beta=\frac{\sin b}{\sin c}, \quad (4)$$

$$\cos\beta=\frac{\tan a}{\tan c}, \quad (5)$$

$$\tan\beta=\frac{\tan b}{\sin a}. \quad (6)$$

Also from the figure,

$$\cos c=\frac{OA}{OB}=\frac{\frac{OA}{OC}}{\frac{OB}{OC}}=\frac{\cos b}{\sec a}=\cos a\cos b. \quad (7)$$

Dividing (1) by (5) and reducing by (7) we have

$$\sin\alpha=\frac{\cos\beta}{\cos b}, \quad (8)$$

and by interchange of letters as before

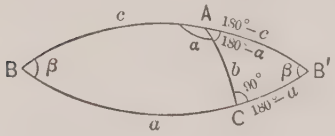
$$\sin\beta=\frac{\cos\alpha}{\cos a}. \quad (9)$$

Substituting the values of $\cos a$ and $\cos b$ from (8) and (9) in (7), we have, after reduction,

$$\cos c = \cot \alpha \cot \beta. \tag{10}$$

In the demonstration of formulas (1) to (10) the parts α, β, a, b, and c were assumed less than 90°. To show that these formulas are true in general, it is necessary to consider two additional cases: viz. (1) when one side and the hypotenuse are each greater than 90°, and (2) when the two sides are each greater than 90°.

1. *When* $a > 90°$ *and* $c > 90°$.



Let ABC be the given spherical right triangle. Draw the lune BB'. Then in the right triangle $AB'C$ each part is less than 90°, and formulas (1) to (10) are applicable.

From (1) $$\sin (180° - a) = \frac{\sin (180° - \alpha)}{\sin (180° - c)}$$

or $$\sin \alpha = \frac{\sin a}{\sin c},$$

which shows that (1) holds when a and c are each greater than 90°.

From (2) $$\cos (180° - a) = \frac{\tan b}{\tan (180° - c)}$$

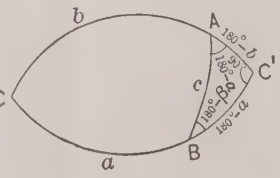
or $$\cos \alpha = \frac{\tan b}{\tan c},$$

which shows that (2) also holds in this case.

Similarly it may be shown that formulas (3) to (10) hold when a and c are each greater than 90°.

2. *When* $a > 90°$ *and* $b > 90°$.

Let ABC be the given spherical right triangle. Draw the lune CC'. Then in the right triangle ABC' each part is less than 90° and formulas (1) to (10) are applicable.



$$\text{From (1)} \quad \sin(180^\circ - a) = \frac{\sin(180^\circ - a)}{\sin c}$$

or $$\sin a = \frac{\sin a}{\sin c},$$

which shows that (1) holds when a and b are each greater than 90°.

$$\text{From (2)} \quad \cos(180^\circ - a) = \frac{\tan(180^\circ - b)}{\tan c}$$

or $$\cos a = \frac{\tan b}{\tan c},$$

which shows that (2) also holds in this case.

Similarly it may be shown that formulas (3) to (10) hold when a and b are each greater than 90°.

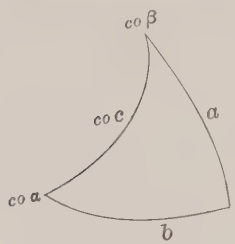
129. Sufficiency of formulas. It will be noticed that the ten formulas of Arts. 127 and 128 contain all possible combinations of the five parts of a spherical right triangle, taken three at a time; hence they are sufficient to solve any spherical right triangle directly from two given parts.

130. Comparison of formulas of plane and spherical right triangles. By rearranging the formulas of the previous article, the analogy between the formulas of the plane and the spherical right triangles is made apparent.

In Plane Right Triangles*	In Spherical Right Triangles
$\sin \alpha = \frac{a}{c}$ $\quad \sin \beta = \frac{b}{c}$	$\sin \alpha = \frac{\sin a}{\sin c}$ $\quad \sin \beta = \frac{\sin b}{\sin c}$
$\cos \alpha = \frac{b}{c}$ $\quad \cos \beta = \frac{a}{c}$	$\cos \alpha = \frac{\tan b}{\tan c}$ $\quad \cos \beta = \frac{\tan a}{\tan c}$
$\tan \alpha = \frac{a}{b}$ $\quad \tan \beta = \frac{b}{a}$	$\tan \alpha = \frac{\tan a}{\sin b}$ $\quad \tan \beta = \frac{\tan b}{\sin a}$
$\sin \alpha = \cos \beta$ $\quad \sin \beta = \cos \alpha$	$\sin \alpha = \frac{\cos \beta}{\cos b}$ $\quad \sin \beta = \frac{\cos \alpha}{\cos a}$
$c^2 = a^2 + b^2$	$\cos c = \cos a \cos b$
$1 = \cot \alpha \cot \beta.$	$\cos c = \cot \alpha \cot \beta.$

*The above comparison is taken from Chauvenet's "Plane and Spherical Trigonometry."

131. Napier's Rules. The ten formulas used in the solution of spherical right triangles can all be expressed by means of two rules, known as Napier's rules of circular parts.

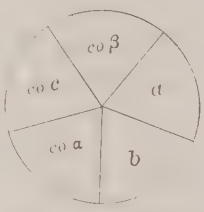


Napier's circular parts are the sides a and b, the complements of the angles opposite or $90° - \alpha$, $90° - \beta$, and the complement of the hypotenuse or $90° - c$.

They are usually written

$$a, \quad b, \quad \text{co}\,\alpha, \quad \text{co}\,\beta, \quad \text{co}\,c.$$

It will be noticed that the right angle is not one of the circular parts.



Let the five circular parts be placed in the sectors of a circle in the order in which they occur in the triangle. Whenever any three parts are considered, it is always possible to select one of them in such a manner that the other two parts will either be *adjacent* to this part, or *opposite* this part. The part having the other two parts adjacent to it or opposite it is called the **middle part.**

Thus let co α, b, and a be the parts under consideration. Then b is the middle part and co α and a are adjacent parts.

If co c, co β, and b are the parts under consideration, b is the middle part and co c and co β are opposite parts.

Napier's rules may now be stated as follows:

The sine of the middle part is equal to the product of the cosines of the opposite parts.

*The sine of the middle part is equal to the product of the tangents of the adjacent parts.**

* To associate *cosine* with *opposite* and *tangent* with *adjacent*, it may be noticed that the words *cosine* and *opposite* have the same vowels; likewise the words *tangent* and *adjacent*.

132. Theorem. *In a spherical right triangle, a side and the angle opposite terminate in the same quadrant.*

From the equation

$$\cos \alpha = \cos a \sin \beta$$

it is seen that $\cos \alpha$ and $\cos a$ must always have the same sign, since $\sin \beta$ is always positive. Hence α and a terminate in the same quadrant.

133. Theorem. *Of the three parts a, b, c, if any two terminate in the same quadrant, the third terminates in the first quadrant; if any two terminate in different quadrants, the third terminates in the second quadrant.*

This follows directly from the equation

$$\cos c = \cos a \cos b$$

by noticing that if any two of the quantities $\cos a$, $\cos b$, and $\cos c$ have like signs, the third is positive; if any two have unlike signs, the third is negative.

134. Two parts determine a triangle. In order to solve a spherical right triangle two parts, in addition to the right angle, must be given. *Each of the required parts should be obtained directly from the given parts.*

Thus, given

$$b = 50^\circ, \quad c = 110^\circ$$

we have

$$\cos a = \frac{\cos 110^\circ}{\cos 50^\circ},$$

$$\cos \alpha = \tan 50^\circ \cot 110^\circ,$$

$$\sin \beta = \frac{\sin 50^\circ}{\sin 110^\circ},$$

using the formulas for spherical right triangles, or Napier's Rules. If, in the solution of a problem, the sine of any required part is found to be negative, no triangle is possible, since no part of a spherical triangle can be greater than 180°.

Likewise if the logarithmic sine or cosine of any required part is found to be greater than zero, the triangle is impossible, since no sine or cosine is numerically greater than unity.

135. The quadrant of any required part. Since the parts of a spherical triangle may have any value between 0° and 180°, it is always necessary to determine whether the required parts are greater or less than 90°. This can be done by the theorems of Arts. 132 and 133.

Thus, given

$$b = 50°, \quad c = 110°,$$

we have $\quad a > 90°, \ \alpha > 90°, \text{ and } \beta < 90°.$

The quadrant in which any required part terminates may also be determined from the formula used in calculating that part, by observing the signs of the functions involved. But when the unknown part is determined from its sine, the part terminates in both quadrants, giving two solutions, unless limited by the theorems of Arts. 132 and 133.

Thus, given

$$b = 50°, \ c = 110°,$$

by writing the signs of each function above the function, we have

$$\overset{-}{\cos}\, a = \frac{\overset{-}{\cos}\, 110°}{\overset{+}{\cos}\, 50°},$$

$$\overset{-}{\cos}\, \alpha = \overset{+}{\tan}\, 50° \ \overset{-}{\cot}\, 110°,$$

$$\overset{+}{\sin}\, \beta = \frac{\overset{+}{\sin}\, 50°}{\overset{+}{\sin}\, 110°}.$$

Then $a > 90°$ and $\alpha > 90°$, since their cosines are negative, and $\beta < 90°$ by Art. 132.

136. Check formula. The formula containing the three computed parts may always be used as a check formula.

Thus, having given

$$b = 50°, \ c = 110°,$$

the check formula is

$$\cos \alpha = \cos a \sin \beta.$$

137. Solution of a right triangle.

Given $b = 77° 35' 16''$ and $\alpha = 112° 19' 42''$

$$\tan a = \frac{\overset{+}{\sin b}}{\overset{-}{\cot \alpha}}$$

b	$77° 35' 16''$
α	$112° 19' 42''$
log sin b	9.98973 − 10
log cot α	9.61353 − 10
log tan a	0.37620
	$67° 11' 30''$
a	$112° 48' 30''$

$$\overset{+}{\cos \beta} = \overset{+}{\cos b}\ \overset{+}{\sin \alpha}$$

log cos b	9.33233 − 10
log sin α	9.96615 − 10
log cos β	9.29848 − 10
β	$78° 31' 53''$

$$\overset{-}{\cot c} = \frac{\overset{-}{\cos \alpha}}{\overset{+}{\tan b}}$$

log cos α	9.57969 − 10
log tan b	0.65740
log cot c	8.92229 − 10
	$85° 13' 13''$
c	$94° 46' 47''$

Check

$$\overset{+}{\cos \beta} = \overset{-}{\tan a}\ \overset{-}{\cot c}$$

log tan a	0.37620
log cot c	8.92229 − 10
log cos β	9.29849 − 10

138. Two solutions or the ambiguous case. Whenever a side and the angle opposite are given, there are two solutions.

Thus if a and α are given, the only formulas by which b, β, and c can be determined are

$$\sin b = \tan a \cot \alpha,$$

$$\sin \beta = \frac{\cos \alpha}{\cos a},$$

$$\sin c = \frac{\sin a}{\sin \alpha}.$$

Since the unknown parts are obtained from their sines, each may have two values, giving two solutions, the theorems of Arts. 132 and 133 not restricting the values of the parts to one solution.

Having found the two values for each part, the theorems

of Arts. 132 and 133 determine the values that belong to each solution.

Thus, given $a = 155° 27' 45''$ to find b
$a = 100° 21' 50''$ c
β

SOLUTION

$$\sin b = \tan a \cot \alpha$$

a	$155° 27' 45''$
α	$100° 21' 50''$
$\log \tan a$	$9.65945 - 10$
$\log \cot \alpha$	$9.26217 - 10$
$\log \sin b$	$8.92162 - 10$
b	$4° 47' 20''$ and
	$175° 12' 40''$

$$\sin \beta = \frac{\cos \alpha}{\cos a}$$

$\log \cos \alpha$	$9.25503 - 10$
$\log \cos a$	$9.95889 - 10$
$\log \sin \beta$	$9.29614 - 10$
β	$11° 24' 22''$ and
	$168° 35' 38''$

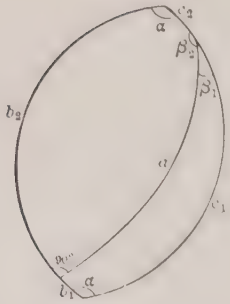
$$\sin c = \frac{\sin a}{\sin \alpha}$$

$\log \sin a$	$9.61835 - 10$
$\log \sin \alpha$	$9.99286 - 10$
$\log \sin c$	$9.62549 - 10$
c	$24° 58' 18''$ and
	$155° 1' 42''$

Check

$$\sin b = \sin c \sin \beta$$

$\log \sin c$	$9.62549 - 10$
$\log \sin \beta$	$9.29614 - 10$
$\log \sin b$	$8.92163 - 10$



FIRST SOLUTION	SECOND SOLUTION
$b_1 = 4° 47' 20''$	$b_2 = 175° 12' 40''$
$\beta_1 = 11° 24' 22''$	$\beta_2 = 168° 35' 38''$
$c_1 = 155° 1' 42''$	$c_2 = 24° 58' 18''$

139. EXAMPLES

Solve the following spherical right triangles, right-angled at γ.

1. $b = 10° 32'$, $\alpha = 12° 3'$
2. $a = 25° 18'$, $b = 32° 41'$
3. $c = 120° 37'$, $\beta = 9° 49'$
4. $c = 46° 40'$, $\alpha = 20° 50'$
5. $a = 115° 6'$, $b = 123° 14'$
6. $a = 112° 43' 30''$, $c = 85° 10' 10''$
7. $a = 15° 18' 20''$, $c = 21° 30' 40''$
8. $b = 168° 13' 45''$, $c = 150° 9' 20''$
9. $a = 132° 25'$, $\alpha = 107° 30'$
10. $c = 80° 3' 20''$, $\beta = 135° 16' 30''$
11. $b = 171° 3' 15''$, $c = 12° 20' 30''$
12. $a = 35° 54' 20''$, $\alpha = 47° 6' 10''$
13. $b = 15° 2' 30''$, $\beta = 20° 11' 40''$
14. $\alpha = 20° 26' 20''$, $\beta = 84° 41' 40''$
15. $a = 25° 41' 30''$, $\alpha = 34° 25' 40''$

140. Quadrantal triangles. A quadrantal triangle is a triangle one of whose sides is 90°. Its polar triangle is then a right triangle. The solution of a quadrantal triangle is effected through the solution of its polar triangle.

141. Isosceles triangles. The solution of an isosceles triangle is effected by solving the two equal right triangles formed by dropping a perpendicular from the vertex to the base.

142. EXAMPLES

Solve the following triangles.

1. $a = 117° 54' 30''$
   $b = 95° 42' 20''$
   $c = 90°$

2. $a = 69° 45'$
   $\beta = 94° 40'$
   $c = 90°$

3. $\beta = 153° 16'$
   $a = 19° 3'$
   $c = 90°$

4. $a = 159° 33' 40''$
   $b = 95° 18' 20''$
   $c = 90°$

5. The base of an isosceles triangle is $51° 8'$. The equal angles are each $41° 57'$. Find the equal sides and the angle at the vertex.

6. The base angles of an isosceles triangle are each $100° 12' 30''$, the vertical angle is $50° 19' 40''$. Find the equal sides and the base.

CHAPTER XII

OBLIQUE SPHERICAL TRIANGLE

143. In the present chapter the general formulas of spherical trigonometry, already developed (Chap. X) are transformed into standard formulas adapted to logarithmic computation; and the problem of the solution of the spherical triangle is discussed.

GENERAL SOLUTION

144. *To find the angles when the three sides are given.*

From Art. 119, we have

$$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha.$$

Therefore $$\cos \alpha = \frac{\cos a - \cos b \cos c}{\sin b \sin c}. \qquad (1)$$

But $$2 \sin^2 \tfrac{1}{2} \alpha = 1 - \cos \alpha$$

$$= \frac{\sin b \sin c - \cos a + \cos b \cos c}{\sin b \sin c}$$

$$= -\frac{\cos a - \cos (b - c)}{\sin b \sin c}.$$

Applying formula (4) of Art. 73, we have

$$\sin^2 \tfrac{1}{2} \alpha = \frac{\sin \tfrac{1}{2} (a + b - c) \sin \tfrac{1}{2} (a - b + c)}{\sin b \sin c}.$$

Letting $2 s = a + b + c$, we have

$$\sin \tfrac{1}{2} \alpha = \sqrt{\frac{\sin (s - b) \sin (s - c)}{\sin b \sin c}}. \qquad (2)$$

Similarly uniting (1) with

$$2\cos^2\tfrac{1}{2}\alpha = 1+\cos\alpha,$$

we have
$$2\cos^2\tfrac{1}{2}\alpha = \frac{\sin b\sin c+\cos a-\cos b\cos c}{\sin b\sin c}$$

$$= -\frac{\cos(b+c)-\cos a}{\sin b\sin c}$$

$$= \frac{2\sin\tfrac{1}{2}(a+b+c)\sin\tfrac{1}{2}(-a+b+c)}{\sin b\sin c}.$$

Therefore
$$\cos\tfrac{1}{2}\alpha = \sqrt{\frac{\sin s\sin(s-a)}{\sin b\sin c}}. \qquad (3)$$

Uniting (2) and (3),

$$\tan\tfrac{1}{2}\alpha = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s\sin(s-a)}} = \frac{\tan r}{\sin(s-a)}. \qquad (4)$$

Similarly
$$\tan\tfrac{1}{2}\beta = \sqrt{\frac{\sin(s-c)\sin(s-a)}{\sin s\sin(s-b)}} = \frac{\tan r}{\sin(s-b)}, \qquad (5)$$

and
$$\tan\tfrac{1}{2}\gamma = \sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin s\sin(s-c)}} = \frac{\tan r}{\sin(s-c)}, \qquad (6)$$

where
$$\tan r = \sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}}. \qquad (7)$$

145. *To find the sides when the three angles are given.*
Following the method of the last article, the equation

$$\cos\alpha = -\cos\beta\cos\gamma+\sin\beta\sin\gamma\cos a$$

gives
$$\sin\tfrac{1}{2}a = \sqrt{\frac{-\cos S\cos(S-\alpha)}{\sin\beta\sin\gamma}},$$

and
$$\cos\tfrac{1}{2}a = \sqrt{\frac{\cos(S-\beta)\cos(S-\gamma)}{\sin\beta\sin\gamma}},$$

and
$$\tan\tfrac{1}{2}a = \sqrt{\frac{-\cos S\cos(S-\alpha)}{\cos(S-\beta)\cos(S-\gamma)}} = \tan R\cos(S-\alpha)$$

where $2S = \alpha + \beta + \gamma$,

and $$\tan R = \sqrt{\frac{-\cos S}{\cos(S-\alpha)\cos(S-\beta)\cos(S-\gamma)}}.$$

146. **Delambre's or Gauss's formulas** express relations between the six parts.

By Art. 68

$$\sin\tfrac{1}{2}(\alpha+\beta) = \sin\tfrac{1}{2}\alpha\cos\tfrac{1}{2}\beta + \cos\tfrac{1}{2}\alpha\sin\tfrac{1}{2}\beta.$$

Substituting for $\sin\frac{1}{2}\alpha$, $\cos\frac{1}{2}\alpha$, $\sin\frac{1}{2}\beta$, and $\cos\frac{1}{2}\beta$ their values in terms of the sides of the triangle, and simplifying, we have

$$\sin\tfrac{1}{2}(\alpha+\beta) = \frac{\sin(s-b)+\sin(s-a)}{\sin c}\sqrt{\frac{\sin s\sin(s-c)}{\sin a\sin b}}$$

$$= \frac{2\sin\frac{1}{2}(2s-a-b)\cos\frac{1}{2}(a-b)}{2\sin\frac{1}{2}c\cos\frac{1}{2}c}\cdot\cos\tfrac{1}{2}\gamma.$$

Then $$\sin\tfrac{1}{2}(\alpha+\beta) = \frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}c}\cdot\cos\tfrac{1}{2}\gamma. \qquad (1)$$

Similarly $$\sin\tfrac{1}{2}(\alpha-\beta) = \frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}c}\cdot\cos\tfrac{1}{2}\gamma, \qquad (2)$$

and $$\cos\tfrac{1}{2}(\alpha+\beta) = \frac{\cos\frac{1}{2}(a+b)}{\cos\frac{1}{2}c}\cdot\sin\tfrac{1}{2}\gamma, \qquad (3)$$

and $$\cos\tfrac{1}{2}(\alpha-\beta) = \frac{\sin\frac{1}{2}(a+b)}{\sin\frac{1}{2}c}\cdot\sin\tfrac{1}{2}\gamma. \qquad (4)$$

147. **Napier's analogies** express relations between five parts of a triangle. They are easily obtained from Gauss's formulas.

Dividing (1) by (2), Art. 146,

$$\frac{\sin\frac{1}{2}(\alpha+\beta)}{\sin\frac{1}{2}(\alpha-\beta)} = \frac{\tan\frac{1}{2}c}{\tan\frac{1}{2}(a-b)}. \qquad (1)$$

Dividing (3) by (4),

$$\frac{\cos\frac{1}{2}(\alpha+\beta)}{\cos\frac{1}{2}(\alpha-\beta)} = \frac{\tan\frac{1}{2}c}{\tan\frac{1}{2}(a+b)}. \qquad (2)$$

Dividing (4) by (2),

$$\frac{\sin\frac{1}{2}(a+b)}{\sin\frac{1}{2}(a-b)}=\frac{\cot\frac{1}{2}\gamma}{\tan\frac{1}{2}(\alpha-\beta)}. \tag{3}$$

Dividing (3) by (1),

$$\frac{\cos\frac{1}{2}(a+b)}{\cos\frac{1}{2}(a-b)}=\frac{\cot\frac{1}{2}\gamma}{\tan\frac{1}{2}(\alpha+\beta)}. \tag{4}$$

148. Formulas collected. The following formulas are sufficient to solve a spherical triangle when any three parts are given:

$$*\tan\tfrac{1}{2}\alpha=\sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s\sin(s-a)}}=\frac{\tan r}{\sin(s-a)} \qquad \text{I}$$

$$\tan\tfrac{1}{2}a=\sqrt{\frac{-\cos S\cos(S-\alpha)}{\cos(S-\beta)\cos(S-\gamma)}}=\tan R\cos(S-\alpha) \qquad \text{II}$$

$$\frac{\sin\frac{1}{2}(\alpha+\beta)}{\sin\frac{1}{2}(\alpha-\beta)}=\frac{\tan\frac{1}{2}c}{\tan\frac{1}{2}(a-b)} \qquad \text{III}$$

$$\frac{\cos\frac{1}{2}(\alpha+\beta)}{\cos\frac{1}{2}(\alpha-\beta)}=\frac{\tan\frac{1}{2}c}{\tan\frac{1}{2}(a+b)} \qquad \text{IV}$$

$$\frac{\sin\frac{1}{2}(a+b)}{\sin\frac{1}{2}(a-b)}=\frac{\cot\frac{1}{2}\gamma}{\tan\frac{1}{2}(\alpha-\beta)} \qquad \text{V}$$

$$\frac{\cos\frac{1}{2}(a+b)}{\cos\frac{1}{2}(a-b)}=\frac{\cot\frac{1}{2}\gamma}{\tan\frac{1}{2}(\alpha+\beta)} \qquad \text{VI}$$

$$\frac{\sin a}{\sin\alpha}=\frac{\sin b}{\sin\beta}. \qquad \text{VII}$$

Formula I is used to determine an angle when the three sides are given.

Formula II is used to determine a side when the three angles are given.

Formulas III and IV are used when two angles and the

* These formulas are typical. Other formulas of the same type are obtained by a cyclic change of letters.

included side are given. Formula III determines $\frac{1}{2}(a-b)$ and formula IV determines $\frac{1}{2}(a+b)$, from which a and b are obtained. Either formula may also be used to determine the side c when the other two sides and their opposite angles are given.

Formulas V and VI are used when two sides and the included angle are given. Formula V determines $\frac{1}{2}(\alpha-\beta)$ and formula VI determines $\frac{1}{2}(\alpha+\beta)$, from which α and β are obtained. Either formula may also be used to determine the angle γ when the other two angles and their opposite sides are given.

Formula VII is used when an angle and the side opposite are among the given parts.

149. Whenever the formulas I to VI are employed in the solution of a spherical triangle as indicated above, the quadrant in which any part terminates may always be determined by noticing the signs of the functions involved.

But when the law of sines is employed, two values are found for the required part. This leads to two solutions unless limited to one solution by the following principles.

150. **Theorem.** *Half the sum of any two angles is in the same quadrant as half the sum of the sides opposite.*

This follows from a consideration of the signs of the functions involved in

$$\frac{\cos\frac{1}{2}(\alpha+\beta)}{\cos\frac{1}{2}(\alpha-\beta)}=\frac{\tan\frac{1}{2}c}{\tan\frac{1}{2}(a+b)}.$$

Since each part is less than 180°, $\tan\frac{1}{2}c$ and $\cos\frac{1}{2}(\alpha-\beta)$ are always positive. Hence $\cos\frac{1}{2}(\alpha+\beta)$ and $\tan\frac{1}{2}(a+b)$ must always have the same sign. Hence $\frac{1}{2}(\alpha+\beta)$ and $\frac{1}{2}(a+b)$ terminate in the same quadrant.

151. **Theorem.** *A side which differs more from 90° than another side, terminates in the same quadrant as its opposite angle.*

We have from Art. 119

$$\cos \alpha = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

If a differs more from 90° than b, $\cos a$ is numerically greater than $\cos b$. $\cos a$ is also greater than $\cos b \cos c$, since $\cos c$ is not greater than unity. Hence the numerator of this fraction has the same sign as $\cos a$. The denominator being always positive, $\cos a$ and $\cos \alpha$ have the same sign. Hence a and α terminate in the same quadrant.

The negative of this theorem is not true.

Thus given, $\quad a = 165°,\ b = 120°,\ \beta = 135°,$

α terminates in the second quadrant, since a differs more from 90° than b.

152. Theorem. *An angle which differs more from 90° than another angle, terminates in the same quadrant as its opposite side.*

This follows from

$$\cos a = \frac{\cos \alpha + \cos \beta \cos \gamma}{\sin \beta \sin \gamma}$$

by considerations similar to those of the previous article.

Thus given, $\quad \alpha = 80°,\ \gamma = 140°,$ and $a = 120°,$

c terminates in the second quadrant, since γ differs more from 90° than α.

153. Illustrative examples.

1. Given the three sides, $a = 105° 27' 20''$, $b = 83° 14' 40''$, $c = 96° 53' 10''$, to find α, β, and γ.

a	105° 27′ 20″
b	83° 14′ 40″
c	96° 53′ 10″
$2s$	284° 94′ 70″
s	142° 47′ 35″
$s-a$	37° 20′ 15″
$s-b$	59° 32′ 55″
$s-c$	45° 54′ 25″
Check: s	142° 47′ 35″

$$\tan r = \sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}}$$

$$\tan \tfrac{1}{2}\alpha = \frac{\tan r}{\sin(s-a)}$$

$$\tan \tfrac{1}{2}\beta = \frac{\tan r}{\sin(s-b)}$$

$$\tan \tfrac{1}{2}\gamma = \frac{\tan r}{\sin(s-c)}$$

$\log \sin (s-a)$	$9.78284-10$
$\log \sin (s-b)$	$9.93553-10$
$\log \sin (s-c)$	$9.85623-10$
$\operatorname{colog} \sin s$	0.21846
$2 \log \tan r$	$9.79306-10$
$\log \tan r$	$9.89653-10$
$\log \tan \frac{1}{2}\alpha$	0.11369
$\log \tan \frac{1}{2}\beta$	$9.96100-10$
$\log \tan \frac{1}{2}\gamma$	0.04030
$\frac{1}{2}\alpha$	$52^\circ\ 24'\ 55''$
$\frac{1}{2}\beta$	$42^\circ\ 25'\ 51''$
$\frac{1}{2}\gamma$	$47^\circ\ 39'\ 17''$
α	$104^\circ\ 49'\ 50''$
β	$84^\circ\ 51'\ 42''$
γ	$95^\circ\ 18'\ 34''$

Check

$$\cot \tfrac{1}{2}\gamma = \frac{\sin \frac{1}{2}(a+b)}{\sin \frac{1}{2}(a-b)} \tan \tfrac{1}{2}(\alpha-\beta)$$

$a+b$	$188^\circ\ 42'\ 0$
$a-b$	$22^\circ\ 12'\ 40'$
$\frac{1}{2}(a+b)$	$94^\circ\ 21'\ 0'$
$\frac{1}{2}(a-b)$	$11^\circ\ 6'\ 20'$
$\frac{1}{2}(\alpha-\beta)$	$9^\circ\ 59'\ 4'$
$\log \sin \frac{1}{2}(a+b)$	$9.99875-10$
$\log \tan \frac{1}{2}(\alpha-\beta)$	$9.24563-10$
$\operatorname{colog} \sin \frac{1}{2}(a-b)$	0.71531
$\log \cot \frac{1}{2}\gamma$	$9.95969-10$
$\log \tan \frac{1}{2}\gamma$	0.04031

2. Given two sides and the included angle, $a = 29^\circ\ 18'$, $b = 37^\circ\ 30'$, $\gamma = 51^\circ\ 52'$, to find α, β, and c.

$$\tan \tfrac{1}{2}(\beta-\alpha) = \frac{\sin \frac{1}{2}(b-a)}{\sin \frac{1}{2}(b+a)} \cot \tfrac{1}{2}\gamma$$

$$\tan \tfrac{1}{2}(\beta+\alpha) = \frac{\cos \frac{1}{2}(b-a)}{\cos \frac{1}{2}(b+a)} \cot \tfrac{1}{2}\gamma$$

$$\tan \tfrac{1}{2}c = \frac{\sin \frac{1}{2}(\beta+\alpha)}{\sin \frac{1}{2}(\beta-\alpha)} \tan \tfrac{1}{2}(b-a)$$

b	$37^\circ\ 30'$
a	$29^\circ\ 18'$
γ	$51^\circ\ 52'$
$b-a$	$8^\circ\ 12'$
$b+a$	$66^\circ\ 48'$
$\frac{1}{2}(b-a)$	$4^\circ\ 6'$
$\frac{1}{2}(b+a)$	$33^\circ\ 24'$
$\frac{1}{2}\gamma$	$25^\circ\ 56'$
$\log \sin \frac{1}{2}(b-a)$	$8.85429-10$
$\log \cot \frac{1}{2}\gamma$	0.31310
$\operatorname{colog} \sin \frac{1}{2}(b+a)$	0.25926
$\log \tan \frac{1}{2}(\beta-\alpha)$	$9.42665-10$
$\frac{1}{2}(\beta-\alpha)$	$14^\circ\ 57'\ 14''$

$\log \cos \frac{1}{2}(b-a)$	$9.99889-10$
$\log \cot \frac{1}{2}\gamma$	0.31310
$\operatorname{colog} \cos \frac{1}{2}(b+a)$	0.07839
$\log \tan \frac{1}{2}(\beta+\alpha)$	0.39038
$\frac{1}{2}(\beta+\alpha)$	$67^\circ\ 51'\ 8''$
$\frac{1}{2}(\beta-\alpha)$	$14^\circ\ 57'\ 14''$
β	$82^\circ\ 48'\ 22''$
α	$52^\circ\ 53'\ 54''$

$\log \tan \frac{1}{2}(b-a)$	$8.85540-10$
$\log \sin \frac{1}{2}(\beta+\alpha)$	$9.96671-10$
$\operatorname{colog} \sin \frac{1}{2}(\beta-\alpha)$	0.58831
$\log \tan \frac{1}{2}c$	$9.41042-10$
$\frac{1}{2}c$	$14^\circ\ 25'\ 43''$
c	$28^\circ\ 51'\ 26''$

Check

$$\frac{\sin a}{\sin \alpha} = \frac{\sin b}{\sin \beta} = \frac{\sin c}{\sin \gamma}$$

$\log \sin a$	$9.68965 - 10$	$\log \sin b$	$9.78445 - 10$	$\log \sin c$	$9.68361 - 10$
$\log \sin \alpha$	$9.90177 - 10$	$\log \sin \beta$	$9.99656 - 10$	$\log \sin \gamma$	$9.89574 - 10$
	$9.78788 - 10$		$9.78789 - 10$		$9.78787 - 10$

3. Given two sides, and an angle opposite one of them, $a = 63° 24' 50''$, $b = 17° 36' 40''$, $a = 44° 48' 20''$, to find β, γ, and c.

$$\sin \beta = \frac{\sin \alpha}{\sin a} \sin b$$

$$\cot \tfrac{1}{2}\gamma = \frac{\sin \frac{1}{2}(a+b)}{\sin \frac{1}{2}(a-b)} \tan \tfrac{1}{2}(\alpha - \beta)$$

$$\tan \tfrac{1}{2} c = \frac{\sin \frac{1}{2}(\alpha+\beta)}{\sin \frac{1}{2}(\alpha-\beta)} \tan \tfrac{1}{2}(a-b)$$

α	$63° 24' 50''$
a	$44° 48' 20''$
b	$17° 36' 40''$
$a + b$	$62° 25' 0''$
$a - b$	$27° 11' 40''$
$\frac{1}{2}(a+b)$	$31° 12' 30''$
$\frac{1}{2}(a-b)$	$13° 35' 50''$
$\log \sin \alpha$	$9.95147 - 10$
$\log \sin b$	$9.48081 - 10$
$\operatorname{colog} \sin a$	0.15200
$\log \sin \beta$	$9.58428 - 10$
β	$22° 34' 44''$
$\alpha - \beta$	$40° 50' 6''$
$\alpha + \beta$	$85° 59' 34''$
$\frac{1}{2}(\alpha - \beta)$	$20° 25' 3''$
$\frac{1}{2}(\alpha + \beta)$	$42° 59' 47''$

$\log \sin \frac{1}{2}(a+b)$	$9.71445 - 10$
$\log \tan \frac{1}{2}(\alpha - \beta)$	$9.57083 - 10$
$\operatorname{colog} \sin \frac{1}{2}(a-b)$	0.62876
$\log \cot \frac{1}{2}\gamma$	$9.91404 - 10$
$\frac{1}{2}\gamma$	$50° 38' 0''$
γ	$101° 16' 0'$

$\log \sin \frac{1}{2}(\alpha+\beta)$	$9.83375 - 10$
$\log \tan \frac{1}{2}(a-b)$	$9.38359 - 10$
$\operatorname{colog} \sin \frac{1}{2}(\alpha-\beta)$	0.45735
$\log \tan \frac{1}{2} c$	$9.67469 - 10$
$\frac{1}{2} c$	$25° 18' 20''$
c	$50° 36' 40'$

Check

$$\frac{\sin b}{\sin \beta} = \frac{\sin c}{\sin \gamma}$$

$\log \sin b$	$9.48081 - 10$	$\log \sin c$	$9.88810 - 10$
$\log \sin \beta$	$9.58428 - 10$	$\log \sin \gamma$	$9.99155 - 10$
	$9.89653 - 10$		$9.89655 - 10$

154. Two solutions. There are two solutions, if any, whenever two sides and an angle opposite one of them, or two angles and a side opposite one of them, are given, unless limited to one solution by the principles of Arts. 150, 151, and 152.

Thus, having given $\beta = 45^\circ\,15'\,12''$, $b = 56^\circ\,49'\,46''$, $a = 68^\circ\,52'\,48''$, to find α, γ, and c.

$$\sin\alpha = \frac{\sin\beta}{\sin b}\sin a$$

a	$68^\circ\,52'\,48''$
b	$56^\circ\,49'\,46''$
β	$45^\circ\,15'\,12''$
$\log\sin\beta$	9.85140
$\log\sin a$	9.96980
$\operatorname{colog}\sin b$	0.07725
$\log\sin\alpha$	9.89845
α	$52^\circ\,19'\,33''$, or $127^\circ\,40'\,27''$

α	$52^\circ\,19'\,33''$	$127^\circ\,40'\,27''$
$\alpha+\beta$	$97^\circ\,34'\,45''$	$172^\circ\,55'\,39''$
$\alpha-\beta$	$7^\circ\,4'\,21''$	$82^\circ\,25'\,15''$
$\frac{1}{2}(\alpha+\beta)$	$48^\circ\,47'\,22''$	$86^\circ\,27'\,50''$
$\frac{1}{2}(\alpha-\beta)$	$3^\circ\,32'\,10''$	$41^\circ\,12'\,38''$
$a+b$	$125^\circ\,42'\,34''$	
$a-b$	$12^\circ\,3'\,2''$	
$\frac{1}{2}(a+b)$	$62^\circ\,51'\,17''$	
$\frac{1}{2}(a-b)$	$6^\circ\,1'\,31''$	

$$\tan\tfrac{1}{2}c = \frac{\sin\frac{1}{2}(\alpha+\beta)}{\sin\frac{1}{2}(\alpha-\beta)}\tan\tfrac{1}{2}(a-b)$$

$\log\sin\frac{1}{2}(\alpha+\beta)$	9.87639	9.99917
$\log\tan\frac{1}{2}(a-b)$	9.02346	9.02346
$\operatorname{colog}\sin\frac{1}{2}(\alpha-\beta)$	1.20984	0.18124
$\log\tan\frac{1}{2}c$	0.10969	9.20387
$\frac{1}{2}c$	$52^\circ\,9'\,35''$	$9^\circ\,5'\,7''$
c	$104^\circ\,19'\,10''$	$18^\circ\,10'\,14''$

$$\cot\tfrac{1}{2}\gamma = \frac{\sin\frac{1}{2}(a+b)}{\sin\frac{1}{2}(a-b)}\tan\tfrac{1}{2}(\alpha-\beta)$$

$\log\sin\frac{1}{2}(a+b)$	9.94932	9.94932
$\log\tan\frac{1}{2}(\alpha-\beta)$	8.79099	9.94238
$\operatorname{colog}\sin\frac{1}{2}(a-b)$	0.97895	0.97895
$\log\cot\frac{1}{2}\gamma$	9.71926	0.87065
$\frac{1}{2}\gamma$	$62^\circ\,21'\,1''$	$7^\circ\,40'\,16''$
γ	$124^\circ\,42'\,2''$	$15^\circ\,20'\,32''$

FIRST SOLUTION

$\alpha = 52° 19' 33''$
$c = 104° 19' 10''$
$\gamma = 124° 42' 2''$

Check

$$\tan \tfrac{1}{2}c = \frac{\cos \tfrac{1}{2}(\alpha+\beta)}{\cos \tfrac{1}{2}(\alpha-\beta)} \tan \tfrac{1}{2}(a+b)$$

$\log \cos \tfrac{1}{2}(\alpha+\beta)$	9.81877	8.79013
$\log \tan \tfrac{1}{2}(a+b)$	0.29012	0.29012
$\operatorname{colog} \cos \tfrac{1}{2}(\alpha-\beta)$	0.00083	0.12361
$\log \tan \tfrac{1}{2}c$	0.10972	9.20386

SECOND SOLUTION

$\alpha = 127° 40' 27''$
$c = 18° 10' 14''$
$\gamma = 15° 20' 32''$

$$\cot \tfrac{1}{2}\gamma = \frac{\cos \tfrac{1}{2}(a+b)}{\cos \tfrac{1}{2}(a-b)} \tan \tfrac{1}{2}(\alpha+\beta)$$

$\log \cos \tfrac{1}{2}(a+b)$	9.65920	9.65920
$\log \tan \tfrac{1}{2}(\alpha+\beta)$	0.05762	0.20904
$\operatorname{colog} \cos \tfrac{1}{2}(a-b)$	0.00241	0.00241
$\log \cot \tfrac{1}{2}\gamma$	9.71923	0.87065

155. Area of spherical triangle. Representing the area of a sphere, S, by 720 spherical degrees, it is demonstrated in geometry that the area of a spherical triangle, A, *in terms of spherical degrees*, is equal to its spherical excess; or

$$A = (\alpha + \beta + \gamma - 180) \text{ spherical degrees}$$

Hence
$$\frac{A}{S} = \frac{\alpha+\beta+\gamma-180}{720}.$$

Let A' and S' represent the area of the triangle and the sphere respectively, in terms of the unit in which r is expressed. Since the ratio of the area of the spherical triangle to the area of the sphere is independent of the units used, we have
$$\frac{A'}{S'} = \frac{A}{S}.$$

Therefore
$$\frac{A'}{S'} = \frac{\alpha+\beta+\gamma-180}{720}.$$

But the area of the sphere expressed in terms of r is $4\pi r^2$, therefore the area of the triangle is given by

$$A' = \pi r^2 \frac{\alpha+\beta+\gamma-180}{180}$$

156. EXAMPLES

Solve the following spherical triangles and check the results:

1. $b = 10^\circ\ 0'\ 10''$
$c = 114^\circ\ 40'\ 40''$
$\alpha = 92^\circ\ 28'\ 20''$

2. $b = 85^\circ\ 4'\ 19''$
$c = 139^\circ\ 58'\ 25''$
$\alpha = 12^\circ\ 20'\ 31''$

3. $c = 82^\circ\ 3'\ 4''$
$a = 70^\circ\ 14'\ 12''$
$\beta = 84^\circ\ 20'\ 9''$

4. $a = 95^\circ\ 3'\ 30''$
$b = 128^\circ\ 38'\ 50''$
$\gamma = 170^\circ\ 52'\ 20''$

5. $a = 29^\circ\ 18'$
$b = 37^\circ\ 30'$
$\gamma = 51^\circ\ 52'$

6. $\alpha = 11^\circ\ 21'\ 10''$
$\gamma = 19^\circ\ 0'\ 20''$
$b = 66^\circ\ 19'\ 30''$

7. $\gamma = 179^\circ\ 22'\ 11''$
$\alpha = 148^\circ\ 17'\ 17''$
$b = 25^\circ\ 39'\ 34''$

8. $a = 108^\circ\ 5'\ 18''$
$b = 170^\circ\ 30'\ 46''$
$c = 85^\circ\ 50'\ 22''$

9. $\alpha = 105^\circ\ 27'\ 20''$
$b = 83^\circ\ 14'\ 40''$
$c = 96^\circ\ 53'\ 10''$

10. $a = 34^\circ\ 19'\ 30''$
$b = 28^\circ\ 37'\ 10''$
$c = 22^\circ\ 44'\ 40''$

11. $\alpha = 61^\circ\ 8'$
$\beta = 59^\circ\ 12'$
$\gamma = 78^\circ\ 25'$

12. $a = 76^\circ\ 43'\ 15''$
$b = 83^\circ\ 35'\ 27''$
$c = 98^\circ\ 26'\ 38''$

13. $\alpha = 110^\circ\ 35'$
$\beta = 135^\circ\ 42'$
$\gamma = 146^\circ\ 8'$

14. $\gamma = 11^\circ\ 34'\ 10''$
$b = 82^\circ\ 56'\ 30''$
$c = 27^\circ\ 9'\ 40''$

15. $\alpha = 32^\circ\ 4'\ 10''$
$\beta = 128^\circ\ 56'\ 20''$
$a = 39^\circ\ 50'\ 30''$

16. $\beta = 80^\circ\ 40'\ 2''$
$c = 75^\circ\ 54'\ 0''$
$b = 100^\circ\ 21'\ 28''$

17. $\alpha = 21^\circ\ 1'\ 10''$
$\gamma = 17^\circ\ 22'\ 50''$
$a = 14^\circ\ 13'\ 30''$

18. $\beta = 77^\circ\ 44'\ 55''$
$\gamma = 92^\circ\ 17'\ 24''$
$a = 26^\circ\ 29'\ 39''$

19. $\alpha = 114^\circ\ 23'\ 9''$
$\beta = 88^\circ\ 41'\ 11''$
$\gamma = 79^\circ\ 0'\ 4''$

20. $\beta = 99^\circ\ 4'\ 12''$
$\gamma = 106^\circ\ 0'\ 9''$
$a = 161^\circ\ 2'\ 10''$

21. $c = 100° 10' 40''$
$a = 65° 20' 30''$
$\gamma = 94° 30' 10''$

23. $b = 42° 15' 20''$
$c = 127° 3' 30''$
$\beta = 31° 44' 20''$

22. $a = 103° 19' 50''$
$\beta = 92° 37' 30''$
$\gamma = 128° 54' 20''$

24. $\gamma = 127° 4' 10''$
$a = 88° 12' 0''$
$c = 141° 20' 30''$

SOLUTION WHEN ONLY ONE PART IS REQUIRED

157. In many problems of astronomy and geodesy, it is required to find only one or two of the unknown parts of a spherical triangle, the other unknown parts being of no importance in the problem. It then becomes desirable to have a method whereby the required parts can be computed without being under the necessity of first computing any part not desired.

It has already been shown that any angle can be found directly from three given sides (Art. 144), and that any side can be found directly from three given angles (Art. 145).

It is evident that any part can be found from any three given parts by the use of the general formulas containing the required part and the three given parts. By the introduction of auxiliary quantities, these formulas will now be adapted to logarithmic computation.

158. *Given two sides and the included angle, to find any one of the remaining parts.*

Let a, b, γ be the given parts.

First. To find c.

The relation between a, b, γ, and c is (Art. 119),

$$\cos c = \cos a \cos b + \sin a \sin b \cos \gamma. \tag{1}$$

To adapt this formula to logarithmic computation, let

$$m \sin M = \sin b \cos \gamma, \tag{2}$$

and

$$m \cos M = \cos b. \tag{3}$$

Then eliminating b by uniting (1), (2), and (3), we have

$$\cos c = m(\cos a \cos M + \sin a \sin M),$$

or
$$\cos c = m \cos(a - M). \tag{4}$$

From (2) and (3)

$$\tan M = \tan b \cos \gamma, \tag{5}$$

and from (3) and (4)

$$\cos c = \frac{\cos b \cos(a - M)}{\cos M}. \tag{6}$$

Equations (5) and (6) enable us to find c.

Illustration. Given $a = 75° 38' 20''$, $b = 54° 54' 38''$, and $\gamma = 30° 17' 43''$.

$\tan M = \tan b \cos \gamma$

a	$75° 38' 20''$
b	$54° 54' 38''$
γ	$30° 17' 43''$
$\log \tan b$	0.15333
$\log \cos \gamma$	9.93623
$\log \tan M$	0.08956
M	$50° 51' 58''$
$a - M$	$24° 46' 22''$

$\cos c = \dfrac{\cos b \cos(a - M)}{\cos M}$

$\log \cos b$	9.75956
$\log \cos(a - M)$	9.95808
$\operatorname{colog} \cos M$	0.19987
$\log \cos c$	9.91751
c	$34° 12' 27''$

Second. To find β.

The relation between a, b, γ, and β is given by Art. 124, (equation 5), from which we have

$$\cot \beta = \frac{\cot b \sin a - \cos a \cos \gamma}{\sin \gamma}. \tag{7}$$

Multiplying numerator and denominator by $\sin b$,

$$\cot \beta = \frac{\cos b \sin a - \sin b \cos a \cos \gamma}{\sin b \sin \gamma}. \tag{8}$$

Again using equations (2) and (3) we have

$$\cot \beta = \frac{m(\cos M \sin a - \sin M \cos a)}{\sin b \sin \gamma}, \tag{9}$$

or
$$\cot \beta = \frac{m \sin(a - M)}{\sin b \sin \gamma}. \tag{10}$$

As before $\tan M = \tan b \cos \gamma$. (11)

From (2) and (10)

$$\cot \beta = \frac{\cot \gamma \sin (a - M)}{\sin M}. \tag{12}$$

Equations (11) and (12) enable us to find β.

Third. To find γ.

By interchanging a and b and consequently α and β in (11) and (12) we have, calling the auxiliary angle N,

$$\tan N = \tan a \cos \gamma \tag{13}$$

and

$$\cot \alpha = \frac{\cot \gamma \sin (b - N)}{\sin N} \tag{14}$$

to determine α.

159. Two parts required. It will be noticed that the same auxiliary quantity M is used to find both c and β. We thus have a convenient method, much used in astronomy, for finding a side and an angle when two sides and the included angle are given.

For finding c and β we have, collecting our formulas,

$$\tan M = \tan b \cos \gamma$$

$$\cos c = \frac{\cos b \cos (a - M)}{\cos M}$$

$$\cot \beta = \frac{\cot \gamma \sin (a - M)}{\sin M}.$$

Dividing equation (10) of Art. 158 by equation (4), we have

$$\frac{\cot \beta}{\cos c} = \frac{\tan (a - M)}{\sin b \sin \gamma},$$

which serves as a check upon c and β.

Similarly, for finding c and α, we have

$$\tan N = \tan a \cos \gamma$$

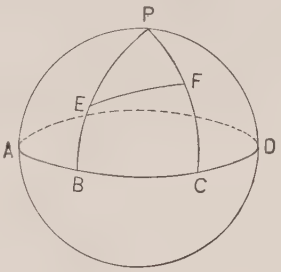
$$\cos c = \frac{\cos a \cos (b - N)}{\cos N}$$

$$\cot \alpha = \frac{\cot \gamma \sin (b - N)}{\sin N}.$$

160. PROBLEMS

An arc of 1′ on the earth's surface is equal to one English geographical mile.

1. Find the distance between Boston, latitude 42° 21′ N., longitude 71° 41′ W., and San Francisco, latitude 37° 48′ N. and longitude 122° 28′ W.



Solution. — Let APD be the meridian of Greenwich from which longitude is measured, $ABCD$ the equator, and P the north pole.

Let the positions of San Francisco and Boston be represented by E and F respectively. The desired distance is EF.

Then
$$\text{angle } DPE = 122° 28',$$
$$\text{angle } DPF = 71° 41',$$
$$\text{arc } EB = 37° 48',$$
$$\text{arc } CF = 42° 21',$$
$$\text{arc } PB = \text{arc } PC = 90°.$$

But
$$PE = PB - EB = 52° 12',$$
$$PF = PC - FC = 47° 39',$$
$$\angle FPE = \angle DPE - \angle DPF = 50° 47'.$$

Letting $\angle FPE = \gamma$, $PE = a$, $PF = b$,

we may find EF or c by the formulas

$$\tan M = \tan b \cos \gamma, \quad \cos c = \frac{\cos b \cos (a - M)}{\cos M}.$$

$\tan M = \tan b \cos \gamma$

a	$52° 12'$
b	$47° 39'$
γ	$50° 47'$
$\log \tan b$	0.04023
$\log \cos \gamma$	$9.80089 - 10$
$\log \tan M$	$9.84112 - 10$
M	$34° 44' 23''$
$a - M$	$17° 27' 37''$

$\cos c = \dfrac{\cos b \cos (a - M)}{\cos M}$

$\log \cos b$	$9.82844 - 10$
$\log \cos (a - M)$	$9.97951 - 10$
$\operatorname{colog} \cos M$	0.08526
$\log \cos c$	$9.89321 - 10$
c	$38° 33' 20''$, or $2313\frac{1}{3}$ miles

2. Find the distance between New York (lat. 40° 43′ N., long. 74° 0′ W.) and San Francisco (lat. 37° 48′ N., long. 122° 28′ W.).

3. Find the distance between Calcutta (lat. 22° 33′ N., long. 88° 19′ E.) and Greenwich (lat. 51° 29′ N.).

4. Find the distance between Baltimore (lat. 39° 17′ N., long. 76° 37′ W.) and Calcutta.

161. *Given two angles and the included side, to find any one of the remaining parts.*

Let α, β, c be the given parts.

First. To find γ.

The relation between α, β, c, and γ is, Art. 121, eq. (3),

$$\cos \gamma = -\cos \alpha \cos \beta + \sin \alpha \sin \beta \cos c. \tag{1}$$

Let $m \sin M = \cos \alpha$, (2)

and $m \cos M = \sin \alpha \cos c$. (3)

Uniting (1), (2), and (3)

$$\cos \gamma = m (-\sin M \cos \beta + \cos M \sin \beta),$$

or $$\cos \gamma = m \sin (\beta - M). \tag{4}$$

From (2) and (3)

$$\cot M = \tan \alpha \cos c, \tag{5}$$

and from (2) and (4)

$$\cos\gamma = \frac{\cos\alpha \sin(\beta - M)}{\sin M}. \tag{6}$$

Equations (5) and (6) enable us to find γ.

Second. To find a.

The relation between α, β, c, and a is given by Art. 124, equation (4), from which

$$\cot a = \frac{\sin\beta \cot\alpha + \cos c \cos\beta}{\sin c}, \tag{7}$$

or

$$\cot a = \frac{\sin\beta \cos\alpha + \sin\alpha \cos c \cos\beta}{\sin\alpha \sin c}. \tag{8}$$

Uniting equations (2), (3), and (8),

$$\cot a = \frac{m(\sin\beta \sin M + \cos\beta \cos M)}{\sin\alpha \sin c}, \tag{9}$$

or

$$\cot a = \frac{m \cos(\beta - M)}{\sin\alpha \sin c}. \tag{10}$$

From (2), (3), and (10)

$$\cot M = \tan\alpha \cos c \tag{11}$$

and

$$\cot a = \frac{\cot c \cos(\beta - M)}{\cos M}, \tag{12}$$

from which a is found.

Third. To find b.

From (11) and (12) by interchanging α and β, and consequently a and b, we have, calling the auxiliary quantity N,

$$\cot N = \tan\beta \cos c \tag{13}$$

and

$$\cot b = \frac{\cot c \cos(\alpha - N)}{\cos N} \tag{14}$$

to determine b.

162. *Given two sides and an angle opposite one of them, to find any one of the remaining parts.*

Let a, b, α be the given parts.

First. To find c.

The relation between a, b, α and c is

$$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha. \tag{1}$$

Let $m \sin M = \sin b \cos \alpha,$ (2)

and $m \cos M = \cos b.$ (3)

Then $$\cos a = m \cos (c - M). \tag{4}$$

From equations (2), (3), and (4),

$$\tan M = \tan b \cos \alpha \tag{5}$$

and $$\cos (c - M) = \frac{\cos a \cos M}{\cos b}. \tag{6}$$

Equation (5) determines M and equation (6) determines $c - M$. Adding these values, we have c.

In general there are 2 solutions for c. We may limit M to positive values less than 180°. By equation (6) $c - M$ may have two values, numerically equal but opposite in sign, giving two values for c unless the sum $M + (c - M)$ is greater than 180° or negative, in which case there is but one solution.

Second. To find γ.

The relation between a, b, α, and γ is

$$\sin \gamma \cot \alpha = \cot a \sin b - \cos b \cos \gamma.$$

Multiplying by $\sin \alpha$ and rearranging, we have

$$\sin \gamma \cos \alpha + \cos \gamma \sin \alpha \cos b = \sin \alpha \cot a \sin b. \tag{7}$$

Let $$n \cos N = \cos \alpha, \tag{8}$$

and $$n \sin N = \sin \alpha \cos b. \tag{9}$$

From (7), (8), and (9) we have

$$n \sin (\gamma + N) = \sin \alpha \cot a \sin b. \tag{10}$$

Then from (8), (9), and (10),

$$\tan N = \tan \alpha \cos b, \tag{11}$$

and $$\sin (\gamma + N) = \sin N \cot a \tan b. \tag{12}$$

Equation (12) determines $\gamma + N$ and equation (11) determines N. Subtracting the second value from the first gives γ.

In general there are two solutions, since $\gamma + N$ may have two values.

Third. To find β.

The angle β is found from

$$\sin \beta = \frac{\sin b \sin \alpha}{\sin a}, \tag{13}$$

which in general gives two values.

163. *Given two angles and a side opposite one of them, to find any one of the remaining parts.*

Let α, β, a be the given parts.

First. To find γ.

The relation between α, β, a, and γ is

$$\cos \alpha = -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cos a. \tag{1}$$

Let $$m \sin M = \cos \beta, \tag{2}$$

and $$m \cos M = \sin \beta \cos a. \tag{3}$$

Then $$\cos \alpha = m \sin (\gamma - M). \tag{4}$$

From (2), (3), and (4)

$$\cot M = \tan \beta \cos a, \tag{5}$$

and $$\sin (\gamma - M) = \frac{\cos \alpha \sin M}{\cos \beta}. \tag{6}$$

Equations (5) and (6) determine M and $\gamma - M$, from which γ is found.

Two solutions may be possible, as in Art. 162.

Second. To find c.

The relation between α, β, a, and c is

$$\sin \beta \cot \alpha = \cot a \sin c - \cos c \cos \beta. \tag{7}$$

Multiplying by $\sin a$ and transposing,

$$\cos a \sin c - \sin a \cos c \cos \beta = \sin \beta \sin a \cot \alpha. \tag{8}$$

Let $$n \cos N = \cos a, \tag{9}$$

and $$n \sin N = \sin a \cos \beta. \tag{10}$$

Then uniting (8), (9), and (10)

$$n \sin(c - N) = \sin\beta \sin\alpha \cot\alpha. \tag{11}$$

From (9), (10), and (11)

$$\tan N = \tan\alpha \cos\beta, \tag{12}$$

and

$$\sin(c - N) = \tan\beta \cot\alpha \sin N. \tag{13}$$

Equations (12) and (13) determine N and $c - N$, from which c is found.

There may be two solutions.

Third. To find b.

We have

$$\sin b = \frac{\sin\beta \sin\alpha}{\sin\alpha}. \tag{14}$$

There are two values of b unless restricted to one solution by the principles of Arts. 150, 151, and 152.

164. The general triangle. The parts of the *general spherical* triangle are not restricted to values less than 180°. It can be shown that all the formulas developed for the oblique spherical triangle are true for the general spherical triangle if the double sign is introduced in the formulas of Arts. 144, 145, and 146.

ANSWERS

Art. 19; Page 10

4. $\frac{\pi}{18}, \frac{\pi}{6}, \frac{\pi}{4}$, etc.

5. 114° 35′ 28″, 286° 28′ 40″, etc.

6. 60°, 135°, −300°, 57° 17′ 44″, 36° 28′ 31″, etc.

7. $7\frac{1}{2}$ ft.

8. $2\frac{2}{5}$ radians, 137° 30′ 34″.

9. 94° 3′ 24″.

10. $\frac{16}{15}\pi$ ft.

11. 2.7216 radians.

12. 247.16 R. P. M.
25.882.

13. 18.33 mi. per sec.

14. 5236.

15. 9.6π
240π ft. per min.

Art. 27; Page 20

1. 3d and 4th. **2.** 1st and 4th. **3.** 1st and 3d

7. 2d. **8.** 3d. **20.** $-\frac{2\pi}{3}$. **21.** $-\frac{4\pi}{3}$.

28. $\sin\alpha_1 = \frac{6}{85}\sqrt{85}$, $\cos\alpha_1 = -\frac{7}{85}\sqrt{85}$, $\cot\alpha_1 = -\frac{7}{6}$,
$\sec\alpha_1 = -\frac{1}{7}\sqrt{85}$, $\csc\alpha_1 = \frac{1}{6}\sqrt{85}$,
$\sin\alpha_2 = -\frac{6}{85}\sqrt{85}$, $\cos\alpha_2 = \frac{7}{85}\sqrt{85}$, $\cot\alpha_2 = -\frac{7}{6}$,
$\sec\alpha_2 = \frac{1}{7}\sqrt{85}$, $\csc\alpha_2 = -\frac{1}{6}\sqrt{85}$.

35. $\frac{16}{17}$.

Art. 36; Page 29

1. $b = 16.5$
$c = 17.5$
$\beta = 70°$.

2. $a = 1.44$
$b = 2.05$
$\beta = 55°$.

3. $c = .869$
$a = .225$
$\alpha = 15°$.

4. $c = 65$
$\alpha = 23°$
$\beta = 67°$

5. $\alpha = 27°$
$\beta = 63°$
$b = 72.6$

6. $\alpha = 18\frac{1}{2}°$
$\beta = 71\frac{1}{2}°$
$b = .00867$

7. $a = 346$, $c = 507$, $\beta = 47°$

8. $\alpha = 27°$, $\beta = 63°$, $a = 3.50$

9. $a = .029$, $b = .089$, $\beta = 72°$

10. $\alpha = 40\frac{1}{2}°$, $a = 8.11$, $b = 9.48$

11. $b = 5161$, $c = 5489$, $\beta = 70° 5'$

12. $a = .1384$, $b = .2878$, $\beta = 64° 19'$

13. $a = 1.446$, $c = 1.719$, $\alpha = 57° 17'$

14. $c = .00006294$, $\alpha = 72° 26'$, $\beta = 17° 34'$

15. $b = 810.80$, $\alpha = 47° 31' 32''$, $\beta = 42° 28' 28''$

16. 17. etc. check your results.

31. Base 1331.1, vertical angle 149° 19′ 10″.

32. Base angles 39° 23′ 56″, base 1477.0.

33. Equal sides 1622.9, base angles 37° 59′ 37″.

34. Equal sides 219.75, base angles 68° 27′ 19″.

35. 37.504.

Art. 38; Page 31

1. 290.83 ft.

2. 405.24 ft.

3. 263.92 ft.
289.93 ft.

4. 21.442 ft.

5. $2\,nr \sin \frac{180°}{n}$.

6. 132.52 ft.

7. $2\,nr \tan \frac{180°}{n}$.

8. 130.99 ft.

9. 226.11 ft.

10. 2572.5 ft.

11. 21.360 in.
22.638 in.
69° 26′ 36″.

12. 54.775 mi.

13. 153.72 lbs.
38° 31′ 46″.

14. 739.38 mi. per hr.

15. 16 ft. $3\frac{5}{32}$ in.

16. 35° 16′.

Art. 50; Page 46

2. $-\cos 10°$, $-\sin 80°$.

3. $-\cot 20°$, $-\tan 70°$.

4. $-\cot 20°$, $-\tan 70°$.

5. $\cos 20°$, $\sin 70°$.

6. $-\tan 80°$, $-\cot 10°$.

7. $-\sin 60°$, $-\cos 30°$.

8. $\cos \theta$.

9. $-\tan \theta$.

10. $-\tan \theta$.

11. $-\cos \theta$.

Art. 56; Page 51

11. $\cos\theta = -\frac{1}{3}\sqrt{5}$, $\tan\theta = -\frac{2}{5}\sqrt{5}$, $\cot\theta = -\frac{1}{2}\sqrt{5}$, $\sec\theta = -\frac{3}{5}\sqrt{5}$, $\csc\theta = \frac{3}{2}$.

12. $\sin\theta = \frac{10}{149}\sqrt{149}$, $\cos\theta = \frac{7}{149}\sqrt{149}$, $\tan\theta = \frac{10}{7}$, $\sec\theta = \frac{1}{7}\sqrt{149}$, $\csc\theta = \frac{1}{10}\sqrt{149}$.

15. $x = 30°$. **16.** $u = 45°$. **17.** $x = 0°, 60°$.

18. $\theta = 45°, 60°$. **19.** $\sin x = \pm\sqrt{-\frac{1}{2} + \frac{1}{2}\sqrt{5}}$.

20. Identity. **22.** $\theta = 120°$. **24.** Identity.

21. $x = 60°, 120°$. **23.** $\theta = 30°$. **25.** $y = 30°, 150°$.

35. $\alpha = 30°, 150°$. **36.** $\alpha = 30°, 60°, 120°, 150°$.

37. $x = 45°, 135°$. **38.** $x = 45°$.

Art. 60; Page 57

33. $\dfrac{\tan^2 x - 1}{\tan^2 x + 1}$. **34.** $\sin x \mp \sqrt{1 - \sin^2 x}$. **35.** $\dfrac{1+\cos^2 x}{1-\cos^2 x}$.

39. $\cos\theta = \pm\sqrt{1-a^2}$, $\tan\theta = \dfrac{a}{\pm\sqrt{1-a^2}}$,

$\cot\theta = \dfrac{\pm\sqrt{1-a^2}}{a}$, $\sec\theta = \dfrac{1}{\pm\sqrt{1-a^2}}$, $\csc\theta = \dfrac{1}{a}$.

41. $\sin\theta = \pm\sqrt{1-\cos^2\theta}$, $\tan\theta = \dfrac{\pm\sqrt{1-\cos^2\theta}}{\cos\theta}$,

$\cot\theta = \dfrac{\cos\theta}{\pm\sqrt{1-\cos^2\theta}}$, $\sec\theta = \dfrac{1}{\cos\theta}$,

$\csc\theta = \dfrac{1}{\pm\sqrt{1-\cos^2\theta}}$.

43. $\dfrac{a}{2}$. **47.** $0°, 90°$. **48.** $135°$. **49.** $0°, 30°, 60°, 150°$.

50. $30°, 150°$. **51.** $45°$. **54.** $36° 52'$. (See tables.)

55. $46° 24', 90°$. **57.** $65° 54', 114° 6'$.

58. $0°, -30°, -150°$.

59. $\tan u = \pm\frac{2}{5}\sqrt{5}, \pm\frac{1}{4}\sqrt{2}$. **61.** $195°, 345°$.

60. $\sin x = \pm\frac{1}{2}\sqrt{3}, \pm\frac{2}{3}\sqrt{2}$. **62.** $54°, 234°$.

Art. 70; Page 66

2. $\frac{1}{4}\sqrt{2}(\sqrt{3}-1)$. **3.** $\frac{1}{4}\sqrt{2}(\sqrt{3}-1)$. **4.** $\frac{1}{4}\sqrt{2}(\sqrt{3}+1)$.

5. $\dfrac{3+\sqrt{3}}{3-\sqrt{3}}$. **7.** $\cos\beta$. **8.** $-\cot\alpha$. **9.** $-\cos\alpha$.

Art. 75; Page 71

1. $\frac{1}{2}\sqrt{2-\sqrt{2}}$, $\frac{1}{2}\sqrt{2+\sqrt{2}}$, $\sqrt{3-2\sqrt{2}}$, $\sqrt{3+2\sqrt{2}}$.

2. $\pm\frac{1}{8}\sqrt{15}$, $-\frac{7}{8}$, $\pm\frac{1}{7}\sqrt{15}$, $\pm\frac{7}{15}\sqrt{15}$.

3. $\pm\sqrt{\dfrac{10\pm\sqrt{10}}{20}}$, etc.

32. $0, \pm\sqrt{\dfrac{3-a}{1-3a}}$. **36.** 67° 30′, 157° 30′.

40. 0°, 65° 42′, 114° 18′. (See tables.)

34. 30°, 90°, 150°. **42.** 0°, 30°, 90°, 150°.

35. 0°, 120°. **43.** 0°, 90°, 120°.

Art. 85; Page 82

1. 60°, 120°, 420°, etc. **2.** 150°, 210°, − 150°, etc.

3. 45°, 225°, − 135°, etc. **4.** $\frac{1}{4}$.

5. $\pm\frac{1}{17}\sqrt{17}$. **6.** $\dfrac{u}{\pm\sqrt{u^2+1}}$.

7. $\dfrac{\pm\sqrt{u^2+1}}{u}$. **9.** $\pm\sqrt{1-u^2}$.

10. $\dfrac{1}{\pm\sqrt{1+u^2}}$. **11.** $\dfrac{1}{u}$.

13. $\pm 2u\sqrt{1-u^2}$.

14. $\dfrac{2u}{1-u^2}$. **15.** $\dfrac{\pm 2\sqrt{u^2-1}}{2-u^2}$.

20. $\pm u\sqrt{1-v^2}\pm v\sqrt{1-u^2}$. **21.** $\pm\sqrt{1-u^2}\sqrt{1-v^2}-uv$.

25. $\pm\sqrt{\frac{1-u}{2}}$.

27. $\pm\sqrt{\frac{u-1}{2u}}$.

29. SOLUTION. $x = \tan \cot^{-1} a = \frac{1}{\cot \cot^{-1} a} = \frac{1}{a}$.

30. $x = \frac{\pm a}{\sqrt{1+a^2}}$.

31. $1 - 2a^2$.

32. $\frac{\sqrt{1-a^2} - a^2}{\sqrt{1+a^2}}$.

34. $\frac{a\sqrt{1-b^2} + b\sqrt{1-a^2}}{\sqrt{1-a^2}\sqrt{1-b^2} - ab}$.

35. $\frac{1}{ab}(1 + \sqrt{a^2-1}\sqrt{b^2-1})$.

Art. 101; Page 98

1. $b = 24.5$
   $c = 31.8$
   $\gamma = 78°$.

2. $\alpha = 50°$
   $b = 54.9$
   $c = 58.6$

3. $a = 61.4$
   $c = 47.7$
   $\gamma = 48°$.

4. $\beta = 48° 20'$
   $\gamma = 62° 40'$
   $c = 76.1$

5. $\beta = 68° 22'$
   $\gamma = 56° 58'$
   $a = 107$

6. $\alpha = 40° 48'$
   $\beta = 104° 52'$
   $c = 141.$

7. $\alpha = 69° 22'$
   $\beta = 38° 38'$
   $c = 42.7$

8. $\beta = 38° 37'$
   $\gamma = 31° 23'$
   $c = 173.$

9. $\alpha = 44° 42'$
   $\beta = 60° 20'$
   $\gamma = 74° 54'$

10. $\alpha = 72° 21'$
    $\beta = 49° 38'$
    $\gamma = 58° 1'$

11. $\beta = 34° 44'$ or $146° 16'$
    $\gamma = 125° 16'$ or $13° 44'$
    $c = 35.8$ or 10.4

12. $\alpha = 51° 44'$ or $128° 16'$
    $\beta = 84° 26'$ or $7° 54'$
    $b = .431$ or $.059$

Art. 101; Page 98

13. $\gamma = 39° 49' 50''$
$a = 41.581$
$c = 41.432$

14. $a = 13.081$
$c = 13.620$
$\beta = 97° 2' 15''$

15. $\alpha = 90° 20' 34''$
$\beta = 65° 17' 34''$
$\gamma = 24° 21' 50''$

16. $\alpha = 126° 59' 18''$
$\beta = 27° 29' 38''$
$\gamma = 25° 31' 4''$

17. $a = 1.9555$
$\alpha = 43° 36' 35''$
$\beta = 15° 37' 7''$

18. $\gamma = 77° 22' 16''$
$\beta = 51° 57' 20''$
$a = 83.732$

19. $\alpha = 87° 33' 58''$
$\gamma = 18° 6' 24''$
$b = 1.4033$

20. $\alpha = 78° 40' 32''$
$\beta = 41° 20' 47''$
$\gamma = 59° 58' 41''$

21. $\alpha = 32° 24' 0''$
$\beta = 55° 0' 28''$
$\gamma = 92° 35' 32''$

22. Impossible.

23. $\alpha = 142° 28' 9''$ or $21° 31' 11''$
$\beta = 29° 31' 31''$ or $150° 28' 29''$
$a = .080746$ or $.04862$

24. $\gamma = 76° 50' 20''$ or $103° 9' 40''$
$\alpha = 69° 39' 35''$ or $43° 20' 15''$
$a = 17.405$ or 12.739

25. $c = 502.28$
$b = 300.25$
$\gamma = 23° 7' 3''$

26. $\alpha = 96° 9' 32''$
$\beta = 41° 11' 10''$
$\gamma = 42° 39' 18''$

27. $A = 150.$

30. $A = 108.61$

33. $A = 368.91.$

35. 172.8 ft.

36. 106.1 ft.

37. 3.710 mi.

38. 97.14
124.59
178.64

39. 60.1 ft.

40. 57.93 mi. per hr.

41. 3888.0 ft.

42. 6328.7 ft.

43. 239600 mi.

Art. 102; Page 102

1. $1\frac{3}{4}$ in. **2.** $60°, 72°, 135°$. **3.** $\frac{2\pi}{9}, \frac{11\pi}{36}, \frac{19\pi}{90}, \frac{196\pi}{675}$.

4. $42° 58' 18''$. **5.** $35° 48' 35''$. **6.** 13.754 in.

7. $\frac{\pi}{9}$. **8.** $\frac{55\pi}{144}, \frac{73\pi}{144}$. **9.** $\frac{2\pi}{9}, \frac{\pi}{3}, \frac{4\pi}{9}$.

10. $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$. **11.** $\frac{4\pi}{77}, \frac{10\pi}{77}, \frac{16\pi}{77}, \frac{22\pi}{77}, \frac{28\pi}{77}, \frac{34\pi}{77}, \frac{40\pi}{77}$.

12. $9\frac{3}{5}$ in. **49.** $0°, 180°$.

50. $30°, 150°, 210°, 330°, 45°, 135°, 225°, 315°$.

51. $30°, 150°$. **52.** $\tan^{-1}(2 \pm \sqrt{3})$. **53.** $0°, 180°, \cos^{-1}\frac{1}{5}$.

54. $45°, 135°, 225°, 215°, \sin^{-1}\sqrt{\frac{1}{5}}$. **55.** $60°, 300°, 90°, 270°$.

56. $60°, 120°, 240°, 300°, 45°, 135°, 225°, 315°$.

57. $\tan^{-1}\frac{3}{4}$. **58.** $90°, 270°, \sin^{-1}(-\frac{21}{29})$.

59. $0°, 45°, 90°, 135°, 180°, 225°, 270°, 315°$.

60. $0°, 45°, 60°, 90°, 120°, 180°, 225°, 240°, 270°, 300°, 315°$.

61. $7\frac{1}{2}°, 37\frac{1}{2}°, 67\frac{1}{2}°, 97\frac{1}{2}°, 127\frac{1}{2}°, 157\frac{1}{2}°$, etc.

62. $30°, 150°, 210°, 330°$.

63. $90°, 270°, 70°, 110°, 190°, 230°, 310°, 350°$.

64. $90°, 180°$. **65.** $210°, 330°$.

66. $45°, 215°, 67\frac{1}{2}°, 157\frac{1}{2}°, 247\frac{1}{2}°, 337\frac{1}{2}°$.

67. $60°, 120°, 240°, 300°, 18°, 54°, 90°, 126°, 162°$, etc.

68. $60°, 90°, 120°, 240°, 270°, 300°$.

69. $\frac{2u}{1+u^2}$. **70.** $0, \pm\sqrt{\frac{1}{2}}$. **71.** $\pm\frac{\sqrt{3}}{2}$. **72.** $\frac{1}{5}\sqrt{10}, -\frac{12}{31}\sqrt{10}$.

73. $\frac{-1 \pm \sqrt{1+m^2}}{m}$. **85.** $\sqrt{3}$. **86.** $\frac{1}{5}\sqrt{5}$. **87.** $0, \frac{1}{3}\sqrt{3}$.

88. $\frac{ab + \sqrt{1-a^2}\cdot\sqrt{1-b^2}}{b\sqrt{1-a^2} + a\sqrt{1-b^2}}$. **97.** $26° 34'$. **98.** 711.7.

99. 532. **100.** 50.0. **101.** 1478.5.

102. 93,470,000 mi. **103.** 51.9 ft. **104.** 27.925.

105. 112.36. **106.** $\frac{r^2}{2}$. **107.** 129.90 ft.

108. 8.2596. **109.** 70° 31′ 43″. **110.** 5296 ft., 251 ft.

111. 196. **112.** 89.431. **114.** $a\sqrt{2}$.

115. 90 ft., 40 ft. **116.** 13.66. **117.** 103.97.

118. 820.54. **119.** 535.4. **120.** 25.43.

121. 567.3. **122.** 132.6. **123.** 36.7 ft.

124. 641. **125.** 962.605. **127.** 16.33 N. 75° 36′ E.

128. a. **129.** 4009. **130.** 1674.3.

Art. 116; Page 124

1. $1, i, -1, -i$.

2. $1, \ \frac{1}{2}+\frac{1}{2}\sqrt{3}\,i, \ -\frac{1}{2}+\frac{1}{2}\sqrt{3}\,i, \ -1, \ -\frac{1}{2}-\frac{1}{2}\sqrt{3}\,i, \ \frac{1}{2}-\frac{1}{2}\sqrt{3}\,i$.

4. $\pm(2+i)$. **5.** $\pm(1-2\,i)$.

6. $\pm(1.272+.786\,i)$.

7. $\frac{1}{2}\sqrt{2}+\frac{1}{2}\sqrt{2}\,i, \ -\frac{1}{2}\sqrt{2}-\frac{1}{2}\sqrt{2}\,i$.

8. $-\frac{1}{2}\sqrt{2}+\frac{1}{2}\sqrt{2}\,i, \ \frac{1}{2}\sqrt{2}-\frac{1}{2}\sqrt{2}\,i$.

10. $1.0842+.29051\,i$,
$-.79370+.79370\,i$,
$-.29051-1.0842\,i$.

Art. 139; Page 143

1. $\beta = 78° 9′ 22″$
$c = 10° 45′ 55″$
$\alpha = 2° 14′ 5″$

2. $\alpha = 41° 11′ 53″$
$\beta = 56° 19′ 56″$
$c = 40° 27′ 11″$

3. $b = 8° 26′ 14″$
$a = 120° 59′ 19″$
$\alpha = 95° 2′ 10″$

4. $\beta = 75° 21′ 53″$
$b = 44° 43′ 49″$
$a = 14° 59′ 33″$

5. $\alpha = 111° 23' 47''$
$\beta = 120° 40' 56''$
$c = 76° 33' 24''$

6. $\beta = 101° 38' 28''$
$\alpha = 112° 13' 48''$
$b = 102° 35' 26''$

7. $\beta = 46° 1' 28''$
$b = 15° 18' 0''$
$\alpha = 46° 2' 40''$

8. $\alpha = 68° 42' 11''$
$\beta = 155° 48' 0''$
$a = 27° 37' 26''$

9. $\beta = 153° 31' 29''$ or $26° 28' 31''$
$c = 50° 43' 22''$ or $129° 16' 38''$
$b = 159° 48' 44''$ or $20° 11' 16''$

Art. 142; Page 144

1. $a = 117° 45' 28''$
$\beta = 96° 27' 1''$
$\gamma = 93° 0' 51''$

2. $\gamma = 88° 23' 11''$
$a = 69° 48' 42''$
$b = 94° 22' 46''$

3. $\alpha = 8° 49' 46''$
$\gamma = 28° 3' 4''$
$b = 106° 56' 53''$

4. $\alpha = 160° 13' 48''$
$\beta = 105° 21' 16''$
$\gamma = 104° 25' 45''$

5. Sides, $32° 45' 6''$
angles, $105° 49' 32''$

6. Sides, $112° 32' 20''$
base, $46° 15' 12''$

Art. 156; Page 154

1. $\beta = 11° 0' 47''$
$\gamma = 92° 8' 27''$
$a = 114° 42' 50''$

2. $\beta = 14° 53' 47''$
$\gamma = 170° 26' 51''$
$a = 55° 56' 0''$

3. $\alpha = 71° 1' 23''$
$\gamma = 84° 22' 25''$
$b = 82° 1' 30''$

6. $a = 24° 35' 10''$
$c = 43° 29' 48''$
$\beta = 154° 19' 20''$

8. $\alpha = 133° 28' 34''$
$\beta = 169° 38' 12''$
$\gamma = 132° 6' 14''$

9. $\alpha = 104° 49' 50''$
$\beta = 84° 51' 42''$
$\gamma = 95° 18' 24''$

10. $\alpha = 84° 57' 8''$
$\beta = 57° 47' 44''$
$\gamma = 43° 4' 36''$

14. $\alpha = 155° 14' 24''$ or $21° 52' 40''$
$\beta = 25° 50' 58''$ or $154° 9' 2''$
$a = 107° 34' 50''$ or $58° 0' 44''$

15. Two solutions. **24.** No solution.

Art. 160 ; Page 159

2. $2229\frac{5}{6}$ miles. **3.** $4291\frac{1}{3}$ miles.

LOGARITHMIC

AND

TRIGONOMETRIC TABLES

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PREFACE

In presenting the accompanying tables we wish especially to call attention to the three- and four-place tables. The three-place tables are included because they enable the student just beginning the subject of trigonometry to work examples with a very small amount of mechanical work. This allows the student to focus his attention upon the principles of trigonometry involved without being distracted by cumbersome computations. Also the principles of interpolation are so clearly brought to light by the use of the three-place table that they are almost self-evident.

The four-place tables were introduced because the conviction is becoming more and more general that four-place tables are sufficient for purposes of instruction. When logarithms were first introduced there was a feeling on the part of many mathematicians that extended tables should be used in numerical computations. Since that time, due to a growing confidence in the accuracy of logarithms, a process of simplification has been going on. Seven-place tables were used extensively some years ago. These gave way to six-place tables, which in turn yielded to five-place tables, which are now being replaced by four-place tables. Some instructors are contenting themselves with three-place tables and with slide rules.

The four-place tables submitted herewith are so arranged that no interpolations are required in those parts of the tables where the variation is the most rapid, provided no greater accuracy is desired than is usually demanded of a four-place table.

Five-place tables have been the standard for a great variety of work. By their use the student is made acquainted with standard tables of sufficient accuracy for practically all computations. It is believed, too, that a student will gain greater independence in the use of a table if he is required to use a variety of tables.

The table of squares, square roots and reciprocals is often not included with the trigonometric tables. We believe, however, that these tables may be used to great advantage. They enable us to dispense with the use of logarithms altogether whenever this seems desirable. By their use the various problems in trigonometry may be solved by means of additions, subtractions and multiplications.

University of Minnesota,
June, 1917.

CONTENTS

	PAGE
Introduction to Tables	1
TABLE I. Natural Trigonometric Functions, to three places .	17
TABLE II. Logarithms of Numbers, to three places . . .	19
TABLE III. Logarithms of Trigonometric Functions, to three places	19
TABLE IV. Natural Trigonometric Functions, to four places .	21
TABLE V. Logarithms of Numbers, to four places . . .	27
TABLE VI. Logarithms of Trigonometric Functions, to four places	33
TABLE VII. Squares, Square Roots, and Reciprocals . . .	45
TABLE VIII. Logarithms of Numbers, to five places . . .	55
TABLE IX. Logarithms of Trigonometric Functions, to five places	77

INTRODUCTION TO TABLES

LOGARITHMS

1. Definition. The logarithm of a number is the exponent to which a fixed number must be raised in order to produce the number. The fixed number affected by the exponent is called the *base* of the logarithmic system.

Thus, if a is the base, n the number, and x the exponent, we have

$$a^x = n,$$

which relation may be expressed by

$$\log_a n = x,$$

which is read the *logarithm of n, to the base a, is x.*

There are two systems of logarithms now in use: (1) the *natural* or *Napierian System* which is characterized by the base e, e being a transcendental number approximately equal to 2.718281828^+; (2) the *common system* which is characterized by the base 10.

The common system was developed by Briggs and is the system presented in the following tables.

2. Laws of the characteristic. The common logarithm of a number consists of two parts, the integral part and the decimal part. Either part may be equal to zero.

The integral part of a logarithm is called the *characteristic*, while the decimal part is called the *mantissa*. Thus, to four decimal places,

$$10^{1.3861} = 24.33 \text{ or } \log_{10} 24.33 = 1.3861.$$

The number 1.3861 is the logarithm of 24.33 to the base 10. The characteristic is 1 and the mantissa is .3861.

The first law of the characteristic is deduced from the following:

$$\begin{array}{lll} 10^0 = 1 & \text{or} & \log_{10} 1 = 0 \\ 10^1 = 10 & & \log_{10} 10 = 1 \\ 10^2 = 100 & & \log_{10} 100 = 2 \\ 10^3 = 1000 & & \log_{10} 1000 = 3 \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \end{array}$$

A number between 100 and 1000 has a logarithm that lies between 2 and 3, hence it is equal to 2 plus some positive decimal. But all numbers between 100 and 1000 have three figures to the left of the decimal point, as 436.259, and the logarithms of all such numbers have a characteristic 2. In general it will be seen that the following law is true.

The First Law of the Characteristic. *All the numbers greater than* 1 *have a positive logarithm, and the characteristic is one less than the number of places to the left of the decimal point.*

The second law of the characteristic is deduced from the following:

$$\begin{array}{lll} 10^0 = 1 & \text{or} & \log_{10} 1 = 0 \\ 10^{-1} = .1 & & \log_{10} .1 = -1 \\ 10^{-2} = .01 & & \log_{10} .01 = -2 \\ 10^{-3} = .001 & & \log_{10} .001 = -3 \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \end{array}$$

All numbers lying between .01 and .001 have logarithms between -2 and -3. The number .00368 lies between .01 and .001. Likewise all decimal numbers having two zeros between the decimal point and the first significant figure lie between .01 and .001, hence the logarithms of all such numbers lie between -2

and -3. Thus -2.4689 is the logarithm of some number between .01 and .001.

In practice it is found convenient to transform all the logarithms so that the mantissas are positive. Thus -2.4689 may be written $3-2.4689-3=+.5311-3$ which may be written $\bar{3}.5311$. This notation means that the characteristic is -3, and that the mantissa is $+.5311$. It is seen that $\bar{3}.5311$ is a composite number part of which is positive and part negative. All numbers that lie between .01 and .001 have logarithms whose characteristic is -3 and whose mantissa is positive. It is easily seen that the following law is true.

Second Law of the Characteristic. *All positive numbers less than* 1 *have a negative characteristic, and it is numerically one greater than the number of zeros between the decimal point and the first significant figure.*

$$\text{Thus } \log .000468 = \bar{4}.67025$$
$$\log .175 \quad = \bar{1}.24304.$$

3. Mantissa not affected by change of decimal point. All numbers which differ from one another only in the position of the decimal point have logarithms which differ from one another only in the characteristic, *i.e.* all such logarithms have the same mantissa. This is illustrated by the example

$$10^{3.24304} = 1750 \tag{1}$$

Multiplying each member of the equation (1) by 10, we have

$$10^{4.24304} = 17500, \tag{2}$$

and dividing each member of (1) by 10^4 gives

$$10^{\bar{1}.24304} = .175 \tag{3}$$

The relations given in (1), (2), and (3) may be written

$$\log 1750 = 3.24304$$
$$\log 17500 = 4.24304$$
$$\log .175 = \bar{1}.24304.$$

The property just illustrated is peculiar to the system of logarithms of which 10 is the base.

4. Laws of logarithms.

(a) *The logarithm of the product of two numbers is equal to the sum of the logarithms of the numbers.*

Given $a^x = m$ (1) or $\log_a m = x$ (3)

$a^y = n$ (2) or $\log_a n = y$. (4)

From the law of exponents

$$a^{x+y} = mn \quad \text{or} \quad \log_a mn = x + y$$

$$\therefore \log_a mn = \log_a m + \log_a n. \quad \text{By (3) and (4).}$$

This law enables us to replace multiplication by addition with the aid of a table of logarithms.

(b) *The logarithm of the quotient of two numbers is equal to the logarithm of the dividend minus the logarithm of the divisor.*

From (1) and (2) above we have, applying a law of exponents,

$$a^{x-y} = \frac{m}{n} \quad \text{or} \quad \log_a \frac{m}{n} = x - y$$

$$\therefore \log_a \frac{m}{n} = \log_a m - \log_a n.$$

This law enables us to replace division by subtraction.

(c) *The logarithm of a number affected by an exponent is equal to the exponent multiplied by the logarithm of the number.*

Given $a^x = m$

there results

$$a^{px} = m^p \quad \text{or} \quad \log_a m^p = px$$

$$\therefore \log_a m^p = p \log_a m.$$

This law is true whether p be a fraction or an integer.

Thus $\log 254^3 = 3 \log 254$

$$\log \sqrt[3]{254} = \log 254^{\frac{1}{3}} = \tfrac{1}{3} \log 254.$$

This law enables us to replace involution by multiplication, and evolution by division.

Application. From the above laws we may write

$$\log \frac{ab^2c^{\frac{1}{3}}}{de^{\frac{2}{3}}} = \log a + 2\log b + \tfrac{1}{3}\log c - \log d - \tfrac{2}{3}\log e.$$

It is left to the student to follow out the individual steps.

5. Computation of logarithms. Logarithms are computed from infinite series.

(*a*) To compute the logarithms of numbers we have

$$\log_e m = \log_e n + 2\left[\left(\frac{m-n}{m+n}\right) + \frac{1}{3}\left(\frac{m-n}{m+n}\right)^3 + \frac{1}{5}\left(\frac{m-n}{m+n}\right)^5 + \cdots\right],$$

a formula developed in algebra.

By letting $n=1$, $m=2$; then $n=2$, $m=3$; etc.; the logarithms of the successive numbers may be computed to the base e. By multiplying these logarithms by 0.43 429 448, which number is called the modulus of the common logarithms and is designated by the letter M, we obtain the logarithms of the numbers to the base 10.

By means of the following formulas common logarithms may be converted into natural logarithms and vice versa:

$$\text{Common } \log n = M \times x \text{ natural } \log n$$

$$\text{Natural } \log n = \frac{1}{M} \text{ common } \log n,$$

where $$M = \log_{10} e = 0.43\ 429\ 448 \cdots$$

or $$\frac{1}{M} = \log_e 10 = 2.30\ 258\ 509 \cdots$$

The values of sin x and cos x may be computed from the following series:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

The logarithms of these functions may then be obtained from the table of the logarithms of numbers.

ARRANGEMENT OF TABLES

Because of the relation between any trigonometric function and its cofunction, it is possible to use a given table for two functions. One function is always designated at the top of the page or column; its cofunction is designated at the bottom.

Angles written in the left-hand column, and at the top of the page, are associated with the function given at the top of the page; angles written in the right-hand column and at the bottom of the page, are associated with the function given at the bottom of the page.

INTERPOLATION

The logarithms of numbers and the values of the trigonometric functions as well as the logarithms of the trigonometric functions are tabulated for certain values taken at regular intervals. To determine them for other values of the numbers or angles not given in the tables, a process known as interpolation is employed. In applying this process it is always assumed that the logarithm of a number is proportional to the number within a small interval to which interpolation is applied.

Likewise the trigonometric function of an angle is assumed to be proportional to the angle within a small interval. While these assumptions are not in harmony with the facts, the values thus obtained are sufficiently accurate for purposes of computation. The result obtained by interpolation is always restricted to the number of decimal places given in the table.

EXPLANATION OF TABLES I, II, AND III

1. To find the value of any trigonometric function when the angle is an integral number of degrees is simply a matter of reading the result from the proper column of Table I.

Thus $\tan 28° = 0.532$

the number 0.532 being found in the column marked Tan. and to the right of 28°.

If the angle is greater than 45° the number is found in the column with the proper designation at the *bottom of the page.*

Thus $\cot 62° = 0.532$

the number 0.532 being found in the column designated Cot. at the bottom of the page and to the left of 62°.

It will be found that

$$\sin 41° = 0.656 \qquad \csc 54° = 1.236$$
$$\cos 17° = 0.956 \qquad \sin 54° = 0.809.$$

2. To find the value of the trigonometric function when the angle contains a given number of *minutes* as well as degrees, it is necessary to interpolate. This is done as follows: Look up the values of the desired trigonometric functions for the two consecutive values given for the angles which are respectively just less and greater than the desired angle. Find the difference between these values and divide this difference proportionally, as indicated by the given number of minutes.

(*a*) To find $\sin 34° 25'$.

$$\sin 34° = .559$$
$$\sin 35° = .574$$
$$\text{Difference for } 1° = .015$$

Since $25'$ is $\frac{25}{60}$ of 1°, the correction for $25'$ is $\frac{25}{60} \times .015 = .006$, and since the sine increases as the angle increases .006 must be added to .559, hence

$$\sin 34° 24' = .565.$$

(*b*) To find $\cos 19° 40'$.

$$\cos 19° = .946$$
$$\cos 20° = .940$$
$$\text{Difference for } 1° = .006$$

The difference for $40'$ or $\frac{2}{3}$ of a degree is $\frac{2}{3} \times .006 = .004$. *Since the cosine decreases as the angle increases it is necessary to subtract* .004 *from* .946 *giving*

$$\cos 19° 40' = .942.$$

3. To find an angle when the trigonometric function is given. This is exceedingly simple provided the given trigonometric function is one of the tabulated values. In this case it is only necessary to read the angle from the table, as illustrated by the following example:

(*a*) To find x when $\cos x = .956$.

In the column marked Cos. the number .956 is found. It is to the right of 17°, hence $x = 17°$.

(*b*) To find x when $\cos x = .629$.

The number .629 is found in the column marked Cos. at the bottom of the page. This signifies that the number of degrees must be read from the right-hand column marked Deg. It is then seen that $x = 51°$.

4. To find the angle when the given trigonometric function is not one of the tabulated values.

In this case the two values that are respectively just larger and smaller than the given function are taken from the table, the difference between the two is found, and the difference between the given function and one of the tabulated values is also found. The ratio of this difference to the difference between the two consecutive values, called the tabular difference, gives the fractional part of a degree that is sought. This will be illustrated by two examples.

(*a*) To find x when $\tan x = 0.111$.

From the table it is seen that

$$\begin{aligned} \tan 6° &= 0.105 \\ \tan 7° &= 0.123 \\ \text{Difference for } 1° &= 0.018 \end{aligned}$$

Also

$$\begin{aligned} \tan 6° &= 0.105 \\ \tan\ x &= 0.111 \\ &\overline{0.006} \end{aligned}$$

It is desired to find the angle that corresponds to 0.006. Since the difference for $1° = 0.018$, the angle corresponding to 0.006 is $\frac{.006}{.018}$ or one third degree. Hence $x = 6°\ 20'$.

The problem might have been solved by finding the difference between 0.111 and 0.123, but in this case it would have been necessary to subtract the resulting correction from 7°.

(*b*) To find x when $\cot x = 0.661$.

From the table it is found that

$$\begin{aligned} \cot 56° &= 0.675 \\ \cot 57° &= 0.649 \\ \text{Difference for } 1° &= 0.026 \end{aligned}$$

Comparing the given cotangent with the cotangent of the smaller angle we have

$$\begin{aligned} \cot 56° &= 0.675 \\ \cot x \quad &= 0.661 \\ &\quad 0.014 \end{aligned}$$

The angle corresponding to this difference is then $\frac{14}{26}$ of one degree, or approximately one half degree. Hence $x = 56°\ 30'$. It should be noted that the cotangent decreases as the angle increases.

The methods explained for Table I may be used in connection with Table III from which the logarithms of the trigonometric functions are found to three places of decimals; it should be noted, however, that all the logarithms given in Table III are too large by 10. By this device no negative characteristics are tabulated.

5. The mantissas of the common logarithms of numbers are given in Table II. This table may be used to find the logarithm of a number containing 1, 2, or 3 significant figures.

a. To find the logarithm of 3. Opposite the number 30 we find the logarithm .477 and since the characteristic of the loga-

rithm of 3 is 0, and the mantissa of the logarithm of 3 is identical with the mantissa of the logarithm of 30 we have

$$\log 3 = 0.477.$$

b. To find log 4700. Using the *first law of the characteristic* and noting that the mantissa of the logarithm of 47 is .672, we have

$$\log 4700 = 3.672$$

c. To find log 478.

$$\begin{aligned} \log 470 &= 2.672 \\ \log 480 &= 2.681 \\ \text{Difference for } 10 &= \quad .009 \end{aligned}$$

The difference .009 corresponds to a difference of 1 in the tenths' place and hence a difference of 1 in the units' place would require a difference of .0009 in the logarithm. Hence 8 in the units' place calls for a correction of .009 × 8 or .0072. We then have

$$\log 478 = 2.679,$$

the logarithm having been contracted to three places.

d. To find x, given $\log x = 1.643$. Since .643 is one of the tabulated logarithms, the value of x is found directly from the table. It is seen that $x = 44.0$. The characteristic 1 determines the position of the decimal point.

e. To find x, given $\log x = 3.646$.

$$\begin{aligned} \log 4400 &= 3.643 \\ \log 4500 &= 3.653 \\ \text{Difference of } 100 &= \quad .010 \end{aligned}$$

Also

$$\begin{aligned} \log 4400 &= 3.643 \\ \log x &= 3.646 \\ &\quad .003 \end{aligned}$$

The correction is then $\frac{3}{10} \times 100$ or 30, from which we have

$$x = 4430.$$

The position of the decimal point was determined by the characteristic 3.

EXPLANATION OF TABLE IV

Table IV may be used to find the natural sine, cosine, tangent, and cotangent of an angle expressed in terms of degrees and minutes. Whenever an angle is expressed in terms of degrees, minutes, and seconds it is advisable to use a five-place table.

Table IV, as well as the following tables of the logarithms of numbers and of the trigonometric functions, is supplied with auxiliary tables of proportional parts which greatly simplify the process of interpolation. The use of Table IV will be illustrated by a few examples.

1. Find $\sin 23° 10'$. From the table designated *sine* we find the value of the function at the intersection of the row containing 23° and the column designated at the top by 10′, giving $\sin 23° 10' = .3934$.

2. Find $\sin 23° 17'$.

$$\begin{aligned} \sin 23° 10' &= .3934 \\ \sin 23° 20' &= .3961 \\ \text{Difference for } 10' &= .0027 \\ \text{Difference for } 1' &= .00027 \\ \text{Difference for } 7' &= .00189 \end{aligned}$$

By reducing the last decimal to 4 places since we are dealing with a 4-place table we have correction for $7' = .0019$.
Then

$$\begin{aligned} \sin 23° 10' &= .3934 \\ \text{Correction for } 7' &= .0019 \\ \therefore \sin 23° 17' &= .3953 \end{aligned}$$

The correction for 7′ can be taken immediately from the small table of proportional parts headed 27 found at the right of the page. The number 27 is the *tabular difference* and is the difference between $\sin 23° 10'$ and $\sin 23° 20'$ as is evident from the above work.

3. Find $\cos 61° 43'$.

$$\begin{aligned} \cos 61° 40' &= .4746 \\ \cos 61° 50' &= .4720 \\ \text{Difference for } 10' &= .0026 \end{aligned}$$

The correction for three minutes found from the table of proportional parts designated by 26 is 7.8 which is understood to be .00078 or to four places .0008. Hence $\cos 61° 43' = .4738$.

The correction .0008 is subtracted from .4746 because the cosine decreases as the angle increases.

4. To find x given $\tan x = 3.412$. From the table, without interpolation, it is seen that

$$x = 73° 40'.$$

5. To find x given $\tan x = 3.432$.

The angle x lies between $73° 40'$ and $73° 50'$ as is evident from the following values.

$$\begin{aligned} \tan 73° 40' &= 3.412 \\ \tan 73° 50' &= 3.450 \\ \text{Difference for } 10' &= .038 \\ \text{Difference for } 1' &= .0038 \\ \text{Also } \tan x &= 3.432 \\ \tan 73° 40' &= 3.412 \\ & .020 \end{aligned}$$

Since .0038 corresponds to $1'$, the number of minutes corresponding to .020 is $\frac{.020}{.0038} = 5.3$, $\therefore x = 73° 45'$. The minor fraction of a minute was discarded.

The result could have been obtained more simply by the use of the proportional part table marked 38. From that table it is seen that a difference of 19 in the tangent gives a difference of $5'$ in the angle.

TABLE V

The four-place table of logarithms may be used whenever the number has not more than four significant figures. In the first part of the table the numbers are given to four figures and no interpolations are necessary. In the second part of the table the numbers are tabulated to three figures. The fourth figure may be found by means of the tables of proportional parts. The first column of figures on the left of the page and the first row of figures across the top of the page are the numbers whose logarithms are tabulated on the rest of the page.

1. Find log 1.836. From the first part of the table it is seen that log 1.836 = 0.2639, the number .2639 being found to the right of 183 and in the column headed 6.

2. Find log 5836.

$$\log 5830 = 3.7657$$
$$\text{Correction for 6 from P.P. table } 7 = \quad 4$$
$$\therefore \log 5836 = \overline{3.7661}$$

3. Find x given $\log x = 1.2366$. The mantissa .2366 is not tabulated, but in the first part of the table where the numbers are given to four figures it is seen that log 17.24 = 1.2365. Since the mantissa .2365 is nearer the given mantissa .2366 than any other, we have $x = 17.24$ without interpolation. In fact, the object of tabulating the numbers to four figures in the first part of the table was to avoid interpolation in that part of the table where the logarithms change most rapidly.

4. Find x given $\log x = \overline{2}.3469$.

$$\log .0222 = \overline{2}.3464$$
$$\log .0223 = \overline{2}.3483$$
$$.0019$$
$$\log .0222 = \overline{2}.3464$$
$$\log x \quad = \overline{2}.3469$$
$$.0005$$

The correction for .0005 in the fourth significant figure of the number is seen, from the P.P. table 19, to be 3. Hence $x = .02223$.

TABLE VI

The logarithms of the trigonometric functions are tabulated to four places. This table may be used conveniently when the angle is expressed in degrees and minutes.

In those parts of the table where the functions vary most rapidly the values are tabulated for every minute. Here no interpolation is necessary. In the main body of the table the angles are expressed at intervals of 10′, the minute being determined by the use of the tables of proportional parts.

The table is used according to the principles outlined for Table IV.

TABLE VII

The squares of all numbers of three significant figures may be found from Table VII without interpolation, if due attention is given to the decimal point. To find the square of a number of four or five significant figures it is necessary to interpolate. This is done according to the principles of interpolation explained in connection with other tables.

The square root of a number is found from either the column $\sqrt{n}$ or $\sqrt{10\,n}$, depending upon the position of the decimal point in the given number. A few examples illustrate the use of the table.

$$2.83^2 = 8.0089$$
$$28.3^2 = 800.89$$
$$\sqrt{2.83} = 1.68226$$
$$\sqrt{28.3} = 5.31977$$
$$\sqrt{283} = 16.8226$$
$$\sqrt{2830} = 53.1977$$

By including the table of squares, square roots, and reciprocals it is possible to work the usual problems of trigonometry quite conveniently without the use of logarithms.

TABLES VIII AND IX

Table VIII gives the logarithms of numbers to five places. It should be used whenever the logarithm of a number of five or six significant figures is desired.

Table IX gives the logarithms of the trigonometric functions to five places of decimals. It should be used whenever the angle is expressed in terms of degrees, minutes, and seconds.

Both tables are supplied with auxiliary tables giving proportional parts, thus simplifying interpolation.

It will not be necessary to illustrate the use of these tables, since the principles involved are identical with those employed in connection with the earlier tables.

TABLE I

THE NATURAL TRIGONOMETRIC FUNCTIONS

FROM

0° TO 90° AT INTERVALS OF ONE DEGREE

ALSO

THE CONVERSION OF DEGREES TO RADIANS

TO

THREE DECIMAL PLACES

NATURAL TRIGONOMETRIC FUNCTIONS

Rad.	Deg.	Sin.	Tan.	Sec.	Csc.	Cot.	Cos.	Deg.	Rad.
0.000	0°	0.000	0.000	1.000	∞	∞	1.000	90°	1.571
0.017	1°	0.017	0.017	1.000	57.30	57.29	1.000	89°	1.553
.035	2°	.035	.035	1.001	28.65	28.64	.999	88°	.536
.052	3°	.052	.052	1.001	19.11	19.08	.999	87°	.518
.070	4°	.070	.070	1.002	14.34	14.30	.998	86°	.501
.087	5°	.087	.087	1.004	11.47	11.43	.996	85°	.484
0.105	6°	0.105	0.105	1.006	9.567	9.514	0.995	84°	1.466
.122	7°	.122	.123	1.008	8.206	8.144	.993	83°	.449
.140	8°	.139	.141	1.010	7.185	7.115	.990	82°	.431
.157	9°	.156	.158	1.012	6.392	6.314	.988	81°	.414
.175	10°	.174	.176	1.015	5.759	5.671	.985	80°	.396
.192	11°	.191	.194	1.019	5.241	5.145	.982	79°	.379
.209	12°	.208	.213	1.022	4.810	4.705	.978	78°	.361
.227	13°	.225	.231	1.026	4.445	4.331	.974	77°	.344
.244	14°	.242	.249	1.031	4.134	4.011	.970	76°	.326
.262	15°	.259	.268	1.035	3.864	3.732	.966	75°	.309
0.279	16°	0.276	0.287	1.040	3.628	3.487	0.961	74°	1.292
.297	17°	.292	.306	1.046	3.420	3.271	.956	73°	.274
.314	18°	.309	.325	1.051	3.236	3.078	.951	72°	.257
.332	19°	.326	.344	1.058	3.072	2.904	.946	71°	.239
.349	20°	.342	.364	1.064	2.924	2.747	.940	70°	.222
.367	21°	.358	.384	1.071	2.790	2.605	.934	69°	.204
.384	22°	.375	.404	1.079	2.669	2.475	.927	68°	.187
.401	23°	.391	.424	1.086	2.559	2.356	.921	67°	.169
.419	24°	.407	.445	1.095	2.459	2.246	.914	66°	.152
.436	25°	.423	.466	1.103	2.366	2.145	.906	65°	.134
0.454	26°	0.438	0.488	1.113	2.281	2.050	0.899	64°	1.117
.471	27°	.454	.510	1.122	2.203	1.963	.891	63°	.100
.489	28°	.469	.532	1.133	2.130	1.881	.883	62°	.082
.506	29°	.485	.554	1.143	2.063	1.804	.875	61°	.065
.524	30°	.500	.577	1.155	2.000	1.732	.866	60°	.047
.541	31°	.515	.601	1.167	1.942	1.664	.857	59°	.030
.559	32°	.530	.625	1.179	1.887	1.600	.848	58°	1.012
.576	33°	.545	.649	1.192	1.836	1.540	.839	57°	.995
.593	34°	.559	.675	1.206	1.788	1.483	.829	56°	.977
.611	35°	.574	.700	1.221	1.743	1.428	.819	55°	.960
0.628	36°	0.588	0.727	1.236	1.701	1.376	0.809	54°	0.942
.646	37°	.602	.754	1.252	1.662	1.327	.799	53°	.925
.663	38°	.616	.781	1.269	1.624	1.280	.788	52°	.908
.681	39°	.629	.810	1.287	1.589	1.235	.777	51°	.890
.698	40°	.643	.839	1.305	1.556	1.192	.766	50°	.873
.716	41°	.656	.869	1.325	1.524	1.150	.755	49°	.855
.733	42°	.669	.900	1.346	1.494	1.111	.743	48°	.838
.750	43°	.682	.933	1.367	1.466	1.072	.731	47°	.820
.768	44°	.695	.966	1.390	1.440	1.036	.719	46°	.803
0.785	45°	0.707	1.000	1.414	1.414	1.000	0.707	45°	0.785
Rad.	Deg.	Cos.	Cot.	Csc.	Sec.	Tan.	Sin	Deg.	Rad.

TABLE II

COMMON LOGARITHMS OF NUMBERS

FROM

10 TO 99

TO

THREE DECIMAL PLACES

TABLE III

COMMON LOGARITHMS

OF THE

TRIGONOMETRIC FUNCTIONS

FROM

0° TO 90° AT INTERVALS OF ONE DEGREE

TO

THREE DECIMAL PLACES

LOGS OF NUMBERS . LOGS OF TRIGONOMETRIC FUNCTIONS

No.	Log.	No.	Log.
..	....	..	....
10	.000	55	.740
11	.041	56	.748
12	.079	57	.756
13	.114	58	.763
14	.146	59	.771
15	.176	60	.778
16	.204	61	.785
17	.230	62	.792
18	.255	63	.799
19	.279	64	.806
20	.301	65	.813
21	.322	66	.820
22	.342	67	.825
23	.362	68	.833
24	.380	69	.839
25	.398	70	.845
26	.415	71	.851
27	.431	72	.857
28	.447	73	.863
29	.462	74	.869
30	.477	75	.875
31	.491	76	.881
32	.505	77	.886
33	.519	78	.892
34	.531	79	.898
35	.544	80	.903
36	.556	81	.908
37	.568	82	.914
38	.580	83	.919
39	.591	84	.924
40	.602	85	.929
41	.613	86	.934
42	.623	87	.940
43	.633	88	.944
44	.643	89	.949
45	.653	90	.954
46	.663	91	.959
47	.672	92	.964
48	.681	93	.968
49	.690	94	.973
50	.699	95	.978
51	.708	96	.982
52	.716	97	.987
53	.724	98	.991
54	.732	99	.996
No.	Log.	No.	Log.

Deg.	Sin.	Tan.	Cot.	Cos.	Deg.
0°	—∞	—∞	∞	10.000	90°
1°	8.242	8.242	11.758	10.000	89°
2°	.543	.543	.457	9.999	88°
3°	.719	.719	.281	.999	87°
4°	.844	.845	.155	.999	86°
5°	8.940	8.942	11.058	.998	85°
6°	9.019	9.022	10.978	9.998	84°
7°	.086	.089	.911	.997	83°
8°	.144	.148	.852	.996	82°
9°	.194	.200	.800	.995	81°
10°	.240	.246	.754	.993	80°
11°	.281	.289	.711	.992	79°
12°	.318	.327	.673	.990	78°
13°	.352	.363	.637	.989	77°
14°	.384	.397	.603	.987	76°
15°	.413	.428	.572	.985	75°
16°	9.440	9.457	10.543	9.983	74°
17°	.466	.485	.515	.981	73°
18°	.490	.512	.488	.978	72°
19°	.513	.537	.463	.976	71°
20°	.534	.561	.439	.973	70°
21°	.554	.584	.416	.970	69°
22°	.574	.606	.394	.967	68°
23°	.592	.628	.372	.964	67°
24°	.609	.649	.351	.961	66°
25°	.626	.669	.331	.957	65°
26°	9.642	9.688	10.312	9.954	64°
27°	.657	.707	.293	.950	63°
28°	.672	.726	.274	.946	62°
29°	.686	.744	.256	.942	61°
30°	.699	.761	.239	.938	60°
31°	.712	.779	.221	.933	59°
32°	.724	.796	.204	.928	58°
33°	.736	.813	.187	.924	57°
34°	.748	.829	.171	.919	56°
35°	.759	.845	.155	.913	55°
36°	9.769	9.861	10.139	9.908	54°
37°	.779	.877	.123	.902	53°
38°	.789	.893	.107	.897	52°
39°	.799	.908	.092	.891	51°
40°	.808	.924	.076	.884	50°
41°	.817	.939	.061	.878	49°
42°	.826	.954	.046	.871	48°
43°	.834	.970	.030	.864	47°
44°	.842	.985	.015	.857	46°
45°	9.849	10.000	10.000	9.849	45°
Deg.	Cos.	Cot.	Tan.	Sin.	Deg.

TABLE IV

THE VALUES

OF

SINE, COSINE, TANGENT, AND COTANGENT

FROM

0° TO 90° AT INTERVALS OF 10 MINUTES

TO

FOUR FIGURES

°		0′	10′	20′	30′	40′	50′	60′		d.
0	0.	0000	0029	0058	0087	0116	0145	0175	89	29
1		0175	0204	0233	0262	0291	0320	0349	88	29
2		0349	0378	0407	0436	0465	0494	0523	87	29
3		0523	0552	0581	0610	0640	0669	0698	86	29
4		0698	0727	0756	0785	0814	0843	0872	**85**	29
5	0.	0872	0901	0929	0958	0987	1016	1045	84	29
6		1045	1074	1103	1132	1161	1190	1219	83	29
7		1219	1248	1276	1305	1334	1363	1392	82	29
8		1392	1421	1449	1478	1507	1536	1564	81	29
9		1564	1593	1622	1650	1679	1708	1736	**80**	29
10	0.	1736	1765	1794	1822	1851	1880	1908	79	29
11		1908	1937	1965	1994	2022	2051	2079	78	28
12		2079	2108	2136	2164	2193	2221	2250	77	28
13		2250	2278	2306	2334	2363	2391	2419	76	28
14		2419	2447	2476	2504	2532	2560	2588	**75**	28
15	0.	2588	2616	2644	2672	2700	2728	2756	74	28
16		2756	2784	2812	2840	2868	2896	2924	73	28
17		2924	2952	2979	3007	3035	3062	3090	72	28
18		3090	3118	3145	3173	3201	3228	3256	71	28
19		3256	3283	3311	3338	3365	3393	3420	**70**	27
20	0.	3420	3448	3475	3502	3529	3557	3584	69	27
21		3584	3611	3638	3665	3692	3719	3746	68	27
22		3746	3773	3800	3827	3854	3881	3907	67	27
23		3907	3934	3961	3987	4014	4041	4067	66	27
24		4067	4094	4120	4147	4173	4200	4226	**65**	26
25	0.	4226	4253	4279	4305	4331	4358	4384	64	26
26		4384	4410	4436	4462	4488	4514	4540	63	26
27		4540	4566	4592	4617	4643	4669	4695	62	26
28		4695	4720	4746	4772	4797	4823	4848	61	26
29		4848	4874	4899	4924	4950	4975	5000	**60**	25
30	0.	5000	5025	5050	5075	5100	5125	5150	59	25
31		5150	5175	5200	5225	5250	5275	5299	58	25
32		5299	5324	5348	5373	5398	5422	5446	57	24
33		5446	5471	5495	5519	5544	5568	5592	56	24
34		5592	5616	5640	5664	5688	5712	5736	**55**	24
35	0.	5736	5760	5783	5807	5831	5854	5878	54	24
36		5878	5901	5925	5948	5972	5995	6018	53	23
37		6018	6041	6065	6088	6111	6134	6157	52	23
38		6157	6180	6202	6225	6248	6271	6293	51	23
39		6293	6316	6338	6361	6383	6406	6428	**50**	22
40	0.	6428	6450	6472	6494	6517	6539	6561	49	22
41		6561	6583	6604	6626	6648	6670	6691	48	22
42		6691	6713	6734	6756	6777	6799	6820	47	22
43		6820	6841	6862	6884	6905	6926	6947	46	21
44		6947	6967	6988	7009	7030	7050	7071	**45**	21
45	0.	7071								
		60′	50′	40′	30′	20′	10′	0′	°	d.

P. P.

	30	**29**
1	3.0	2.9
2	6.0	5.8
3	9.0	8.7
4	12.0	11.6
5	15.0	14.5
6	18.0	17.4
7	21.0	20.3
8	24.0	23.2
9	27.0	26.1

	28	**27**
1	2.8	2.7
2	5.6	5.4
3	8.4	8.1
4	11.2	10.8
5	14.0	13.5
6	16.8	16.2
7	19.6	18.9
8	22.4	21.3
9	25.2	24.3

	26	**25**
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	24	**23**
1	2.4	2.3
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.6	20.7

	22	**21**	**20**
1	2.2	2.1	2.0
2	4.4	4.2	4.0
3	6.6	6.3	6.0
4	8.8	8.4	8.0
5	11.0	10.5	10.0
6	13.2	12.6	12.0
7	15.4	14.7	14.0
8	17.6	16.8	16.0
9	19.8	18.9	18.0

°	0′	10′	20′	30′	40′	50′	60′		d.
45	0. 7071	7092	7112	7133	7153	7173	7193	44	20
46	7193	7214	7234	7254	7274	7294	7314	43	20
47	7314	7333	7353	7373	7392	7412	7431	42	20
48	7431	7451	7470	7490	7509	7528	7547	41	19
49	7547	7566	7585	7604	7623	7642	7660	**40**	19
50	0. 7660	7679	7698	7716	7735	7753	7771	39	18
51	7771	7790	7808	7826	7844	7862	7880	38	18
52	7880	7898	7916	7934	7951	7969	7986	37	18
53	7986	8004	8021	8039	8056	8073	8090	36	17
54	8090	8107	8124	8141	8158	8175	8192	**35**	17
55	0. 8192	8208	8225	8241	8258	8274	8290	34	16
56	8290	8307	8323	8339	8355	8371	8387	33	16
57	8387	8403	8418	8434	8450	8465	8480	32	16
58	8480	8496	8511	8526	8542	8557	8572	31	15
59	8572	8587	8601	8616	8631	8646	8660	**30**	15
60	0. 8660	8675	8689	8704	8718	8732	8746	29	14
61	8746	8760	8774	8788	8802	8816	8829	28	14
62	8829	8843	8857	8870	8884	8897	8910	27	14
63	8910	8923	8936	8949	8962	8975	8988	26	13
64	8988	9001	9013	9026	9038	9051	9063	**25**	12
65	0. 9063	9075	9088	9100	9112	9124	9135	24	12
66	9135	9147	9159	9171	9182	9194	9205	23	12
67	9205	9216	9228	9239	9250	9261	9272	22	11
68	9272	9283	9293	9304	9315	9325	9336	21	11
69	9336	9346	9356	9367	9377	9387	9397	**20**	10
70	0. 9397	9407	9417	9426	9436	9446	9455	19	10
71	9455	9465	9474	9483	9492	9502	9511	18	9
72	9511	9520	9528	9537	9546	9555	9563	17	9
73	9563	9572	9580	9588	9596	9605	9613	16	8
74	9613	9621	9628	9636	9644	9652	9659	**15**	8
75	0. 9659	9667	9674	9681	9689	9696	9703	14	7
76	9703	9710	9717	9724	9730	9737	9744	13	7
77	9744	9750	9757	9763	9769	9775	9781	12	6
78	9781	9787	9793	9799	9805	9811	9816	11	6
79	9816	9822	9827	9833	9838	9843	9848	**10**	5
80	0. 9848	9853	9858	9863	9868	9872	9877	9	5
81	9877	9881	9886	9890	9894	9899	9903	8	4
82	9903	9907	9911	9914	9918	9922	9925	7	4
83	9925	9929	9932	9936	9939	9942	9945	6	3
84	9945	9948	9951	9954	9957	9959	9962	**5**	3
85	0. 9962	9964	9967	9969	9971	9974	9976	4	2
86	9976	9978	9980	9981	9983	9985	9986	3	2
87	9986	9988	9989	9990	9992	9993	9994	2	1
88	9994	9995	9996	9997	9997	9998	9998	1	1
89	9998	9999	9999	*0000	*0000	*0000	*0000	**0**	0
90	1. 0000								
	60′	50′	40′	30′	20′	10′	0′	°	d.

P. P.

	21	**20**	**19**	**18**
1	2.1	2.0	1.9	1.8
2	4.2	4.0	3.8	3.6
3	6.3	6.0	5.7	5.4
4	8.4	8.0	7.6	7.2
5	10.5	10.0	9.5	9.0
6	12.6	12.0	11.4	10.8
7	14.7	14.0	13.3	12.6
8	16.8	16.0	15.2	14.4
9	18.9	18.0	17.1	16.2

	17	**16**	**15**	**14**
1	1.7	1.6	1.5	1.4
2	3.4	3.2	3.0	2.8
3	5.1	4.8	4.5	4.2
4	6.8	6.4	6.0	5.6
5	8.5	8.0	7.5	7.0
6	10.2	9.6	9.0	8.4
7	11.9	11.2	10.5	9.8
8	13.6	12.8	12.0	11.2
9	15.3	14.4	13.5	12.6

	13	**12**	**11**	**10**
1	1.3	1.2	1.1	1.0
2	2.6	2.4	2.2	2.0
3	3.9	3.6	3.3	3.0
4	5.2	4.8	4.4	4.0
5	6.5	6.0	5.5	5.0
6	7.8	7.2	6.6	6.0
7	9.1	8.4	7.7	7.0
8	10.4	9.6	8.8	8.0
9	11.7	10.8	9.9	9.0

	9	**8**	**7**	**6**
1	0.9	0.8	0.7	0.6
2	1.8	1.6	1.4	1.2
3	2.7	2.4	2.1	1.8
4	3.6	3.2	2.8	2.4
5	4.5	4.0	3.5	3.0
6	5.4	4.8	4.2	3.6
7	6.3	5.6	4.9	4.2
8	7.2	6.4	5.6	4.8
9	8.1	7.2	6.3	5.4

	5	**4**	**3**	**2**
1	0.5	0.4	0.3	0.2
2	1.0	0.8	0.6	0.4
3	1.5	1.2	0.9	0.6
4	2.0	1.6	1.2	0.8
5	2.5	2.0	1.5	1.0
6	3.0	2.4	1.8	1.2
7	3.5	2.8	2.1	1.4
8	4.0	3.2	2.4	1.6
9	4.5	3.6	2.7	1.8

P. P.

°	0′	10′	20′	30′	40′	50′	60′		d.
0	0. 0000	0029	0058	0087	0116	0145	0175	89	29
1	0175	0204	0233	0262	0291	0320	0349	88	29
2	0349	0378	0407	0437	0466	0495	0524	87	29
3	0524	0553	0582	0612	0641	0670	0699	86	29
4	0699	0729	0758	0787	0816	0846	0875	**85**	29
5	0. 0875	0904	0934	0963	0992	1022	1051	84	29
6	1051	1080	1110	1139	1169	1198	1228	83	30
7	1228	1257	1287	1317	1346	1376	1405	82	30
8	1405	1435	1465	1495	1524	1554	1584	81	30
9	1584	1614	1644	1673	1703	1733	1763	**80**	30
10	0. 1763	1793	1823	1853	1883	1914	1944	79	30
11	1944	1974	2004	2035	2065	2095	2126	78	30
12	2126	2156	2186	2217	2247	2278	2309	77	30
13	2309	2339	2370	2401	2432	2462	2493	76	31
14	2493	2524	2555	2586	2617	2648	2679	**75**	31
15	0. 2679	2711	2742	2773	2805	2836	2867	74	31
16	2867	2899	2931	2962	2994	3026	3057	73	32
17	3057	3089	3121	3153	3185	3217	3249	72	32
18	3249	3281	3314	3346	3378	3411	3443	71	32
19	3443	3476	3508	3541	3574	3607	3640	**70**	33
20	0. 3640	3673	3706	3739	3772	3805	3839	69	33
21	3839	3872	3906	3939	3973	4006	4040	68	34
22	4040	4074	4108	4142	4176	4210	4245	67	34
23	4245	4279	4314	4348	4383	4417	4452	66	34
24	4452	4487	4522	4557	4592	4628	4663	**65**	35
25	0. 4663	4699	4734	4770	4806	4841	4877	64	36
26	4877	4913	4950	4986	5022	5059	5095	63	36
27	5095	5132	5169	5206	5243	5280	5317	62	37
28	5317	5354	5392	5430	5467	5505	5543	61	38
29	5543	5581	5619	5658	5696	5735	5774	**60**	38
30	0. 5774	5812	5851	5890	5930	5969	6009	59	39
31	6009	6048	6088	6128	6168	6208	6249	58	40
32	6249	6289	6330	6371	6412	6453	6494	57	41
33	6494	6536	6577	6619	6661	6703	6745	56	42
34	6745	6787	6830	6873	6916	6959	7002	**55**	43
35	0. 7002	7046	7089	7133	7177	7221	7265	54	44
36	7265	7310	7355	7400	7445	7490	7536	53	45
37	7536	7581	7627	7673	7720	7766	7813	52	46
38	7813	7860	7907	7954	8002	8050	8098	51	48
39	8098	8146	8195	8243	8292	8342	8391	**50**	49
40	0. 8391	8441	8491	8541	8591	8642	8693	49	50
41	8693	8744	8796	8847	8899	8952	9004	48	52
42	9004	9057	9110	9163	9217	9271	9325	47	54
43	9325	9380	9435	9490	9545	9601	9657	46	55
44	9657	9713	9770	9827	9884	9942	*0000	**45**	57
45	1. 0000								
	60′	50′	40′	30′	20′	10′	0′	°	d.

P. P.

	29	**30**	**31**	**32**	**33**
1	2.9	3.0	3.1	3.2	3.3
2	5.8	6.0	6.2	6.4	6.6
3	8.7	9.0	9.3	9.6	9.9
4	11.6	12.0	12.4	12.8	13.2
5	14.5	15.0	15.5	16.0	16.5
6	17.4	18.0	18.6	19.2	19.8
7	20.3	21.0	21.7	22.4	23.1
8	23.2	24.0	24.8	25.6	26.4
9	26.1	27.0	27.9	28.8	29.7

	34	**35**	**36**	**37**	**38**
1	3.4	3.5	3.6	3.7	3.8
2	6.8	7.0	7.2	7.4	7.6
3	10.2	10.5	10.8	11.1	11.4
4	13.6	14.0	14.4	14.8	15.2
5	17.0	17.5	18.0	18.5	19.0
6	20.4	21.0	21.6	22.2	22.8
7	23.8	24.5	25.2	25.9	26.6
8	27.2	28.0	28.8	29.6	30.4
9	30.6	31.5	32.4	33.3	34.2

	39	**40**	**41**	**42**	**43**
1	3.9	4.0	4.1	4.2	4.3
2	7.8	8.0	8.2	8.4	8.6
3	11.7	12.0	12.3	12.6	12.9
4	15.6	16.0	16.4	16.8	17.2
5	19.5	20.0	20.5	21.0	21.5
6	23.4	24.0	24.6	25.2	25.8
7	27.3	28.0	28.7	29.4	30.1
8	31.2	32.0	32.8	33.6	34.4
9	35.1	36.0	36.9	37.8	38.7

	44	**45**	**46**	**47**	**48**
1	4.4	4.5	4.6	4.7	4.8
2	8.8	9.0	9.2	9.4	9.6
3	13.2	13.5	13.8	14.1	14.4
4	17.6	18.0	18.4	18.8	19.2
5	22.0	22.5	23.0	23.5	24.0
6	26.4	27.0	27.6	28.2	28.8
7	30.8	31.5	32.2	32.9	33.6
8	35.2	36.0	36.8	37.6	38.4
9	39.6	40.5	41.4	42.3	43.2

	49	**50**	**51**	**52**	**53**
1	4.9	5.0	5.1	5.2	5.3
2	9.8	10.0	10.2	10.4	10.6
3	14.7	15.0	15.3	15.6	15.9
4	19.6	20.0	20.4	20.8	21.2
5	24.5	25.0	25.5	26.0	26.5
6	29.4	30.0	30.6	31.2	31.8
7	34.3	35.0	35.7	36.4	37.1
8	39.2	40.0	40.8	41.6	42.4
9	44.1	45.0	45.9	46.8	47.7

	54	**55**	**56**	**57**	**58**
1	5.4	5.5	5.6	5.7	5.8
2	10.8	11.0	11.2	11.4	11.6
3	16.2	16.5	16.8	17.1	17.4
4	21.6	22.0	22.4	22.8	23.2
5	27.0	27.5	28.0	28.5	29.0
6	32.4	33.0	33.6	34.2	34.8
7	37.8	38.5	39.2	39.9	40.6
8	43.2	44.0	44.8	45.6	46.4
9	48.6	49.5	50.4	51.3	52.2

P. P.

°	0′	10′	20′	30′	40′	50′	60′		d.
45	1.000	1.006	1.012	1.018	1.024	1.030	1.036	44	6
46	1.036	1.042	1.048	1.054	1.060	1.066	1.072	43	6
47	1.072	1.079	1.085	1.091	1.098	1.104	1.111	42	6
48	1.111	1.117	1.124	1.130	1.137	1.144	1.150	41	6
49	1.150	1.157	1.164	1.171	1.178	1.185	1.192	**40**	7
50	1.192	1.199	1.206	1.213	1.220	1.228	1.235	39	7
51	1.235	1.242	1.250	1.257	1.265	1.272	1.280	38	8
52	1.280	1.288	1.295	1.303	1.311	1.319	1.327	37	8
53	1.327	1.335	1.343	1.351	1.360	1.368	1.376	36	8
54	1.376	1.385	1.393	1.402	1.411	1.419	1.428	**35**	9
55	1.428	1.437	1.446	1.455	1.464	1.473	1.483	34	9
56	1.483	1.492	1.501	1.511	1.520	1.530	1.540	33	10
57	1.540	1.550	1.560	1.570	1.580	1.590	1.600	32	10
58	1.600	1.611	1.621	1.632	1.643	1.653	1.664	31	11
59	1.664	1.675	1.686	1.698	1.709	1.720	1.732	**30**	11
60	1.732	1.744	1.756	1.767	1.780	1.792	1.804	29	12
61	1.804	1.816	1.829	1.842	1.855	1.868	1.881	28	13
62	1.881	1.894	1.907	1.921	1.935	1.949	1.963	27	14
63	1.963	1.977	1.991	2.006	2.020	2.035	2.050	26	14
64	2.050	2.066	2.081	2.097	2.112	2.128	2.145	**25**	16
65	2.145	2.161	2.177	2.194	2.211	2.229	2.246	24	17
66	2.246	2.264	2.282	2.300	2.318	2.337	2.356	23	18
67	2.356	2.375	2.394	2.414	2.434	2.455	2.475	22	20
68	2.475	2.496	2.517	2.539	2.560	2.583	2.605	21	22
69	2.605	2.628	2.651	2.675	2.699	2.723	2.747	**20**	24
70	2.747	2.773	2.798	2.824	2.850	2.877	2.904	19	26
71	2.904	2.932	2.960	2.989	3.018	3.047	3.078	18	29
72	3.078	3.108	3.140	3.172	3.204	3.237	3.271	17	32
73	3.271	3.305	3.340	3.376	3.412	3.450	3.487	16	36
74	3.487	3.526	3.566	3.606	3.647	3.689	3.732	**15**	41
75	3.732	3.776	3.821	3.867	3.914	3.962	4.011	14	46
76	4.011	4.016	4.113	4.165	4.219	4.275	4.331	13	53
77	4.331	4.390	4.449	4.511	4.574	4.638	4.705	12	62
78	4.705	4.773	4.843	4.915	4.989	5.066	5.145	11	73
79	5.145	5.226	5.309	5.396	5.485	5.576	5.671	**10**	88
80	5.671	5.769	5.871	5.976	6.084	6.197	6.314	9	
81	6.314	6.435	6.561	6.691	6.827	6.968	7.115	8	
82	7.115	7.269	7.429	7.596	7.770	7.953	8.144	7	
83	8.144	8.345	8.556	8.777	9.010	9.255	9.514	6	
84	9.415	9.788	10.078	10.385	10.712	11.059	11.430	**5**	
85	11.430	11.826	12.251	12.706	13.197	13.727	14.301	4	
86	14.301	14.924	15.605	16.350	17.169	18.075	19.081	3	
87	19.081	20.206	21.470	22.904	24.542	26.432	28.636	2	
88	28.636	31.242	34.368	38.188	42.964	49.104	57.290	1	
89	57.290	68.750	85.940	114.59	171.89	343.77	infinit.	**0**	
90	infinit.								

	60′	50′	40′	30′	20′	10′	0′	° d.

P. P.

	6	**7**	**8**	**9**	**10**	**11**
1	0.6	0.7	0.8	0.9	1.0	1.1
2	1.2	1.4	1.6	1.8	2.0	2.2
3	1.8	2.1	2.4	2.7	3.0	3.3
4	2.4	2.8	3.2	3.6	4.0	4.4
5	3.0	3.5	4.0	4.5	5.0	5.5
6	3.6	4.2	4.8	5.4	6.0	6.6
7	4.2	4.9	5.6	6.3	7.0	7.7
8	4.8	5.6	6.4	7.2	8.0	8.8
9	5.4	6.3	7.2	8.1	9.0	9.9

	12	**13**	**14**	**15**	**16**
1	1.2	1.3	1.4	1.5	1.6
2	2.4	2.6	2.8	3.0	3.2
3	3.6	3.9	4.2	4.5	4.8
4	4.8	5.2	5.6	6.0	6.4
5	6.0	6.5	7.0	7.5	8.0
6	7.2	7.8	8.4	9.0	9.6
7	8.4	9.1	9.8	10.5	11.2
8	9.6	10.4	11.2	12.0	12.8
9	10.8	11.7	12.6	13.5	14.4

	17	**18**	**19**	**20**	**21**
1	1.7	1.8	1.9	2.0	2.1
2	3.4	3.6	3.8	4.0	4.2
3	5.1	5.4	5.7	6.0	6.3
4	6.8	7.2	7.6	8.0	8.4
5	8.5	9.0	9.5	10.0	10.5
6	10.2	10.8	11.4	12.0	12.6
7	11.9	12.6	13.3	14.0	14.7
8	13.6	14.4	15.2	16.0	16.8
9	15.3	16.2	17.1	18.0	18.9

	22	**23**	**24**	**25**	**26**
1	2.2	2.3	2.4	2.5	2.6
2	4.4	4.6	4.8	5.0	5.2
3	6.6	6.9	7.2	7.5	7.8
4	8.8	9.2	9.6	10.0	10.4
5	11.0	11.5	12.0	12.5	13.0
6	13.2	13.8	14.4	15.0	15.6
7	15.4	16.1	16.8	17.5	18.2
8	17.6	18.4	19.2	20.0	20.8
9	19.8	20.7	21.6	22.5	23.4

	27	**28**	**59**	**62**	**63**
1	2.7	2.8	5.9	6.2	6.3
2	5.4	5.6	11.8	12.4	12.6
3	8.1	8.4	17.7	18.6	18.9
4	10.8	11.2	23.6	24.8	25.2
5	13.5	14.0	29.5	31.0	31.5
6	16.2	16.8	35.4	37.2	37.8
7	18.9	19.6	41.3	43.4	44.1
8	21.6	22.4	47.2	49.6	50.4
9	24.3	25.2	53.1	55.8	56.7

	64	**67**	**68**	**70**	**72**
1	6.4	6.7	6.8	7.0	7.2
2	12.8	13.4	13.6	14.0	14.4
3	19.2	20.1	20.4	21.0	21.6
4	25.6	26.8	27.2	28.0	28.8
5	32.0	33.5	34.0	35.0	36.0
6	38.4	40.2	40.8	42.0	43.2
7	44.8	46.9	47.6	49.0	50.4
8	51.2	53.6	54.4	56.0	57.6
9	57.6	60.3	61.2	63.0	64.8

P. P.

TABLE V

COMMON LOGARITHMS OF NUMBERS

TO

FOUR DECIMAL PLACES

	0	1	2	3	4	5	6	7	8	9
1 00	0000	0004	0009	0013	0017	0022	0026	0030	0035	0039
1 01	0043	0048	0052	0056	0060	0065	0069	0073	0077	0082
1 02	0086	0090	0095	0099	0103	0107	0111	0116	0120	0124
1 03	0128	0133	0137	0141	0145	0149	0154	0158	0162	0166
1 04	0170	0175	0179	0183	0187	0191	0195	0199	0204	0208
1 05	0212	0216	0220	0224	0228	0233	0237	0241	0245	0249
1 06	0253	0257	0261	0265	0269	0273	0278	0282	0286	0290
1 07	0294	0298	0302	0306	0310	0314	0318	0322	0326	0330
1 08	0334	0338	0342	0346	0350	0354	0358	0362	0366	0370
1 09	0374	0378	0382	0386	0390	0394	0398	0402	0406	0410
1 10	0414	0418	0422	0426	0430	0434	0438	0441	0445	0449
1 11	0453	0457	0461	0465	0469	0473	0477	0481	0484	0488
1 12	0492	0496	0500	0504	0508	0512	0515	0519	0523	0527
1 13	0531	0535	0538	0542	0546	0550	0554	0558	0561	0565
1 14	0569	0573	0577	0580	0584	0588	0592	0596	0599	0603
1 15	0607	0611	0615	0618	0622	0626	0630	0633	0637	0641
1 16	0645	0648	0652	0656	0660	0663	0667	0671	0674	0678
1 17	0682	0686	0689	0693	0697	0700	0704	0708	0711	0715
1 18	0719	0722	0726	0730	0734	0737	0741	0745	0748	0752
1 19	0755	0759	0763	0766	0770	0774	0777	0781	0785	0788
1 20	0792	0795	0799	0803	0806	0810	0813	0817	0821	0824
1 21	0828	0831	0835	0839	0842	0846	0849	0853	0856	0860
1 22	0864	0867	0871	0874	0878	0881	0885	0888	0892	0896
1 23	0899	0903	0906	0910	0913	0917	0920	0924	0927	0931
1 24	0934	0938	0941	0945	0948	0952	0955	0959	0962	0966
1 25	0969	0973	0976	0980	0983	0986	0990	0993	0997	1000
1 26	1004	1007	1011	1014	1017	1021	1024	1028	1031	1035
1 27	1038	1041	1045	1048	1052	1055	1059	1062	1065	1069
1 28	1072	1075	1079	1082	1086	1089	1092	1096	1099	1103
1 29	1106	1109	1113	1116	1119	1123	1126	1129	1133	1136
1 30	1139	1143	1146	1149	1153	1156	1159	1163	1166	1169
1 31	1173	1176	1179	1183	1186	1189	1193	1196	1199	1202
1 32	1206	1209	1212	1216	1219	1222	1225	1229	1232	1235
1 33	1239	1242	1245	1248	1252	1255	1258	1261	1265	1268
1 34	1271	1274	1278	1281	1284	1287	1290	1294	1297	1300
1 35	1303	1307	1310	1313	1316	1319	1323	1326	1329	1332
1 36	1335	1339	1342	1345	1348	1351	1355	1358	1361	1364
1 37	1367	1370	1374	1377	1380	1383	1386	1389	1392	1396
1 38	1399	1402	1405	1408	1411	1414	1418	1421	1424	1427
1 39	1430	1433	1436	1440	1443	1446	1449	1452	1455	1458
1 40	1461	1464	1467	1471	1474	1477	1480	1483	1486	1489
1 41	1492	1495	1498	1501	1504	1508	1511	1514	1517	1520
1 42	1523	1526	1529	1532	1535	1538	1541	1544	1547	1550
1 43	1553	1556	1559	1562	1565	1569	1572	1575	1578	1581
1 44	1584	1587	1590	1593	1596	1599	1602	1605	1608	1611
1 45	1614	1617	1620	1623	1626	1629	1632	1635	1638	1641
1 46	1644	1647	1649	1652	1655	1658	1661	1664	1667	1670
1 47	1673	1676	1679	1682	1685	1688	1691	1694	1697	1700
1 48	1703	1706	1708	1711	1714	1717	1720	1723	1726	1729
1 49	1732	1735	1738	1741	1744	1746	1749	1752	1755	1758

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1 50	1761	1764	1767	1770	1772	1775	1778	1781	1784	1787
1 51	1790	1793	1796	1798	1801	1804	1807	1810	1813	1816
1 52	1818	1821	1824	1827	1830	1833	1836	1838	1841	1844
1 53	1847	1850	1853	1855	1858	1861	1864	1867	1870	1872
1 54	1875	1878	1881	1884	1886	1889	1892	1895	1898	1901
1 55	1903	1906	1909	1912	1915	1917	1920	1923	1926	1928
1 56	1931	1934	1937	1940	1942	1945	1948	1951	1953	1956
1 57	1959	1962	1965	1967	1970	1973	1976	1978	1981	1984
1 58	1987	1989	1992	1995	1998	2000	2003	2006	2009	2011
1 59	2014	2017	2019	2022	2025	2028	2030	2033	2036	2038
1 60	2041	2044	2047	2049	2052	2055	2057	2060	2063	2066
1 61	2068	2071	2074	2076	2079	2082	2084	2087	2090	2092
1 62	2095	2098	2101	2103	2106	2109	2111	2114	2117	2119
1 63	2122	2125	2127	2130	2133	2135	2138	2140	2143	2146
1 64	2148	2151	2154	2156	2159	2162	2164	2167	2170	2172
1 65	2175	2177	2180	2183	2185	2188	2191	2193	2196	2198
1 66	2201	2204	2206	2209	2212	2214	2217	2219	2222	2225
1 67	2227	2230	2232	2235	2238	2240	2243	2245	2248	2251
1 68	2253	2256	2258	2261	2263	2266	2269	2271	2274	2276
1 69	2279	2281	2284	2287	2289	2292	2294	2297	2299	2302
1 70	2304	2307	2310	2312	2315	2317	2320	2322	2325	2327
1 71	2330	2333	2335	2338	2340	2343	2345	2348	2350	2353
1 72	2355	2358	2360	2363	2365	2368	2370	2373	2375	2378
1 73	2380	2383	2385	2388	2390	2393	2395	2398	2400	2403
1 74	2405	2408	2410	2413	2415	2418	2420	2423	2425	2428
1 75	2430	2433	2435	2438	2440	2443	2445	2448	2450	2453
1 76	2455	2458	2460	2463	2465	2467	2470	2472	2475	2477
1 77	2480	2482	2485	2487	2490	2492	2494	2497	2499	2502
1 78	2504	2507	2509	2512	2514	2516	2519	2521	2524	2526
1 79	2529	2531	2533	2536	2538	2541	2543	2545	2548	2550
1 80	2553	2555	2558	2560	2562	2565	2567	2570	2572	2574
1 81	2577	2579	2582	2584	2586	2589	2591	2594	2596	2598
1 82	2601	2603	2605	2608	2610	2613	2615	2617	2620	2622
1 83	2625	2627	2629	2632	2634	2636	2639	2641	2643	2646
1 84	2648	2651	2653	2655	2658	2660	2662	2665	2667	2669
1 85	2672	2674	2676	2679	2681	2683	2686	2688	2690	2693
1 86	2695	2697	2700	2702	2704	2707	2709	2711	2714	2716
1 87	2718	2721	2723	2725	2728	2730	2732	2735	2737	2739
1 88	2742	2744	2746	2749	2751	2753	2755	2758	2760	2762
1 89	2765	2767	2769	2772	2774	2776	2778	2781	2783	2785
1 90	2788	2790	2792	2794	2797	2799	2801	2804	2806	2808
1 91	2810	2813	2815	2817	2819	2822	2824	2826	2828	2831
1 92	2833	2835	2838	2840	2842	2844	2847	2849	2851	2853
1 93	2856	2858	2860	2862	2865	2867	2869	2871	2874	2876
1 94	2878	2880	2882	2885	2887	2889	2891	2894	2896	2898
1 95	2900	2903	2905	2907	2909	2911	2914	2916	2918	2920
1 96	2923	2925	2927	2929	2931	2934	2936	2938	2940	2942
1 97	2945	2947	2949	2951	2953	2956	2958	2960	2962	2964
1 98	2967	2969	2971	2973	2975	2978	2980	2982	2984	2986
1 99	2989	2991	2993	2995	2997	2999	3002	3004	3006	3008

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10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

To avoid interpolation in the first ten lines, use table on the preceding pages.

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7	15.4
8	17.6
9	19.8

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2	4.2	4.0	3.8
3	6.3	6.0	5.7
4	8.4	8.0	7.6
5	10.5	10.0	9.5
6	12.6	12.0	11.4
7	14.7	14.0	13.3
8	16.8	16.0	15.2
9	18.9	18.0	17.1

	18	17	16
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2	3.6	3.4	3.2
3	5.4	5.1	4.8
4	7.2	6.8	6.4
5	9.0	8.5	8.0
6	10.8	10.2	9.6
7	12.6	11.9	11.2
8	14.4	13.6	12.8
9	16.2	15.3	14.4

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4	6.0	5.6	5.2
5	7.5	7.0	6.5
6	9.0	8.4	7.8
7	10.5	9.8	9.1
8	12.0	11.2	10.4
9	13.5	12.6	11.7

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7	8.4
8	9.6
9	10.8

	11
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8	8.8
9	9.9

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9	9.0

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6	5.4
7	6.3
8	7.2
9	8.1

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

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55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

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7	4.9
8	5.6
9	6.3

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8	4.8
9	5.4

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8	4.0
9	4.5

4	
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2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

TABLE VI

THE LOGARITHMS

OF THE

SINE, COSINE, TANGENT, AND COTANGENT

FROM

0° TO 90° TO FOUR FIGURES

°	′		0′	1′	2′	3′	4′	5′	6′	7′	8′	9′	10′		
0	0	6.	——	4637	7648	9408	*0658	*1627	*2419	*3088	*3668	*4180	*4637	50	
	10	7.	4637	5051	5429	5777	6099	6398	6678	6942	7190	7425	7648	40	
	20		7648	7859	8061	8255	8439	8617	8787	8951	*109	*261	*408	30	
	30	7.9	408	551	689	822	952	*078	*200	*319	*435	*548	*658	20	
	40	8.0	658	765	870	972	*072	*169	*265	*358	*450	*539	*627	10	
	50	8.1	627	713	797	880	961	*041	*119	*196	*271	*346	*419	0	**89**
1	0	8.2	419	490	561	630	699	766	832	898	962	*025	*088	50	
	10	8.3	088	150	210	270	329	388	445	502	558	613	668	40	
	20		668	722	775	828	880	931	982	*032	*082	*131	*179	30	
	30	8.4	179	227	275	322	368	414	459	504	549	593	637	20	
	40		637	680	723	765	807	848	890	930	971	*011	*050	10	
	50	8.5	050	090	129	167	206	243	281	318	355	392	428	0	**88**
2	0	8.5	428	464	500	535	571	605	640	674	708	742	776	50	
	10		776	809	842	875	907	939	972	*003	*035	*066	*097	40	
	20	8.6	097	128	159	189	220	250	279	309	339	368	397	30	
	30		397	426	454	483	511	539	567	595	622	650	677	20	
	40		677	704	731	758	784	810	837	863	889	914	940	10	
	50		940	965	991	*016	*041	*066	*090	*115	*140	*164	*188	0	**87**
3	0	8.7	188	212	236	260	283	307	330	354	377	400	423	50	
	10		423	445	468	491	513	535	557	580	602	623	645	40	
	20		645	667	688	710	731	752	773	794	815	836	857	30	
	30		857	877	898	918	939	959	979	999	*019	*039	*059	20	
	40	8.8	059	078	098	117	137	156	175	194	213	232	251	10	
	50		251	270	289	307	326	345	363	381	400	418	436	0	**86**
4	0	8.8	436	454	472	490	508	525	543	560	578	595	613	50	
	10		613	630	647	665	682	699	716	733	749	766	783	40	
	20		783	799	816	833	849	865	882	898	914	930	946	30	
	30		946	962	978	994	*010	*026	*042	*057	*073	*089	*104	20	
	40	8.9	104	119	135	150	166	181	196	211	226	241	256	10	
	50		256	271	286	301	315	330	345	359	374	388	403	0	**85**
5	0	8.9	403	417	432	446	460	475	489	503	517	531	545	50	
	10		545	559	573	587	601	614	628	642	655	669	682	40	
	20		682	696	709	723	736	750	763	776	789	803	816	30	
	30		816	829	842	855	868	881	894	907	919	932	945	20	
	40		945	958	970	983	996	*008	*021	*033	*046	*058	*070	10	
	50	9.0	070	083	095	107	120	132	144	156	168	180	192	0	**84**
6	0	9.0	192	204	216	228	240	252	264	276	287	299	311	50	
	10		311	323	334	346	357	369	380	392	403	415	426	40	
	20		426	438	449	460	472	483	494	505	516	527	539	30	
	30		539	550	561	572	583	594	605	616	626	637	648	20	
	40		648	659	670	680	691	702	712	723	734	744	755	10	
	50		755	765	776	786	797	807	818	828	838	849	859	0	**83**
			10′	**9′**	**8′**	**7′**	**6′**	**5′**	**4′**	**3′**	**2′**	**1′**	**0′**		°

°	′		0′	1′	2′	3′	4′	5′	6′	7′	8′	9′	10′		
7	0	9.0	859	869	879	890	900	910	920	930	940	951	961	50	
	10		961	971	981	991	*001	*011	*020	*030	*040	*050	*060	40	
	20	9.1	060	070	080	089	099	109	118	128	138	147	157	30	
	30		157	167	176	186	195	205	214	224	233	242	252	20	
	40		252	261	271	280	289	299	308	317	326	336	345	10	
	50		345	354	363	372	381	390	399	409	418	427	436	0	**82**
8	0	9.1	436	445	453	462	471	480	489	498	507	516	525	50	
	10		525	533	542	551	560	568	577	586	594	603	612	40	
	20		612	620	629	637	646	655	663	672	680	689	697	30	
	30		697	705	714	722	731	739	747	756	764	772	781	20	
	40		781	789	797	806	814	822	830	838	847	855	863	10	
	50		863	871	879	887	895	903	911	919	927	935	943	0	**81**
9	0	9.1	943	951	959	967	975	983	991	999	*007	*015	*022	50	
	10	9.2	022	030	038	046	054	061	069	077	085	092	100	40	
	20		100	108	115	123	131	138	146	153	161	169	176	30	
	30		176	184	191	199	206	214	221	229	236	243	251	20	
	40		251	258	266	273	280	288	295	303	310	317	324	10	
	50		324	332	339	346	353	361	368	375	382	390	397	0	**80**
10	0	9.2	397	404	411	418	425	432	439	447	454	461	468	50	
	10		468	475	482	489	496	503	510	517	524	531	538	40	
	20		538	545	551	558	565	572	579	586	593	600	606	30	
	30		606	613	620	627	634	640	647	654	661	667	674	20	
	40		674	681	687	694	701	707	714	721	727	734	740	10	
	50		740	747	754	760	767	773	780	786	793	799	806	0	**79**
11	0	9.2	806	812	819	825	832	838	845	851	858	864	870	50	
	10		870	877	883	890	896	902	909	915	921	928	934	40	
	20		934	940	947	953	959	965	972	978	984	990	997	30	
	30		997	*003	*009	*015	*021	*027	*034	*040	*046	*052	*058	20	
	40	9.3	058	064	070	077	083	089	095	101	107	113	119	10	
	50		119	125	131	137	143	149	155	161	167	173	179	0	**78**
12	0	9.3	179	185	191	197	202	208	214	220	226	232	238	50	
	10		238	244	250	255	261	267	273	279	284	290	296	40	
	20		296	302	308	313	319	325	331	336	342	348	353	30	
	30		353	359	365	370	376	382	387	393	399	404	410	20	
	40		410	416	421	427	432	438	444	449	455	460	466	10	
	50		466	471	477	482	488	493	499	504	510	515	521	0	**77**
13	0	9.3	521	526	532	537	543	548	554	559	564	570	575	50	
	10		575	581	586	591	597	602	608	613	618	624	629	40	
	20		629	634	640	645	650	655	661	666	671	677	682	30	
	30		682	687	692	698	703	708	713	719	724	729	734	20	
	40		734	739	745	750	755	760	765	770	775	781	786	10	
	50		786	791	796	801	806	811	816	822	827	832	837	0	**76**
			10′	**9′**	**8′**	**7′**	**6′**	**5′**	**4′**	**3′**	**2′**	**1′**	**0′**	′	°

°		0′	10′	20′	30′	40′	50′	60′		d.
14	9.3	837	887	937	986	*035	*083	*130	**75**	
15	9.4	130	177	223	269	314	359	403	74	
16		403	447	491	533	576	618	659	73	
17		659	700	741	781	821	861	900	72	
18		900	939	977	*015	*052	*090	*126	71	
19	9.5	126	163	199	235	270	306	341	**70**	
20	9.5	341	375	409	443	477	510	543	69	34
21		543	576	609	641	673	704	736	68	32
22		736	767	798	828	859	889	919	67	31
23		919	948	978	*007	*036	*065	*093	66	29
24	9.6	093	121	149	177	205	232	259	**65**	28
25	9.6	259	286	313	340	366	392	418	64	27
26		418	444	470	495	521	546	570	63	25
27		570	595	620	644	668	692	716	62	24
28		716	740	763	787	810	833	856	61	23
29		856	878	901	923	946	968	990	**60**	23
30	9.6	990	*012	*033	*055	*076	*097	*118	59	22
31	9.7	118	139	160	181	201	222	242	58	21
32		242	262	282	302	322	342	361	57	20
33		361	380	400	419	438	457	476	56	19
34		476	494	513	531	550	568	586	**55**	18
35	9.7	586	604	622	640	657	675	692	54	18
36		692	710	727	744	761	778	795	53	17
37		795	811	828	844	861	877	893	52	16
38		893	910	926	941	957	973	989	51	16
39		989	*004	*020	*035	*050	*066	*081	**50**	15
40	9.8	081	096	111	125	140	155	169	49	15
41		169	184	198	213	227	241	255	48	14
42		255	269	283	297	311	324	338	47	14
43		338	351	365	378	391	405	418	46	13
44		418	431	444	457	469	482	495	**45**	13
45	9.8	495	507	520	532	545	557	569	44	12
46		569	582	594	606	618	629	641	43	12
47		641	653	665	676	688	699	711	42	12
48		711	722	733	745	756	767	778	41	11
49		778	789	800	810	821	832	843	**40**	11
		60′	**50′**	**40′**	**30′**	**20′**	**10′**	**0′**	°	d.

	50	**49**	**48**	**47**	**46**
1	5.0	4.9	4.8	4.7	4.6
2	10.0	9.8	9.6	9.4	9.2
3	15.0	14.7	14.4	14.1	13.8
4	20.0	19.6	19.2	18.8	18.4
5	25.0	24.5	24.0	23.5	23.0
6	30.0	29.4	28.8	28.2	27 6
7	35.0	34.3	33.6	32.9	32.2
8	40.0	39.2	38.4	37.6	36.8
9	45.0	44.1	43.2	42.3	41.4

	45	**44**	**43**	**42**	**41**
1	4.5	4.4	4.3	4.2	4.1
2	9.0	8.8	8.6	8.4	8.2
3	13.5	13.2	12.9	12.6	12.3
4	18.0	17.6	17.2	16.8	16.4
5	22.5	22.0	21.5	21.0	20.5
6	27.0	26.4	25.8	25.2	24.6
7	31.5	30.8	30.1	29.4	28.7
8	36.0	35.2	34.4	33.6	32.8
9	40.5	39.6	38.7	37.8	36.9

	40	**39**	**38**	**37**	**36**
1	4.0	3.9	3.8	3.7	3.6
2	8.0	7.8	7.6	7.4	7.2
3	12.0	11.7	11.4	11.1	10.8
4	16.0	15.6	15.2	14.8	14.4
5	20.0	19.5	19.0	18.5	18.0
6	24.0	23.4	22.8	22.2	21.6
7	28.0	27.3	26.6	25.9	25.2
8	32.0	31.2	30.4	29.6	28.8
9	36.0	35.1	34.2	33.3	32.4

	35	**34**	**33**	**32**	**31**
1	3.5	3.4	3.3	3.2	3.1
2	7.0	6.8	6.6	6.4	6.2
3	10.5	10.2	9.9	9.6	9.3
4	14.0	13.6	13.2	12.8	12.4
5	17.5	17.0	16.5	16.0	15.5
6	21.0	20.4	19.8	19.2	18.6
7	24.5	23.8	23.1	22.4	21.7
8	28.0	27.2	26.4	25.6	24.8
9	31.5	30.6	29.7	28.8	27.9

	30	**29**	**28**	**27**	**26**
1	3.0	2.9	2.8	2.7	2.6
2	6.0	5.8	5.6	5.4	5.2
3	9.0	8.7	8.4	8.1	7.8
4	12.0	11.6	11.2	10.8	10.4
5	15.0	14.5	14.0	13.5	13.0
6	18.0	17.4	16.8	16.2	15.6
7	21.0	20.3	19.6	18.9	18.2
8	24.0	23.2	22.4	21.6	20.8
9	27.0	26.1	25.2	24.3	23.4

	25	**24**	**23**	**22**	**21**
1	2.5	2.4	2.3	2.2	2.1
2	5.0	4.8	4.6	4.4	4.2
3	7.5	7.2	6.9	6.6	6.3
4	10.0	9.6	9.2	8.8	8.4
5	12.5	12.0	11.5	11.0	10.5
6	15.0	14.4	13.8	13.2	12.6
7	17.5	16.8	16.1	15.4	14.7
8	20.0	19.2	18.4	17.6	16.8
9	22.5	21.6	20.7	19.8	18.9

	20	**19**	**18**	**17**	**16**
1	2.0	1.9	1.8	1.7	1.6
2	4.0	3.8	3.6	3.4	3.2
3	6.0	5.7	5.4	5.1	4.8
4	8.0	7.6	7.2	6.8	6.4
5	10.0	9.5	9.0	8.5	8.0
6	12.0	11.4	10.8	10.2	9.6
7	14.0	13.3	12.6	11.9	11.2
8	16.0	15.2	14.4	13.6	12.8
9	18.0	17.1	16.2	15.3	14.4

	15	**14**	**13**	**12**	**11**
1	1.5	1.4	1.3	1.2	1.1
2	3.0	2.8	2.6	2.4	2.2
3	4.5	4.2	3.9	3.6	3.3
4	6.0	5.6	5.2	4.8	4.4
5	7.5	7.0	6.5	6.0	5.5
6	9.0	8.4	7.8	7.2	6.6
7	10.5	9.8	9.1	8.4	7.7
8	12.0	11.2	10.4	9.6	8.8
9	13.5	12.6	11.7	10.8	9.9

°		0′	10′	20′	30′	40′	50′	60′		d.
50	9.8	843	853	864	874	884	895	905	39	10
51		905	915	925	935	945	955	965	38	10
52		965	975	985	995	*004	*014	*023	37	10
53	9.9	023	033	042	052	061	070	080	36	9
54		080	089	098	107	116	125	134	**35**	9
55	9.9	134	142	151	160	169	177	186	34	8
56		186	194	203	211	219	228	236	33	8
57		236	244	252	260	268	276	284	32	8
58		284	292	300	308	315	323	331	31	8
59		331	338	346	353	361	368	375	**30**	7
60	9.9	375	383	390	397	404	411	418	29	7
61		418	425	432	439	446	453	459	28	7
62		459	466	473	479	486	492	499	27	7
63		499	505	512	518	524	530	537	26	6
64		537	543	549	555	561	567	573	**25**	6
65	9.9	573	579	584	590	596	602	607	24	6
66		607	613	618	624	629	635	640	23	6
67		640	646	651	656	661	667	672	22	5
68		672	677	682	687	692	697	702	21	5
69		702	706	711	716	721	725	730	**20**	5
70	9.9	730	734	739	743	748	752	757	19	5
71		757	761	765	770	774	778	782	18	4
72		782	786	790	794	798	802	806	17	4
73		806	810	814	817	821	825	828	16	4
74		828	832	836	839	843	846	849	**15**	4
75	9.9	849	853	856	859	863	866	869	14	3
76		869	872	875	878	881	884	887	13	3
77		887	890	893	896	899	901	904	12	3
78		904	907	909	912	914	917	919	11	3
79		919	922	924	927	929	931	934	**10**	3
80	9.9	934	936	938	940	942	944	946	9	2
81		946	948	950	952	954	956	958	8	2
82		958	959	961	963	964	966	968	7	2
83		968	969	971	972	973	975	976	6	2
84		976	977	979	980	981	982	983	**5**	1
85	9.9	983	985	986	987	988	989	989	4	1
86		989	990	991	992	993	993	994	3	1
87		994	995	995	996	996	997	997	2	0
88		997	998	998	999	999	999	999	1	0
89		999	*000	*000	*000	*000	*000	*000	**0**	0
90	10.0	000								
		60′	**50′**	**40′**	**30′**	**20′**	**10′**	**0′**	°	d.

10

1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

9

1	.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

8

1	.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

7

1	.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

6

1	.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

5

1	.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

4

1	.4
2	.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

3

1	.3
2	.6
3	.9
4	1.2
5	1.5
6	1.8
7	2.1
8	2.4
9	2.7

2

1	.2
2	.4
3	.6
4	.8
5	1.0
6	1.2
7	1.4
8	1.6
9	1.8

1

1	.1
2	.2
3	.3
4	.4
5	.5
6	.6
7	.7
8	.8
9	.9

°	′		0′	1′	2′	3′	4′	5′	6′	7′	8′	9′	10′		
0	0	6.		4637	7648	9408	*0658	*1627	*2419	*3088	*3668	*4180	*4637	50	
	10	7.	4637	5051	5429	5777	6099	6398	6678	6942	7190	7425	7648	40	
	20		7648	7860	8062	8255	8439	8617	8787	8951	*109	*261	*409	30	
	30	7.9	409	551	689	823	952	*078	*200	*319	*435	*548	*658	20	
	40	8.0	658	765	870	972	*072	*170	*265	*359	*450	*540	*627	10	
	50	8.1	627	713	798	880	962	*041	*120	*196	*272	*346	*419	0	**89**
1	0	8.2	419	491	562	631	700	767	833	899	963	*026	*089	50	
	10	8.3	089	150	211	271	330	389	446	505	559	614	669	40	
	20		669	723	776	829	881	932	983	*033	*083	132	181	30	
	30	8.4	181	229	276	323	370	416	461	506	551	595	638	20	
	40		638	682	725	767	809	851	892	933	973	*013	*053	10	
	50	8.5	053	092	131	170	208	246	283	321	358	394	431	0	**88**
2	0	8.5	431	467	503	538	573	608	643	677	711	745	779	50	
	10		779	812	845	878	911	943	975	*007	*038	*070	*101	40	
	20	8.6	101	132	163	193	223	254	283	313	343	372	401	30	
	30		401	430	459	487	515	544	571	599	627	654	682	20	
	40		682	709	736	762	789	815	842	868	894	920	945	10	
	50		945	971	996	*021	*046	*071	*096	*121	*145	*170	*194	0	**87**
3	0	8.7	194	218	242	266	290	313	337	360	383	406	429	50	
	10		429	452	475	497	520	542	565	587	609	631	652	40	
	20		652	674	696	717	739	760	781	802	823	844	865	30	
	30		865	886	906	927	947	967	988	*008	*028	*048	*067	20	
	40	8.8	067	087	107	126	146	165	185	204	223	242	261	10	
	50		261	280	299	317	336	355	373	392	410	428	446	0	**86**
4	0	8.8	446	465	483	501	518	536	554	572	589	607	624	50	
	10		624	642	659	676	694	711	728	745	762	778	795	40	
	20		795	812	829	845	862	878	895	911	927	944	960	30	
	30		960	976	992	*008	*024	*040	*056	*071	*087	*103	*118	20	
	40	8.9	118	134	150	165	180	196	211	226	241	256	272	10	
	50		272	287	302	316	331	346	361	376	390	405	420	0	**85**
5	0	8.9	420	434	449	463	477	492	506	520	534	549	563	50	
	10		563	577	591	605	619	633	646	660	674	688	701	40	
	20		701	715	729	742	756	769	782	796	809	823	836	30	
	30		836	849	862	875	888	901	915	927	940	953	966	20	
	40		966	979	992	*005	*017	*030	*043	*055	*068	*080	*093	10	
	50	9.0	093	105	118	130	143	155	167	180	192	204	216	0	**84**
6	0	9.0	216	228	240	253	265	277	289	300	312	324	336	50	
	10		336	348	360	371	383	395	407	418	430	441	453	40	
	20		453	464	476	487	499	510	521	533	544	555	567	30	
	30		567	578	589	600	611	622	633	645	656	667	678	20	
	40		678	688	699	710	721	732	743	754	764	775	786	10	
	50		786	796	807	818	828	839	849	860	871	881	891	0	**83**
			10′	9′	8′	7′	6′	5′	4′	3′	2′	1′	0′	′	°

LOG COTANGENT

°	′		0′	1′	2′	3′	4′	5′	6′	7′	8′	9′	10′		
7	0	9.0	891	902	912	923	933	943	954	964	974	984	995	50	
	10		995	*005	*015	*025	*035	*045	*055	*066	*076	*086	*096	40	
	20	9.1	096	106	116	125	135	145	155	165	175	185	194	30	
	30		194	204	214	223	233	243	252	262	272	281	291	20	
	40		291	300	310	319	329	338	348	357	367	376	385	10	
	50		385	395	404	413	423	432	441	450	460	469	478	0	**82**
8	0	9.1	478	487	496	505	515	524	533	542	551	560	569	50	
	10		569	578	587	596	605	613	622	631	640	649	658	40	
	20		658	667	675	684	693	702	710	719	728	736	745	30	
	30		745	754	762	771	779	788	797	805	814	822	831	20	
	40		831	839	848	856	864	873	881	890	898	906	915	10	
	50		915	923	931	940	948	956	964	973	981	989	997	0	**81**
9	0	9.1	997	*005	*013	*022	*030	*038	*046	*054	*062	*070	*078	50	
	10	9.2	078	086	094	102	110	118	126	134	142	150	158	40	
	20		158	166	174	181	189	197	205	213	221	228	236	30	
	30		236	244	252	259	267	275	282	290	298	305	313	20	
	40		313	321	328	336	343	351	359	366	374	381	389	10	
	50		389	396	404	411	419	426	434	441	448	456	463	0	**80**
10	0	9.2	463	471	478	485	493	500	507	515	522	529	536	50	
	10		536	544	551	558	565	573	580	587	594	601	609	40	
	20		609	616	623	630	637	644	651	658	666	673	680	30	
	30		680	687	694	701	708	715	722	729	736	743	750	20	
	40		750	757	764	770	777	784	791	798	805	812	819	10	
	50		819	825	832	839	846	853	859	866	873	880	887	0	**79**
11	0	9.2	887	893	900	907	913	920	927	933	940	947	953	50	
	10		953	960	967	973	980	987	993	*000	*006	*013	*020	40	
	20	9.3	020	026	033	039	046	052	059	065	072	078	085	30	
	30		085	091	098	104	110	117	123	130	136	142	149	20	
	40		149	155	162	168	174	181	187	193	200	206	212	10	
	50		212	219	225	231	237	244	250	256	262	269	275	0	**78**
12	0	9.3	275	281	287	293	300	306	312	318	324	330	336	50	
	10		336	343	349	355	361	367	373	379	385	391	397	40	
	20		397	403	409	416	422	428	434	440	446	452	458	30	
	30		458	464	469	475	481	487	493	499	505	511	517	20	
	40		517	523	529	535	541	546	552	558	564	570	576	10	
	50		576	581	587	593	599	605	611	616	622	628	634	0	**77**
13	0	9.3	634	639	645	651	657	662	668	674	680	685	691	50	
	10		691	697	702	708	714	719	725	731	736	742	748	40	
	20		748	753	759	764	770	776	781	787	792	798	804	30	
	30		804	809	815	820	826	831	837	842	848	853	859	20	
	40		859	864	870	875	881	886	892	897	903	908	914	10	
	50		914	919	924	930	935	941	946	952	957	962	968	0	**76**
			10′	**9′**	**8′**	**7′**	**6′**	**5′**	**4′**	**3′**	**2′**	**1′**	**0′**	**′**	**°**

LOG COTANGENT

°		0′	10′	20′	30′	40′	50′	60′		d.
14	9.3	968	*021	*074	*127	*178	*230	*281	75	53
15	9.4	281	331	381	430	479	527	575	74	49
16		575	622	669	716	762	808	853	73	46
17		853	898	943	987	*031	*075	*118	72	44
18	9.5	118	161	203	245	287	329	370	71	42
19		370	411	451	491	531	571	611	**70**	40
20	9.5	611	650	689	727	766	804	842	69	38
21		842	879	917	954	991	*028	*064	68	37
22	9.6	064	100	136	172	208	243	279	67	36
23		279	314	348	383	417	452	486	66	35
24		486	520	553	587	620	654	687	**65**	34
25	9.6	687	720	752	785	817	850	882	64	33
26		882	914	946	977	*009	*040	*072	63	32
27	9.7	072	103	134	165	196	226	257	62	31
28		257	287	317	348	378	408	438	61	30
29		438	467	497	526	556	585	614	**60**	29
30	9.7	614	644	673	701	730	759	788	59	29
31		788	816	845	873	902	930	958	58	28
32		958	986	*014	*042	*070	*097	*125	57	28
33	9.8	125	153	180	208	235	263	290	56	27
34		290	317	344	371	398	425	452	**55**	27
35	9.8	452	479	506	533	559	586	613	54	27
36		613	639	666	692	718	745	771	53	26
37		771	797	824	850	876	902	928	52	26
38		928	954	980	*006	*032	*058	*084	51	26
39	9.9	084	110	135	161	187	212	238	**50**	26
40	9.9	238	264	289	315	341	366	392	49	26
41		392	417	443	468	494	519	544	48	25
42		544	570	595	621	646	671	697	47	25
43		697	722	747	772	798	823	848	46	25
44		848	874	899	924	949	975	*000	**45**	25
		60′	**50′**	**40′**	**30′**	**20′**	**10′**	**0′**	°	d.

	53	**52**	**51**	**50**	**49**
1	5.3	5.2	5.1	5.0	4.9
2	10.6	10.4	10.2	10.0	9.8
3	15.9	15.6	15.3	15.0	14.7
4	21.2	20.8	20.4	20.0	19.6
5	26.5	26.0	25.5	25.0	24.5
6	31.8	31.2	30.6	30.0	29.4
7	37.1	36.4	35.7	35.0	34.3
8	42.4	41.6	40.8	40.0	39.2
9	47.7	46.8	45.9	45.0	44.1

	48	**47**	**46**	**45**	**44**
1	4.8	4.7	4.6	4.5	4.4
2	9.6	9.4	9.2	9.0	8.8
3	14.4	14.1	13.8	13.5	13.2
4	19.2	18.8	18.4	18.0	17.6
5	24.0	23.5	23.0	22.5	22.0
6	28.8	28.2	27.6	27.0	26.4
7	33.6	32.9	32.2	31.5	30.8
8	38.4	37.6	36.8	36.0	35.2
9	43.2	42.3	41.4	40.5	39.6

	43	**42**	**41**	**40**	**39**
1	4.3	4.2	4.1	4.0	3.9
2	8.6	8.4	8.2	8.0	7.8
3	12.9	12.6	12.3	12.0	11.7
4	17.2	16.8	16.4	16.0	15.6
5	21.5	21.0	20.5	20.0	19.5
6	25.8	25.2	24.6	24.0	23.4
7	30.1	29.4	28.7	28.0	27.3
8	34.4	33.6	32.8	32.0	31.2
9	38.7	37.8	36.9	36.0	35.1

	38	**37**	**36**	**35**	**34**
1	3.8	3.7	3.6	3.5	3.4
2	7.6	7.4	7.2	7.0	6.8
3	11.4	11.1	10.8	10.5	10.2
4	15.2	14.8	14.4	14.0	13.6
5	19.0	18.5	18.0	17.5	17.0
6	22.8	22.2	21.6	21.0	20.4
7	26.6	25.9	25.2	24.5	23.8
8	30.4	29.6	28.8	28.0	27.2
9	34.2	33.3	32.4	31.5	30.6

	33	**32**	**31**	**30**	**29**
1	3.3	3.2	3.1	3.0	2.9
2	6.6	6.4	6.2	6.0	5.8
3	9.9	9.6	9.3	9.0	8.7
4	13.2	12.8	12.4	12.0	11.6
5	16.5	16.0	15.5	15.0	14.5
6	19.8	19.2	18.6	18.0	17.4
7	23.1	22.4	21.7	21.0	20.3
8	26.4	25.6	24.8	24.0	23.2
9	29.7	28.8	27.9	27.0	26.1

	28	**27**	**26**	**25**
1	2.8	2.7	2.6	2.5
2	5.6	5.4	5.2	5.0
3	8.4	8.1	7.8	7.5
4	11.2	10.8	10.4	10.0
5	14.0	13.5	13.0	12.5
6	16.8	16.2	15.6	15.0
7	19.6	18.9	18.2	17.5
8	22.4	21.6	20.8	20.0
9	25.2	24.3	23.4	22.5

°		0′	10′	20′	30′	40′	50′	60′		d.
45	0.0	000	025	051	076	101	126	152	44	25
46		152	177	202	228	253	278	303	43	25
47		303	329	354	379	405	430	456	42	25
48		456	481	506	532	557	583	608	41	25
49		608	634	659	685	711	736	762	**40**	25
50	0.0	762	788	813	839	865	890	916	39	26
51		916	942	968	994	*020	*046	*072	38	26
52	0.1	072	098	124	150	176	203	229	37	26
53		229	255	282	308	334	361	387	36	26
54		387	414	441	467	494	521	548	**35**	27
55	0.1	548	575	602	629	656	683	710	34	27
56		710	737	765	792	820	847	875	33	27
57		875	903	930	958	986	*014	*042	32	28
58	0.2	042	070	098	127	155	184	212	31	28
59		212	241	270	299	327	356	386	**30**	29
60	0.2	386	415	444	474	503	533	562	29	29
61		562	592	622	652	683	713	743	28	30
62		743	774	804	835	866	897	928	27	31
63		928	960	991	*023	*054	*086	118	26	32
64	0.3	118	150	183	215	248	280	313	**25**	33
65	0.3	313	346	380	413	447	480	514	24	34
66		514	548	583	617	652	686	721	23	35
67		721	757	792	828	864	900	936	22	36
68		936	972	*009	*046	*083	*121	*158	21	37
69	0.4	158	196	234	273	311	350	389	**20**	38
70	0.4	389	429	469	509	549	589	630	19	40
71		630	671	713	755	797	839	882	18	42
72		882	925	969	*013	*057	*102	*147	17	44
73	0.5	147	192	238	284	331	378	425	16	46
74		425	473	521	570	619	669	719	**15**	50
75	0.5	719	770	822	873	926	979	*032	14	52
		60′	**50′**	**40′**	**30′**	**20′**	**10′**	**0′**	°	d.

	25	**26**	**27**	**28**	**29**
1	2.5	2.6	2.7	2.8	2.9
2	5.0	5.2	5.4	5.6	5.8
3	7.5	7.8	8.1	8.4	8.7
4	10.0	10.4	10.8	11.2	11.6
5	12.5	13.0	13.5	14.0	14.5
6	15.0	15.6	16.2	16.8	17.4
7	17.5	18.2	18.9	19.6	20.3
8	20.0	20.8	21.6	22.4	23.2
9	22.5	23.4	24.3	25.2	26.1

	30	**31**	**32**	**33**	**34**
1	3.0	3.1	3.2	3.3	3.4
2	6.0	6.2	6.4	6.6	6.8
3	9.0	9.3	9.6	9.9	10.2
4	12.0	12.4	12.8	13.2	13.6
5	15.0	15.5	16.0	16.5	17.0
6	18.0	18.6	19.2	19.8	20.4
7	21.0	21.7	22.4	23.1	23.8
8	24.0	24.8	25.6	26.4	27.2
9	27.0	27.9	28.8	29.7	30.6

	35	**36**	**37**	**38**	**39**
1	3.5	3.6	3.7	3.8	3.9
2	7.0	7.2	7.4	7.6	7.8
3	10.5	10.8	11.1	11.4	11.7
4	14.0	14.4	14.8	15.2	15.6
5	17.5	18.0	18.5	19.0	19.5
6	21.0	21.6	22.2	22.8	23.4
7	24.5	25.2	25.9	26.6	27.3
8	28.0	28.8	29.6	30.4	31.2
9	31.5	32.4	33.3	34.2	35.1

	40	**41**	**42**	**43**	**44**
1	4.0	4.1	4.2	4.3	4.4
2	8.0	8.2	8.4	8.6	8.8
3	12.0	12.3	12.6	12.9	13.2
4	16.0	16.4	16.8	17.2	17.6
5	20.0	20.5	21.0	21.5	22.0
6	24.0	24.6	25.2	25.8	26.4
7	28.0	28.7	29.4	30.1	30.8
8	32.0	32.8	33.6	34.4	35.2
9	36.0	36.9	37.8	38.7	39.6

	45	**46**	**47**	**48**	**49**
1	4.5	4.6	4.7	4.8	4.9
2	9.0	9.2	9.4	9.6	9.8
3	13.5	13.8	14.1	14.4	14.7
4	18.0	18.4	18.8	19.2	19.6
5	22.5	23.0	23.5	24.0	24.5
6	27.0	27.6	28.2	28.8	29.4
7	31.5	32.2	32.9	33.6	34.3
8	36.0	36.8	37.6	38.4	39.2
9	40.5	41.4	42.3	43.2	44.1

	50	**51**	**52**	**53**
1	5.0	5.1	5.2	5.3
2	10.0	10.2	10.4	10.6
3	15.0	15.3	15.6	15.9
4	20.0	20.4	20.8	21.2
5	25.0	25.5	26.0	26.5
6	30.0	30.6	31.2	31.8
7	35.0	35.7	36.4	37.1
8	40.0	40.8	41.6	42.4
9	45.0	45.9	46.8	47.7

°	′		0′	1′	2′	3′	4′	5′	6′	7′	8′	9′	10′		
76	0	0.6	032	038	043	048	054	059	065	070	076	081	086	50	
	10		086	092	097	103	108	114	119	125	130	136	141	40	
	20		141	147	152	158	163	169	174	180	185	191	196	30	
	30		196	202	208	213	219	224	230	236	241	247	252	20	
	40		252	258	264	269	275	281	286	292	298	303	309	10	
	50		309	315	320	326	332	338	343	349	355	361	366	0	**13**
77	0	0.6	366	372	378	384	389	395	401	407	413	419	424	50	
	10		424	430	436	442	448	454	459	465	471	477	483	40	
	20		483	489	495	501	507	513	519	525	531	536	542	30	
	30		542	548	554	560	566	572	578	584	591	597	603	20	
	40		603	609	615	621	627	633	639	645	651	657	664	10	
	50		664	670	676	682	688	694	700	707	713	719	725	0	**12**
78	0	0.6	725	731	738	744	750	756	763	769	775	781	788	50	
	10		788	794	800	807	813	819	826	832	838	845	851	40	
	20		851	858	864	870	877	883	890	896	902	909	915	30	
	30		915	922	928	935	941	948	954	961	967	974	980	20	
	40		980	987	994	*000	*007	*013	*020	*027	*033	*040	*047	10	
	50	0.7	047	053	060	067	073	080	087	093	100	107	113	0	**11**
79	0	0.7	113	120	127	134	141	147	154	161	168	175	181	50	
	10		181	188	195	202	209	216	223	230	236	243	250	40	
	20		250	257	264	271	278	285	292	299	306	313	320	30	
	30		320	327	334	342	349	356	363	370	377	384	391	20	
	40		391	399	406	413	420	427	435	442	449	456	464	10	
	50		464	471	478	485	493	500	507	515	522	529	537	0	**10**
80	0	0.7	537	544	552	559	566	574	581	589	596	604	611	50	
	10		611	619	626	634	641	649	657	664	672	679	687	40	
	20		687	695	702	710	718	725	733	741	748	756	764	30	
	30		764	772	779	787	795	803	811	819	826	834	842	20	
	40		842	850	858	866	874	882	890	898	906	914	922	10	
	50		922	930	938	946	954	962	970	978	987	995	*003	0	**9**
81	0	0.8	003	011	019	027	036	044	052	060	069	077	085	50	
	10		085	094	102	110	119	127	136	144	152	161	169	40	
	20		169	178	186	195	203	212	221	229	238	246	255	30	
	30		255	264	272	281	290	298	307	316	325	333	342	20	
	40		342	351	360	369	378	387	395	404	413	422	431	10	
	50		431	440	449	458	467	476	485	495	504	513	522	0	**8**
82	0	0.8	522	531	540	550	559	568	577	587	596	605	615	50	
	10		615	624	633	643	652	662	671	681	690	700	709	40	
	20		709	719	728	738	748	757	767	777	786	796	806	30	
	30		806	815	825	835	845	855	865	875	884	894	904	20	
	40		904	914	924	934	945	955	965	975	985	995	*005	10	
	50	0.9	005	016	026	036	046	057	067	077	088	098	109	0	**7**
			10′	**9′**	**8′**	**7′**	**6′**	**5′**	**4′**	**3′**	**2′**	**1′**	**0′**	**′**	**°**

LOG COTANGENT

°	′		0′	1′	2′	3′	4′	5′	6′	7′	8′	9′	10′		
83	0	0.9	109	119	129	140	151	161	172	182	193	204	214	50	
	10		214	225	236	246	257	268	279	290	301	312	322	40	
	20		322	333	344	355	367	378	389	400	411	422	433	30	
	30		433	445	456	467	479	490	501	513	524	536	547	20	
	40		547	559	570	582	593	605	617	629	640	652	664	10	
	50		664	676	688	700	711	723	735	747	760	772	784	0	**6**
84	0	0.9	784	796	808	820	833	845	857	870	882	895	907	50	
	10		907	920	932	945	957	970	983	995	*008	*021	*034	40	
	20	1.0	034	047	060	072	085	099	112	125	138	151	164	30	
	30		164	177	191	204	218	231	244	258	271	285	299	20	
	40		299	312	326	340	354	367	381	395	409	423	437	10	
	50		437	451	466	480	494	508	523	537	551	566	580	0	**5**
85	0	1.0	580	595	610	624	639	654	669	683	698	713	728	50	
	10		728	744	759	774	789	804	820	835	850	866	882	40	
	20		882	897	913	929	944	960	976	992	*008	*024	*040	30	
	30	1.1	040	056	073	089	105	122	138	155	171	188	205	20	
	40		205	222	238	255	272	289	306	324	341	358	376	10	
	50		376	393	411	428	446	464	482	499	517	535	554	0	**4**
86	0	1.1	554	572	590	608	627	645	664	682	701	720	739	50	
	10		739	758	777	796	815	835	854	874	893	913	933	40	
	20		933	952	972	992	*012	*033	*053	*073	*094	*114	*135	30	
	30	1.2	135	156	177	198	219	240	261	283	304	326	348	20	
	40		348	369	391	413	435	458	480	503	525	548	571	10	
	50		571	594	617	640	663	687	710	734	758	782	806	0	**3**
87	0	1.2	806	830	855	879	904	929	954	979	*004	*029	*055	50	
	10	1.3	055	080	106	132	158	185	211	238	264	291	318	40	
	20		318	346	373	401	429	456	485	513	541	570	599	30	
	30		599	628	657	687	717	746	777	807	837	868	899	20	
	40		899	930	962	993	*025	*057	*089	*122	*155	*188	*221	10	
	50	1.4	221	255	289	323	357	392	427	462	497	533	569	0	**2**
88	0	1.4	569	606	642	679	717	754	792	830	869	908	947	50	
	10		947	987	*027	*067	*108	*149	*191	*233	*275	*318	*362	40	
	20	1.5	362	405	449	494	539	584	630	677	724	771	819	30	
	30		819	868	917	967	*017	*068	*119	*171	*224	*277	*331	20	
	40	1.6	331	386	441	497	554	611	670	729	789	850	911	10	
	50		911	974	*037	*101	*167	*233	*300	*369	*438	*509	*581	0	**1**
89	0	1.7	581	654	728	804	880	959	*038	*120	*202	*287	*373	50	
	10	1.8	373	460	550	641	735	830	928	*028	*130	*235	*342	40	
	20	1.9	342	452	565	681	800	922	*0048	*0177	*0314	*0449	*0591	30	
	30	2.	0591	0739	0891	1049	1213	1383	1561	1745	1938	2140	2352–	20	
	40		2352	2575	2810	3058	3322	3602	3901	4223	4571	4949	5363	10	
	50		5363	5820	6332	6912	7581	8373	9342	3.0592	3.2352	3.5363	—	0	**0**
			10′	**9′**	**8′**	**7′**	**6′**	**5′**	**4′**	**3′**	**2′**	**1′**	**0′**	′	°

TABLE VII

SQUARES, SQUARE ROOTS

AND

RECIPROCALS

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$	n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$
1.00	1.0000	1.00000	3.16228	1.00000	**1.50**	2.2500	1.22474	3.87298	.666667
1.01	1.0201	1.00499	3.17805	.990099	1.51	2.2801	1.22882	3.88587	.662252
1.02	1.0404	1.00995	3.19374	.980392	1.52	2.3104	1.23288	3.89872	.657895
1.03	1.0609	1.01489	3.20936	.970874	1.53	2.3409	1.23693	3.91152	.653595
1.04	1.0816	1.01980	3.22490	.961538	1.54	2.3716	1.24097	3.92428	.649351
1.05	1.1025	1.02470	3.24037	.952381	1.55	2.4025	1.24499	3.93700	.645161
1.06	1.1236	1.02956	3.25576	.943396	1.56	2.4336	1.24900	3.94968	.641026
1.07	1.1449	1.03441	3.27109	.934579	1.57	2.4649	1.25300	3.96232	.636943
1.08	1.1664	1.03923	3.28634	.925926	1.58	2.4964	1.25698	3.97492	.632911
1.09	1.1881	1.04403	3.30151	.917431	1.59	2.5281	1.26095	3.98748	.628931
1.10	1.2100	1.04881	3.31662	.909091	**1.60**	2.5600	1.26491	4.00000	.625000
1.11	1.2321	1.05357	3.33167	.900901	1.61	2.5921	1.26886	4.01248	.621118
1.12	1.2544	1.05830	3.34664	.892857	1.62	2.6244	1.27279	4.02492	.617284
1.13	1.2769	1.06301	3.36155	.884956	1.63	2.6569	1.27671	4.03733	.613497
1.14	1.2996	1.06771	3.37639	.877193	1.64	2.6896	1.28062	4.04969	.609756
1.15	1.3225	1.07238	3.39116	.869565	1.65	2.7225	1.28452	4.06202	.606061
1.16	1.3456	1.07703	3.40588	.862069	1.66	2.7556	1.28841	4.07431	.602410
1.17	1.3689	1.08167	3.42053	.854701	1.67	2.7889	1.29228	4.08656	.598802
1.18	1.3924	1.08628	3.43511	.847458	1.68	2.8224	1.29615	4.09878	.595238
1.19	1.4161	1.09087	3.44964	.840336	1.69	2.8561	1.30000	4.11096	.591716
1.20	1.4400	1.09545	3.46410	.833333	**1.70**	2.8900	1.30384	4.12311	.588235
1.21	1.4641	1.10000	3.47851	.826446	1.71	2.9241	1.30767	4.13521	.584795
1.22	1.4884	1.10454	3.49285	.819672	1.72	2.9584	1.31149	4.14729	.581395
1.23	1.5129	1.10905	3.50714	.813008	1.73	2.9929	1.31529	4.15933	.578035
1.24	1.5376	1.11355	3.52136	.806452	1.74	3.0276	1.31909	4.17133	.574713
1.25	1.5625	1.11803	3.53553	.800000	1.75	3.0625	1.32288	4.18330	.571429
1.26	1.5876	1.12250	3.54965	.793651	1.76	3.0976	1.32665	4.19524	.568182
1.27	1.6129	1.12694	3.56371	.787402	1.77	3.1329	1.33041	4.20714	.564972
1.28	1.6384	1.13137	3.57771	.781250	1.78	3.1684	1.33417	4.21900	.561798
1.29	1.6641	1.13578	3.59166	.775194	1.79	3.2041	1.33791	4.23084	.558659
1.30	1.6900	1.14018	3.60555	.769231	**1.80**	3.2400	1.34164	4.24264	.555556
1.31	1.7161	1.14455	3.61939	.763359	1.81	3.2761	1.34536	4.25441	.552486
1.32	1.7424	1.14891	3.63318	.757576	1.82	3.3124	1.34907	4.26615	.549451
1.33	1.7689	1.15326	3.64692	.751880	1.83	3.3489	1.35277	4.27785	.546448
1.34	1.7956	1.15758	3.66060	.746269	1.84	3.3856	1.35647	4.28952	.543478
1.35	1.8225	1.16190	3.67423	.740741	1.85	3.4225	1.36015	4.30116	.540541
1.36	1.8496	1.16619	3.68782	.735294	1.86	3.4596	1.36382	4.31277	.537634
1.37	1.8769	1.17047	3.70135	.729927	1.87	3.4969	1.36748	4.32435	.534759
1.38	1.9044	1.17473	3.71484	.724638	1.88	3.5344	1.37113	4.33590	.531915
1.39	1.9321	1.17898	3.72827	.719424	1.89	3.5721	1.37477	4.34741	.529101
1.40	1.9600	1.18322	3.74166	.714286	**1.90**	3.6100	1.37840	4.35890	.526316
1.41	1.9881	1.18743	3.75500	.709220	1.91	3.6481	1.38203	4.37035	.523560
1.42	2.0164	1.19164	3.76829	.704225	1.92	3.6864	1.38564	4.38178	.520833
1.43	2.0449	1.19583	3.78153	.699301	1.93	3.7249	1.38924	4.39318	.518135
1.44	2.0736	1.20000	3.79473	.694444	1.94	3.7636	1.39284	4.40454	.515464
1.45	2.1025	1.20416	3.80789	.689655	1.95	3.8025	1.39642	4.41588	.512821
1.46	2.1316	1.20830	3.82099	.684932	1.96	3.8416	1.40000	4.42719	.510204
1.47	2.1609	1.21244	3.83406	.680272	1.97	3.8809	1.40357	4.43847	.507614
1.48	2.1904	1.21655	3.84708	.675676	1.98	3.9204	1.40712	4.44972	.505051
1.49	2.2201	1.22066	3.86005	.671141	1.99	3.9601	1.41067	4.46094	.502513
1.50	2.2500	1.22474	3.87298	.666667	**2.00**	4.0000	1.41421	4.47214	.500000
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$	n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$	n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$
2.00	4.0000	1.41421	4.47214	.500000	**2.50**	6.2500	1.58114	5.00000	.400000
2.01	4.0401	1.41774	4.48330	.497512	2.51	6.3001	1.58430	5.00999	.398406
2.02	4.0804	1.42127	4.49444	.495050	2.52	6.3504	1.58745	5.01996	.396825
2.03	4.1209	1.42478	4.50555	.492611	2.53	6.4009	1.59060	5.02991	.395257
2.04	4.1616	1.42829	4.51664	.490196	2.54	6.4516	1.59374	5.03984	.393701
2.05	4.2025	1.43178	4.52769	.487805	2.55	6.5025	1.59687	5.04975	.392157
2.06	4.2436	1.43527	4.53872	.485437	2.56	6.5536	1.60000	5.05964	.390625
2.07	4.2849	1.43875	4.54973	.483092	2.57	6.6049	1.60312	5.06952	.389105
2.08	4.3264	1.44222	4.56070	.480769	2.58	6.6564	1.60624	5.07937	.387597
2.09	4.3681	1.44568	4.57165	.478469	2.59	6.7081	1.60935	5.08920	.386100
2.10	4.4100	1.44914	4.58258	.476190	**2.60**	6.7600	1.61245	5.09902	.384615
2.11	4.4521	1.45258	4.59347	.473934	2.61	6.8121	1.61555	5.10882	.383142
2.12	4.4944	1.45602	4.60435	.471698	2.62	6.8644	1.61864	5.11859	.381679
2.13	4.5369	1.45945	4.61519	.469434	2.63	6.9169	1.62173	5.12835	.380228
2.14	4.5796	1.46287	4.62601	.467290	2.64	6.9696	1.62481	5.13809	.378788
2.15	4.6225	1.46629	4.63681	.465116	2.65	7.0225	1.62788	5.14782	.377358
2.16	4.6656	1.46969	4.64758	.462963	2.66	7.0756	1.63095	5.15752	.375940
2.17	4.7089	1.47309	4.65833	.460829	2.67	7.1289	1.63401	5.16720	.374532
2.18	4.7524	1.47648	4.66905	.458716	2.68	7.1824	1.63707	5.17687	.373134
2.19	4.7961	1.47986	4.67974	.456621	2.69	7.2361	1.64012	5.18652	.371747
2.20	4.8400	1.48324	4.69042	.454545	**2.70**	7.2900	1.64317	5.19615	.370370
2.21	4.8841	1.48661	4.70106	.452489	2.71	7.3441	1.64621	5.20577	.369004
2.22	4.9284	1.48997	4.71169	.450450	2.72	7.3984	1.64924	5.21536	.367647
2.23	4.9729	1.49332	4.72229	.448430	2.73	7.4529	1.65227	5.22494	.366300
2.24	5.0176	1.49666	4.73286	.446429	2.74	7.5076	1.65529	5.23450	.364964
2.25	5.0625	1.50000	4.74342	.444444	2.75	7.5625	1.65831	5.24404	.363636
2.26	5.1076	1.50333	4.75395	.442478	2.76	7.6176	1.66132	5.25357	.362319
2.27	5.1529	1.50665	4.76445	.440529	2.77	7.6729	1.66433	5.26308	.361011
2.28	5.1984	1.50997	4.77493	.438596	2.78	7.7284	1.66733	5.27257	.359712
2.29	5.2441	1.51327	4.78539	.436681	2.79	7.7841	1.67033	5.28205	.358423
2.30	5.2900	1.51658	4.79583	.434783	**2.80**	7.8400	1.67332	5.29150	.357143
2.31	5.3361	1.51987	4.80625	.432900	2.81	7.8961	1.67631	5.30094	.355872
2.32	5.3824	1.52315	4.81664	.431034	2.82	7.9524	1.67929	5.31037	.354610
2.33	5.4289	1.52643	4.82701	.429185	2.83	8.0089	1.68226	5.31977	.353357
2.34	5.4756	1.52971	4.83735	.427350	2.84	8.0656	1.68523	5.32917	.352113
2.35	5.5225	1.53297	4.84768	.425532	2.85	8.1225	1.68819	5.33854	.350877
2.36	5.5696	1.53623	4.85798	.423729	2.86	8.1796	1.69115	5.34790	.349650
2.37	5.6169	1.53948	4.86826	.421941	2.87	8.2369	1.69411	5.35724	.348432
2.38	5.6644	1.54272	4.87852	.420168	2.88	8.2944	1.69706	5.36656	.347222
2.39	5.7121	1.54596	4.88876	.418410	2.89	8.3521	1.70000	5.37587	.346021
2.40	5.7600	1.54919	4.89898	.416667	**2.90**	8.4100	1.70294	5.38516	.344828
2.41	5.8081	1.55242	4.90918	.414938	2.91	8.4681	1.70587	5.39444	.343643
2.42	5.8564	1.55563	4.91935	.413223	2.92	8.5264	1.70880	5.40370	.342466
2.43	5.9049	1.55885	4.92950	.411523	2.93	8.5849	1.71172	5.41295	.341297
2.44	5.9536	1.56205	4.93964	.409836	2.94	8.6436	1.71464	5.42218	.340136
2.45	6.0025	1.56525	4.94975	.408163	2.95	8.7025	1.71756	5.43139	.338983
2.46	6.0516	1.56844	4.95984	.406504	2.96	8.7616	1.72047	5.44059	.337838
2.47	6.1009	1.57162	4.96991	.404858	2.97	8.8209	1.72337	5.44977	.336700
2.48	6.1504	1.57480	4.97996	.403226	2.98	8.8804	1.72627	5.45894	.335570
2.49	6.2001	1.57797	4.98999	.401606	2.99	8.9401	1.72916	5.46809	.334448
2.50	6.2500	1.58114	5.00000	.400000	**3.00**	9.0000	1.73205	5.47723	.333333
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$	n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$
3.00	9.0000	1.73205	5.47723	.333333
3.01	9.0601	1.73494	5.48635	.332226
3.02	9.1204	1.73781	5.49545	.331126
3.03	9.1809	1.74069	5.50454	.330033
3.04	9.2416	1.74356	5.51362	.328947
3.05	9.3025	1.74642	5.52268	.327869
3.06	9.3636	1.74929	5.53173	.326797
3.07	9.4249	1.75214	5.54076	.325733
3.08	9.4864	1.75499	5.54977	.324675
3.09	9.5481	1.75784	5.55878	.323625
3.10	9.6100	1.76068	5.56776	.322581
3.11	9.6721	1.76352	5.57674	.321543
3.12	9.7344	1.76635	5.58570	.320513
3.13	9.7969	1.76918	5.59464	.319489
3.14	9.8596	1.77200	5.60357	.318471
3.15	9.9225	1.77482	5.61249	.317460
3.16	9.9856	1.77764	5.62139	.316456
3.17	10.0489	1.78045	5.63028	.315457
3.18	10.1124	1.78326	5.63915	.314465
3.19	10.1761	1.78606	5.64801	.313480
3.20	10.2400	1.78885	5.65685	.312500
3.21	10.3041	1.79165	5.66569	.311526
3.22	10.3684	1.79444	5.67450	.310559
3.23	10.4329	1.79722	5.68331	.309598
3.24	10.4976	1.80000	5.69210	.308642
3.25	10.5625	1.80278	5.70088	.307692
3.26	10.6276	1.80555	5.70964	.306748
3.27	10.6929	1.80831	5.71839	.305810
3.28	10.7584	1.81108	5.72713	.304878
3.29	10.8241	1.81384	5.73585	.303951
3.30	10.8900	1.81659	5.74456	.303030
3.31	10.9561	1.81934	5.75326	.302115
3.32	11.0224	1.82209	5.76194	.301205
3.33	11.0889	1.82483	5.77062	.300300
3.34	11.1556	1.82757	5.77927	.299401
3.35	11.2225	1.83030	5.78792	.298507
3.36	11.2896	1.83303	5.79655	.297619
3.37	11.3569	1.83576	5.80517	.296736
3.38	11.4244	1.83848	5.81378	.295858
3.39	11.4921	1.84120	5.82237	.294985
3.40	11.5600	1.84391	5.83095	.294118
3.41	11.6281	1.84662	5.83952	.293255
3.42	11.6964	1.84932	5.84808	.292398
3.43	11.7649	1.85203	5.85662	.291545
3.44	11.8336	1.85472	5.86515	.290698
3.45	11.9025	1.85742	5.87367	.289855
3.46	11.9716	1.86011	5.88218	.289017
3.47	12.0409	1.86279	5.89067	.288184
3.48	12.1104	1.86548	5.89915	.287356
3.49	12.1801	1.86815	5.90762	.286533
3.50	12.2500	1.87083	5.91608	.285714
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$
3.50	12.2500	1.87083	5.91608	.285714
3.51	12.3201	1.87350	5.92453	.284900
3.52	12.3904	1.87617	5.93296	.284091
3.53	12.4609	1.87883	5.94138	.283286
3.54	12.5316	1.88149	5.94979	.282486
3.55	12.6025	1.88414	5.95819	.281690
3.56	12.6736	1.88680	5.96657	.280899
3.57	12.7449	1.88944	5.97495	.280112
3.58	12.8164	1.89209	5.98331	.279330
3.59	12.8881	1.89473	5.99166	.278552
3.60	12.9600	1.89737	6.00000	.277778
3.61	13.0321	1.90000	6.00833	.277008
3.62	13.1044	1.90263	6.01664	.276243
3.63	13.1769	1.90526	6.02495	.275482
3.64	13.2496	1.90788	6.03324	.274725
3.65	13.3225	1.91050	6.04152	.273973
3.66	13.3956	1.91311	6.04979	.273224
3.67	13.4689	1.91572	6.05805	.272480
3.68	13.5424	1.91833	6.06630	.271739
3.69	13.6161	1.92094	6.07454	.271003
3.70	13.6900	1.92354	6.08276	.270270
3.71	13.7641	1.92614	6.09098	.269542
3.72	13.8384	1.92873	6.09918	.268817
3.73	13.9129	1.93132	6.10737	.268097
3.74	13.9876	1.93391	6.11555	.267380
3.75	14.0625	1.93649	6.12372	.266667
3.76	14.1376	1.93907	6.13188	.265957
3.77	14.2129	1.94165	6.14003	.265252
3.78	14.2884	1.94422	6.14817	.264550
3.79	14.3641	1.94679	6.15630	.263852
3.80	14.4400	1.94936	6.16441	.263158
3.81	14.5161	1.95192	6.17252	.262467
3.82	14.5924	1.95448	6.18061	.261780
3.83	14.6689	1.95704	6.18870	.261097
3.84	14.7456	1.95959	6.19677	.260417
3.85	14.8225	1.96214	6.20484	.259740
3.86	14.8996	1.96469	6.21289	.259067
3.87	14.9769	1.96723	6.22093	.258398
3.88	15.0544	1.96977	6.22896	.257732
3.89	15.1321	1.97231	6.23699	.257069
3.90	15.2100	1.97484	6.24500	.256410
3.91	15.2881	1.97737	6.25300	.255754
3.92	15.3664	1.97990	6.26099	.255102
3.93	15.4449	1.98242	6.26897	.254453
3.94	15.5236	1.98494	6.27694	.253807
3.95	15.6025	1.98746	6.28490	.253165
3.96	15.6816	1.98997	6.29285	.252525
3.97	15.7609	1.99249	6.30079	.251889
3.98	15.8404	1.99499	6.30872	.251256
3.99	15.9201	1.99750	6.31664	.250627
4.00	16.0000	2.00000	6.32456	.250000
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$
4.00	16.0000	2.00000	6.32456	.250000
4.01	16.0801	2.00250	6.33246	.249377
4.02	16.1604	2.00499	6.34035	.248756
4.03	16.2409	2.00749	6.34823	.248139
4.04	16.3216	2.00998	6.35610	.247525
4.05	16.4025	2.01246	6.36396	.246914
4.06	16.4836	2.01494	6.37181	.246305
4.07	16.5649	2.01742	6.37966	.245700
4.08	16.6464	2.01990	6.38749	.245098
4.09	16.7281	2.02237	6.39531	.244499
4.10	16.8100	2.02485	6.40312	.243902
4.11	16.8921	2.02731	6.41093	.243309
4.12	16.9744	2.02978	6.41872	.242718
4.13	17.0569	2.03224	6.42651	.242131
4.14	17.1396	2.03470	6.43428	.241546
4.15	17.2225	2.03715	6.44205	.240964
4.16	17.3056	2.03961	6.44981	.240385
4.17	17.3889	2.04206	6.45755	.239808
4.18	17.4724	2.04450	6.46529	.239234
4.19	17.5561	2.04695	6.47302	.238663
4.20	17.6400	2.04939	6.48074	.238095
4.21	17.7241	2.05183	6.48845	.237530
4.22	17.8084	2.05426	6.49615	.236967
4.23	17.8929	2.05670	6.50384	.236407
4.24	17.9776	2.05913	6.51153	.235849
4.25	18.0625	2.06155	6.51920	.235294
4.26	18.1476	2.06398	6.52687	.234742
4.27	18.2329	2.06640	6.53452	.234192
4.28	18.3184	2.06882	6.54217	.233645
4.29	18.4041	2.07123	6.54981	.233100
4.30	18.4900	2.07364	6.55744	.232558
4.31	18.5761	2.07605	6.56506	.232019
4.32	18.6624	2.07846	6.57267	.231481
4.33	18.7489	2.08087	6.58027	.230947
4.34	18.8356	2.08327	6.58787	.230415
4.35	18.9225	2.08567	6.59545	.229885
4.36	19.0096	2.08806	6.60303	.229358
4.37	19.0969	2.09045	6.61060	.228833
4.38	19.1844	2.09284	6.61816	.228311
4.39	19.2721	2.09523	6.62571	.227790
4.40	19.3600	2.09762	6.63325	.227273
4.41	19.4481	2.10000	6.64078	.226757
4.42	19.5364	2.10238	6.64831	.226244
4.43	19.6249	2.10476	6.65582	.225734
4.44	19.7136	2.10713	6.66333	.225225
4.45	19.8025	2.10950	6.67083	.224719
4.46	19.8916	2.11187	6.67832	.224215
4.47	19.9809	2.11424	6.68581	.223714
4.48	20.0704	2.11660	6.69328	.223214
4.49	20.1601	2.11896	6.70075	.222717
4.50	20.2500	2.12132	6.70820	.222222
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$
4.50	20.2500	2.12132	6.70820	.222222
4.51	20.3401	2.12368	6.71565	.221729
4.52	20.4304	2.12603	6.72309	.221239
4.53	20.5209	2.12838	6.73053	.220751
4.54	20.6116	2.13073	6.73795	.220264
4.55	20.7025	2.13307	6.74537	.219780
4.56	20.7936	2.13542	6.75278	.219298
4.57	20.8849	2.13776	6.76018	.218818
4.58	20.9764	2.14009	6.76757	.218341
4.59	21.0681	2.14243	6.77495	.217865
4.60	21.1600	2.14476	6.78233	.217391
4.61	21.2521	2.14709	6.78970	.216920
4.62	21.3444	2.14942	6.79706	.216450
4.63	21.4369	2.15174	6.80441	.215983
4.64	21.5296	2.15407	6.81175	.215517
4.65	21.6225	2.15639	6.81909	.215054
4.66	21.7156	2.15870	6.82642	.214592
4.67	21.8089	2.16102	6.83374	.214133
4.68	21.9024	2.16333	6.84105	.213675
4.69	21.9961	2.16564	6.84836	.213220
4.70	22.0900	2.16795	6.85565	.212766
4.71	22.1841	2.17025	6.86294	.212314
4.72	22.2784	2.17256	6.87023	.211864
4.73	22.3729	2.17486	6.87750	.211416
4.74	22.4676	2.17715	6.88477	.210970
4.75	22.5625	2.17945	6.89202	.210526
4.76	22.6576	2.18174	6.89928	.210084
4.77	22.7529	2.18403	6.90652	.209644
4.78	22.8484	2.18632	6.91375	.209205
4.79	22.9441	2.18861	6.92098	.208768
4.80	23.0400	2.19089	6.92820	.208333
4.81	23.1361	2.19317	6.93542	.207900
4.82	23.2324	2.19545	6.94262	.207469
4.83	23.3289	2.19773	6.94982	.207039
4.84	23.4256	2.20000	6.95701	.206612
4.85	23.5225	2.20227	6.96419	.206186
4.86	23.6196	2.20454	6.97137	.205761
4.87	23.7169	2.20681	6.97854	.205339
4.88	23.8144	2.20907	6.98570	.204918
4.89	23.9121	2.21133	6.99285	.204499
4.90	24.0100	2.21359	7.00000	.204082
4.91	24.1081	2.21585	7.00714	.203666
4.92	24.2064	2.21811	7.01427	.203252
4.93	24.3049	2.22036	7.02140	.202840
4.94	24.4036	2.22261	7.02851	.202429
4.95	24.5025	2.22486	7.03562	.202020
4.96	24.6016	2.22711	7.04273	.201613
4.97	24.7009	2.22935	7.04982	.201207
4.98	24.8004	2.23159	7.05691	.200803
4.99	24.9001	2.23383	7.06399	.200401
5.00	25.0000	2.23607	7.07107	.200000
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$
5.00	25.0000	2.23607	7.07107	.200000
5.01	25.1001	2.23830	7.07814	.199601
5.02	25.2004	2.24054	7.08520	.199203
5.03	25.3009	2.24277	7.09225	.198807
5.04	25.4016	2.24499	7.09930	.198413
5.05	25.5025	2.24722	7.10634	.198020
5.06	25.6036	2.24944	7.11337	.197628
5.07	25.7049	2.25167	7.12039	.197239
5.08	25.8064	2.25389	7.12741	.196850
5.09	25.9081	2.25610	7.13442	.196464
5.10	26.0100	2.25832	7.14143	.196078
5.11	26.1121	2.26053	7.14843	.195695
5.12	26.2144	2.26274	7.15542	.195312
5.13	26.3169	2.26495	7.16240	.194932
5.14	26.4196	2.26716	7.16938	.194553
5.15	26.5225	2.26936	7.17635	.194175
5.16	26.6256	2.27156	7.18331	.193798
5.17	26.7289	2.27376	7.19027	.193424
5.18	26.8324	2.27596	7.19722	.193050
5.19	26.9361	2.27816	7.20417	.192678
5.20	27.0400	2.28035	7.21110	.192308
5.21	27.1441	2.28254	7.21803	.191939
5.22	27.2484	2.28473	7.22496	.191571
5.23	27.3529	2.28692	7.23187	.191205
5.24	27.4576	2.28910	7.23878	.190840
5.25	27.5625	2.29129	7.24569	.190476
5.26	27.6676	2.29347	7.25259	.190114
5.27	27.7729	2.29565	7.25948	.189753
5.28	27.8784	2.29783	7.26636	.189394
5.29	27.9841	2.30000	7.27324	.189036
5.30	28.0900	2.30217	7.28011	.188679
5.31	28.1961	2.30434	7.28697	.188324
5.32	28.3024	2.30651	7.29383	.187970
5.33	28.4089	2.30868	7.30068	.187617
5.34	28.5156	2.31084	7.30753	.187266
5.35	28.6225	2.31301	7.31437	.186916
5.36	28.7296	2.31517	7.32120	.186567
5.37	28.8369	2.31733	7.32803	.186220
5.38	28.9444	2.31948	7.33485	.185874
5.39	29.0521	2.32164	7.34166	.185529
5.40	29.1600	2.32379	7.34847	.185185
5.41	29.2681	2.32594	7.35527	.184843
5.42	29.3764	2.32809	7.36206	.184502
5.43	29.4849	2.33024	7.36885	.184162
5.44	29.5936	2.33238	7.37564	.183824
5.45	29.7025	2.33452	7.38241	.183486
5.46	29.8116	2.33666	7.38918	.183150
5.47	29.9209	2.33880	7.39594	.182815
5.48	30.0304	2.34094	7.40270	.182482
5.49	30.1401	2.34307	7.40945	.182149
5.50	30.2500	2.34521	7.41620	.181818
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$
5.50	30.2500	2.34521	7.41620	.181818
5.51	30.3601	2.34734	7.42294	.181488
5.52	30.4704	2.34947	7.42967	.181159
5.53	30.5809	2.35160	7.43640	.180832
5.54	30.6916	2.35372	7.44312	.180505
5.55	30.8025	2.35584	7.44983	.180180
5.56	30.9136	2.35797	7.45654	.179856
5.57	31.0249	2.36008	7.46324	.179533
5.58	31.1364	2.36220	7.46994	.179211
5.59	31.2481	2.36432	7.47663	.178891
5.60	31.3600	2.36643	7.48331	.178571
5.61	31.4721	2.36854	7.48999	.178253
5.62	31.5844	2.37065	7.49667	.177936
5.63	31.6969	2.37276	7.50333	.177620
5.64	31.8096	2.37487	7.50999	.177305
5.65	31.9225	2.37697	7.51665	.176991
5.66	32.0356	2.37908	7.52330	.176678
5.67	32.1489	2.38118	7.52994	.176367
5.68	32.2624	2.38328	7.53658	.176056
5.69	32.3761	2.38537	7.54321	.175747
5.70	32.4900	2.38747	7.54983	.175439
5.71	32.6041	2.38956	7.55645	.175131
5.72	32.7184	2.39165	7.56307	.174825
5.73	32.8329	2.39374	7.56968	.174520
5.74	32.9476	2.39583	7.57628	.174216
5.75	33.0625	2.39792	7.58288	.173913
5.76	33.1776	2.40000	7.58947	.173611
5.77	33.2929	2.40208	7.59605	.173310
5.78	33.4084	2.40416	7.60263	.173010
5.79	33.5241	2.40624	7.60920	.172712
5.80	33.6400	2.40832	7.61577	.172414
5.81	33.7561	2.41039	7.62234	.172117
5.82	33.8724	2.41247	7.62889	.171821
5.83	33.9889	2.41454	7.63544	.171527
5.84	34.1056	2.41661	7.64199	.171233
5.85	34.2225	2.41868	7.64853	.170940
5.86	34.3396	2.42074	7.65506	.170649
5.87	34.4569	2.42281	7.66159	.170358
5.88	34.5744	2.42487	7.66812	.170068
5.89	34.6921	2.42693	7.67463	.169779
5.90	34.8100	2.42899	7.68115	.169492
5.91	34.9281	2.43105	7.68765	.169205
5.92	35.0464	2.43311	7.69415	.168919
5.93	35.1649	2.43516	7.70065	.168634
5.94	35.2836	2.43721	7.70714	.168350
5.95	35.4025	2.43926	7.71362	.168067
5.96	35.5216	2.44131	7.72010	.167785
5.97	35.6409	2.44336	7.72658	.167504
5.98	35.7604	2.44540	7.73305	.167224
5.99	35.8801	2.44745	7.73951	.166945
6.00	36.0000	2.44949	7.74597	.166667
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$
6.00	36.0000	2.44949	7.74597	.166667
6.01	36.1201	2.45153	7.75242	.166389
6.02	36.2404	2.45357	7.75887	.166113
6.03	36.3609	2.45561	7.76531	.165837
6.04	36.4816	2.45764	7.77174	.165563
6.05	36.6025	2.45967	7.77817	.165289
6.06	36.7236	2.46171	7.78460	.165017
6.07	36.8449	2.46374	7.79102	.164745
6.08	36.9664	2.46577	7.79744	.164474
6.09	37.0881	2.46779	7.80385	.164204
6.10	37.2100	2.46982	7.81025	.163934
6.11	37.3321	2.47184	7.81665	.163666
6.12	37.4544	2.47386	7.82304	.163399
6.13	37.5769	2.47588	7.82943	.163132
6.14	37.6996	2.47790	7.83582	.162866
6.15	37.8225	2.47992	7.84219	.162602
6.16	37.9456	2.48193	7.84857	.162338
6.17	38.0689	2.48395	7.85493	.162075
6.18	38.1924	2.48596	7.86130	.161812
6.19	38.3161	2.48797	7.86766	.161551
6.20	38.4400	2.48998	7.87401	.161290
6.21	38.5641	2.49199	7.88036	.161031
6.22	38.6884	2.49399	7.88670	.160772
6.23	38.8129	2.49600	7.89303	.160514
6.24	38.9376	2.49800	7.89937	.160256
6.25	39.0625	2.50000	7.90569	.160000
6.26	39.1876	2.50200	7.91202	.159744
6.27	39.3129	2.50400	7.91833	.159490
6.28	39.4384	2.50599	7.92465	.159236
6.29	39.5641	2.50799	7.93095	.158983
6.30	39.6900	2.50998	7.93725	.158730
6.31	39.8161	2.51197	7.94355	.158479
6.32	39.9424	2.51396	7.94984	.158228
6.33	40.0689	2.51595	7.95613	.157978
6.34	40.1956	2.51794	7.96241	.157729
6.35	40.3225	2.51992	7.96869	.157480
6.36	40.4496	2.52190	7.97496	.157233
6.37	40.5769	2.52389	7.98123	.156986
6.38	40.7044	2.52587	7.98749	.156740
6.39	40.8321	2.52784	7.99375	.156495
6.40	40.9600	2.52982	8.00000	.156250
6.41	41.0881	2.53180	8.00625	.156006
6.42	41.2164	2.53377	8.01249	.155763
6.43	41.3449	2.53574	8.01873	.155521
6.44	41.4736	2.53772	8.02496	.155280
6.45	41.6025	2.53969	8.03119	.155039
6.46	41.7316	2.54165	8.03741	.154799
6.47	41.8609	2.54362	8.04363	.154560
6.48	41.9904	2.54558	8.04984	.154321
6.49	42.1201	2.54755	8.05605	.154083
6.50	42.2500	2.54951	8.06226	.153846
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$
6.50	42.2500	2.54951	8.06226	.153846
6.51	42.3801	2.55147	8.06846	.153610
6.52	42.5104	2.55343	8.07465	.153374
6.53	42.6409	2.55539	8.08084	.153139
6.54	42.7716	2.55734	8.08703	.152905
6.55	42.9025	2.55930	8.09321	.152672
6.56	43.0336	2.56125	8.09938	.152439
6.57	43.1649	2.56320	8.10555	.152207
6.58	43.2964	2.56515	8.11172	.151976
6.59	43.4281	2.56710	8.11788	.151745
6.60	43.5600	2.56905	8.12404	.151515
6.61	43.6921	2.57099	8.13019	.151286
6.62	43.8244	2.57294	8.13634	.151057
6.63	43.9569	2.57488	8.14248	.150830
6.64	44.0896	2.57682	8.14862	.150602
6.65	44.2225	2.57876	8.15475	.150376
6.66	44.3556	2.58070	8.16088	.150150
6.67	44.4889	2.58263	8.16701	.149925
6.68	44.6224	2.58457	8.17313	.149701
6.69	44.7561	2.58650	8.17924	.149477
6.70	44.8900	2.58844	8.18535	.149254
6.71	45.0241	2.59037	8.19146	.149031
6.72	45.1584	2.59230	8.19756	.148810
6.73	45.2929	2.59422	8.20366	.148588
6.74	45.4276	2.59615	8.20975	.148368
6.75	45.5625	2.59808	8.21584	.148148
6.76	45.6976	2.60000	8.22192	.147929
6.77	45.8329	2.60192	8.22800	.147710
6.78	45.9684	2.60384	8.23408	.147493
6.79	46.1041	2.60576	8.24015	.147275
6.80	46.2400	2.60768	8.24621	.147059
6.81	46.3761	2.60960	8.25227	.146843
6.82	46.5124	2.61151	8.25833	.146628
6.83	46.6489	2.61343	8.26438	.146413
6.84	46.7856	2.61534	8.27043	.146199
6.85	46.9225	2.61725	8.27647	.145985
6.86	47.0596	2.61916	8.28251	.145773
6.87	47.1969	2.62107	8.28855	.145560
6.88	47.3344	2.62298	8.29458	.145349
6.89	47.4721	2.62488	8.30060	.145138
6.90	47.6100	2.62679	8.30662	.144928
6.91	47.7481	2.62869	8.31264	.144718
6.92	47.8864	2.63059	8.31865	.144509
6.93	48.0249	2.63249	8.32466	.144300
6.94	48.1636	2.63439	8.33067	.144092
6.95	48.3025	2.63629	8.33667	.143885
6.96	48.4416	2.63818	8.34266	.143678
6.97	48.5809	2.64008	8.34865	.143472
6.98	48.7204	2.64197	8.35464	.143266
6.99	48.8601	2.64386	8.36062	.143062
7.00	49.0000	2.64575	8.36660	.142857
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$	n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$
7.00	49.0000	2.64575	8.36660	.142857	**7.50**	56.2500	2.73861	8.66025	.133333
7.01	49.1401	2.64764	8.37257	.142653	7.51	56.4001	2.74044	8.66603	.133156
7.02	49.2804	2.64953	8.37854	.142450	7.52	56.5504	2.74226	8.67179	.132979
7.03	49.4209	2.65141	8.38451	.142248	7.53	56.7009	2.74408	8.67756	.132802
7.04	49.5616	2.65330	8.39047	.142045	7.54	56.8516	2.74591	8.68332	.132626
7.05	49.7025	2.65518	8.39643	.141844	7.55	57.0025	2.74773	8.68907	.132450
7.06	49.8436	2.65707	8.40238	.141643	7.56	57.1536	2.74955	8.69483	.132275
7.07	49.9849	2.65895	8.40833	.141443	7.57	57.3049	2.75136	8.70057	.132100
7.08	50.1264	2.66083	8.41427	.141243	7.58	57.4564	2.75318	8.70632	.131926
7.09	50.2681	2.66271	8.42021	.141044	7.59	57.6081	2.75500	8.71206	.131752
7.10	50.4100	2.66458	8.42615	.140845	**7.60**	57.7600	2.75681	8.71780	.131579
7.11	50.5521	2.66646	8.43208	.140647	7.61	57.9121	2.75862	8.72353	.131406
7.12	50.6944	2.66833	8.43801	.140449	7.62	58.0644	2.76043	8.72926	.131234
7.13	50.8369	2.67021	8.44393	.140252	7.63	58.2169	2.76225	8.73499	.131062
7.14	50.9796	2.67208	8.44985	.140056	7.64	58.3696	2.76405	8.74071	.130890
7.15	51.1225	2.67395	8.45577	.139860	7.65	58.5225	2.76586	8.74643	.130719
7.16	51.2656	2.67582	8.46168	.139665	7.66	58.6756	2.76767	8.75214	.130548
7.17	51.4089	2.67769	8.46759	.139470	7.67	58.8289	2.76948	8.75785	.130378
7.18	51.5524	2.67955	8.47349	.139276	7.68	58.9824	2.77128	8.76356	.130208
7.19	51.6961	2.68142	8.47939	.139082	7.69	59.1361	2.77308	8.76926	.130039
7.20	51.8400	2.68328	8.48528	.138889	**7.70**	59.2900	2.77489	8.77496	.129870
7.21	51.9841	2.68514	8.49117	.138696	7.71	59.4441	2.77669	8.78066	.129702
7.22	52.1284	2.68701	8.49706	.138504	7.72	59.5984	2.77849	8.78635	.129534
7.23	52.2729	2.68887	8.50294	.138313	7.73	59.7529	2.78029	8.79204	.129366
7.24	52.4176	2.69072	8.50882	.138122	7.74	59.9076	2.78209	8.79773	.129199
7.25	52.5625	2.69258	8.51469	.137931	7.75	60.0625	2.78388	8.80341	.129032
7.26	52.7076	2.69444	8.52056	.137741	7.76	60.2176	2.78568	8.80909	.128866
7.27	52.8529	2.69629	8.52643	.137552	7.77	60.3729	2.78747	8.81476	.128700
7.28	52.9984	2.69815	8.53229	.137363	7.78	60.5284	2.78927	8.82043	.128535
7.29	53.1441	2.70000	8.53815	.137174	7.79	60.6841	2.79106	8.82610	.128370
7.30	53.2900	2.70185	8.54400	.136986	**7.80**	60.8400	2.79285	8.83176	.128205
7.31	53.4361	2.70370	8.54985	.136799	7.81	60.9961	2.79464	8.83742	.128041
7.32	53.5824	2.70555	8.55570	.136612	7.82	61.1524	2.79643	8.84308	.127877
7.33	53.7289	2.70740	8.56154	.136426	7.83	61.3089	2.79821	8.84873	.127714
7.34	53.8756	2.70924	8.56738	.136240	7.84	61.4656	2.80000	8.85438	.127551
7.35	54.0225	2.71109	8.57321	.136054	7.85	61.6225	2.80179	8.86002	.127389
7.36	54.1696	2.71293	8.57904	.135870	7.86	61.7796	2.80357	8.86566	.127226
7.37	54.3169	2.71477	8.58487	.135685	7.87	61.9369	2.80535	8.87130	.127065
7.38	54.4644	2.71662	8.59069	.135501	7.88	62.0944	2.80713	8.87694	.126904
7.39	54.6121	2.71846	8.59651	.135318	7.89	62.2521	2.80891	8.88257	.126743
7.40	54.7600	2.72029	8.60233	.135135	**7.90**	62.4100	2.81069	8.88819	.126582
7.41	54.9081	2.72213	8.60814	.134953	7.91	62.5681	2.81247	8.89382	.126422
7.42	55.0564	2.72397	8.61394	.134771	7.92	62.7264	2.81425	8.89944	.126263
7.43	55.2049	2.72580	8.61974	.134590	7.93	62.8849	2.81603	8.90505	.126103
7.44	55.3536	2.72764	8.62554	.134409	7.94	63.0436	2.81780	8.91067	.125945
7.45	55.5025	2.72947	8.63134	.134228	7.95	63.2025	2.81957	8.91628	.125786
7.46	55.6516	2.73130	8.63713	.134048	7.96	63.3616	2.82135	8.92188	.125628
7.47	55.8009	2.73313	8.64292	.133869	7.97	63.5209	2.82312	8.92749	.125471
7.48	55.9504	2.73496	8.64870	.133690	7.98	63.6804	2.82489	8.93308	.125313
7.49	56.1001	2.73679	8.65448	.133511	7.99	63.8401	2.82666	8.93868	.125156
7.50	56.2500	2.73861	8.66025	.133333	**8.00**	64.0000	2.82843	8.94427	.125000
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$	n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$
8.00	64.0000	2.82843	8.94427	.125000
8.01	64.1601	2.83019	8.94986	.124844
8.02	64.3204	2.83196	8.95545	.124688
8.03	64.4809	2.83373	8.96103	.124533
8.04	64.6416	2.83549	8.96660	.124378
8.05	64.8025	2.83725	8.97218	.124224
8.06	64.9636	2.83901	8.97775	.124069
8.07	65.1249	2.84077	8.98332	.123916
8.08	65.2864	2.84253	8.98888	.123762
8.09	65.4481	2.84429	8.99444	.123609
8.10	65.6100	2.84605	9.00000	.123457
8.11	65.7721	2.84781	9.00555	.123305
8.12	65.9344	2.84956	9.01110	.123153
8.13	66.0969	2.85132	9.01665	.123001
8.14	66.2596	2.85307	9.02219	.122850
8.15	66.4225	2.85482	9.02774	.122699
8.16	66.5856	2.85657	9.03327	.122549
8.17	66.7489	2.85832	9.03881	.122399
8.18	66.9124	2.86007	9.04434	.122249
8.19	67.0761	2.86182	9.04986	.122100
8.20	67.2400	2.86356	9.05539	.121951
8.21	67.4041	2.86531	9.06091	.121803
8.22	67.5684	2.86705	9.06642	.121655
8.23	67.7329	2.86880	9.07193	.121507
8.24	67.8976	2.87054	9.07744	.121359
8.25	68.0625	2.87228	9.08295	.121212
8.26	68.2276	2.87402	9.08845	.121065
8.27	68.3929	2.87576	9.09395	.120919
8.28	68.5584	2.87750	9.09945	.120773
8.29	68.7241	2.87924	9.10494	.120627
8.30	68.8900	2.88097	9.11043	.120482
8.31	69.0561	2.88271	9.11592	.120337
8.32	69.2224	2.88444	9.12140	.120192
8.33	69.3889	2.88617	9.12688	.120048
8.34	69.5556	2.88791	9.13236	.119904
8.35	69.7225	2.88964	9.13783	.119760
8.36	69.8896	2.89137	9.14330	.119617
8.37	70.0569	2.89310	9.14877	.119474
8.38	70.2244	2.89482	9.15423	.119332
8.39	70.3921	2.89655	9.15969	.119190
8.40	70.5600	2.89828	9.16515	.119048
8.41	70.7281	2.90000	9.17061	.118906
8.42	70.8964	2.90172	9.17606	.118765
8.43	71.0649	2.90345	9.18150	.118624
8.44	71.2336	2.90517	9.18695	.118483
8.45	71.4025	2.90689	9.19239	.118343
8.46	71.5716	2.90861	9.19783	.118203
8.47	71.7409	2.91033	9.20326	.118064
8.48	71.9104	2.91204	9.20869	.117925
8.49	72.0801	2.91376	9.21412	.117786
8.50	72.2500	2.91548	9.21954	.117647
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$
8.50	72.2500	2.91548	9.21954	.117647
8.51	72.4201	2.91719	9.22497	.117509
8.52	72.5904	2.91890	9.23038	.117371
8.53	72.7609	2.92062	9.23580	.117233
8.54	72.9316	2.92233	9.24121	.117096
8.55	73.1025	2.92404	9.24662	.116959
8.56	73.2736	2.92575	9.25203	.116822
8.57	73.4449	2.92746	9.25743	.116686
8.58	73.6164	2.92916	9.26283	.116550
8.59	73.7881	2.93087	9.26823	.116414
8.60	73.9600	2.93258	9.27362	.116279
8.61	74.1321	2.93428	9.27901	.116144
8.62	74.3044	2.93598	9.28440	.116009
8.63	74.4769	2.93769	9.28978	.115875
8.64	74.6496	2.93939	9.29516	.115741
8.65	74.8225	2.94109	9.30054	.115607
8.66	74.9956	2.94279	9.30591	.115473
8.67	75.1689	2.94449	9.31128	.115340
8.68	75.3424	2.94618	9.31665	.115207
8.69	75.5161	2.94788	9.32202	.115075
8.70	75.6900	2.94958	9.32738	.114943
8.71	75.8641	2.95127	9.33274	.114811
8.72	76.0384	2.95296	9.33809	.114679
8.73	76.2129	2.95466	9.34345	.114548
8.74	76.3876	2.95635	9.34880	.114416
8.75	76.5625	2.95804	9.35414	.114286
8.76	76.7376	2.95973	9.35949	.114155
8.77	76.9129	2.96142	9.36483	.114025
8.78	77.0884	2.96311	9.37017	.113895
8.79	77.2641	2.96479	9.37550	.113766
8.80	77.4400	2.96648	9.38083	.113636
8.81	77.6161	2.96816	9.38616	.113507
8.82	77.7924	2.96985	9.39149	.113379
8.83	77.9689	2.97153	9.39681	.113250
8.84	78.1456	2.97321	9.40213	.113122
8.85	78.3225	2.97489	9.40744	.112994
8.86	78.4996	2.97658	9.41276	.112867
8.87	78.6769	2.97825	9.41807	.112740
8.88	78.8544	2.97993	9.42338	.112613
8.89	79.0321	2.98161	9.42868	.112486
8.90	79.2100	2.98329	9.43398	.112360
8.91	79.3881	2.98496	9.43928	.112233
8.92	79.5664	2.98664	9.44458	.112108
8.93	79.7449	2.98831	9.44987	.111982
8.94	79.9236	2.98998	9.45516	.111857
8.95	80.1025	2.99166	9.46044	.111732
8.96	80.2816	2.99333	9.46573	.111607
8.97	80.4609	2.99500	9.47101	.111483
8.98	80.6404	2.99666	9.47629	.111359
8.99	80.8201	2.99833	9.48156	.111235
9.00	81.0000	3.00000	9.48683	.111111
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$
9.00	81.0000	3.00000	9.48683	.111111
9.01	81.1801	3.00167	9.49210	.110988
9.02	81.3604	3.00333	9.49737	.110865
9.03	81.5409	3.00500	9.50263	.110742
9.04	81.7216	3.00666	9.50789	.110619
9.05	81.9025	3.00832	9.51315	.110497
9.06	82.0836	3.00998	9.51840	.110375
9.07	82.2649	3.01164	9.52365	.110254
9.08	82.4464	3.01330	9.52890	.110132
9.09	82.6281	3.01496	9.53415	.110011
9.10	82.8100	3.01662	9.53939	.109890
9.11	82.9921	3.01828	9.54463	.109769
9.12	83.1744	3.01993	9.54987	.109649
9.13	83.3569	3.02159	9.55510	.109529
9.14	83.5396	3.02324	9.56033	.109409
9.15	83.7225	3.02490	9.56556	.109290
9.16	83.9056	3.02655	9.57079	.109170
9.17	84.0889	3.02820	9.57601	.109051
9.18	84.2724	3.02985	9.58123	.108932
9.19	84.4561	3.03150	9.58645	.108814
9.20	84.6400	3.03315	9.59166	.108696
9.21	84.8241	3.03480	9.59687	.108578
9.22	85.0084	3.03645	9.60208	.108460
9.23	85.1929	3.03809	9.60729	.108342
9.24	85.3776	3.03974	9.61249	.108225
9.25	85.5625	3.04138	9.61769	.108108
9.26	85.7476	3.04302	9.62289	.107991
9.27	85.9329	3.04467	9.62808	.107875
9.28	86.1184	3.04631	9.63328	.107759
9.29	86.3041	3.04795	9.63846	.107643
9.30	86.4900	3.04959	9.64365	.107527
9.31	86.6761	3.05123	9.64883	.107411
9.32	86.8624	3.05287	9.65401	.107296
9.33	87.0489	3.05450	9.65919	.107181
9.34	87.2356	3.05614	9.66437	.107066
9.35	87.4225	3.05778	9.66954	.106952
9.36	87.6096	3.05941	9.67471	.106838
9.37	87.7969	3.06105	9.67988	.106724
9.38	87.9844	3.06268	9.68504	.106610
9.39	88.1721	3.06431	9.69020	.106496
9.40	88.3600	3.06594	9.69536	.106383
9.41	88.5481	3.06757	9.70052	.106270
9.42	88.7364	3.06920	9.70567	.106157
9.43	88.9249	3.07083	9.71082	.106045
9.44	89.1136	3.07246	9.71597	.105932
9.45	89.3025	3.07409	9.72111	.105820
9.46	89.4916	3.07571	9.72625	.105708
9.47	89.6809	3.07734	9.73139	.105597
9.48	89.8704	3.07896	9.73653	.105485
9.49	90.0601	3.08058	9.74166	.105374
9.50	90.2500	3.08221	9.74679	.105263
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$
9.50	90.2500	3.08221	9.74679	.105263
9.51	90.4401	3.08383	9.75192	.105152
9.52	90.6304	3.08545	9.75705	.105042
9.53	90.8209	3.08707	9.76217	.104932
9.54	91.0116	3.08869	9.76729	.104822
9.55	91.2025	3.09031	9.77241	.104712
9.56	91.3936	3.09192	9.77753	.104603
9.57	91.5849	3.09354	9.78264	.104493
9.58	91.7764	3.09516	9.78775	.104384
9.59	91.9681	3.09677	9.79285	.104275
9.60	92.1600	3.09839	9.79796	.104167
9.61	92.3521	3.10000	9.80306	.104058
9.62	92.5444	3.10161	9.80816	.103950
9.63	92.7369	3.10322	9.81326	.103842
9.64	92.9296	3.10483	9.81835	.103734
9.65	93.1225	3.10644	9.82344	.103627
9.66	93.3156	3.10805	9.82853	.103520
9.67	93.5089	3.10966	9.83362	.103413
9.68	93.7024	3.11127	9.83870	.103306
9.69	93.8961	3.11288	9.84378	.103199
9.70	94.0900	3.11448	9.84886	.103093
9.71	94.2841	3.11609	9.85393	.102987
9.72	94.4784	3.11769	9.85901	.102881
9.73	94.6729	3.11929	9.86408	.102775
9.74	94.8676	3.12090	9.86914	.102669
9.75	95.0625	3.12250	9.87421	.102564
9.76	95.2576	3.12410	9.87927	.102459
9.77	95.4529	3.12570	9.88433	.102354
9.78	95.6484	3.12730	9.88939	.102249
9.79	95.8441	3.12890	9.89444	.102145
9.80	96.0400	3.13050	9.89949	.102041
9.81	96.2361	3.13209	9.90454	.101937
9.82	96.4324	3.13369	9.90959	.101833
9.83	96.6289	3.13528	9.91464	.101729
9.84	96.8256	3.13688	9.91968	.101626
9.85	97.0225	3.13847	9.92472	.101523
9.86	97.2196	3.14006	9.92975	.101420
9.87	97.4169	3.14166	9.93479	.101317
9.88	97.6144	3.14325	9.93982	.101215
9.89	97.8121	3.14484	9.94485	.101112
9.90	98.0100	3.14643	9.94987	.101010
9.91	98.2081	3.14802	9.95490	.100908
9.92	98.4064	3.14960	9.95992	.100806
9.93	98.6049	3.15119	9.96494	.100705
9.94	98.8036	3.15278	9.96995	.100604
9.95	99.0025	3.15436	9.97497	.100503
9.96	99.2016	3.15595	9.97998	.100402
9.97	99.4009	3.15753	9.98499	.100301
9.98	99.6004	3.15911	9.98999	.100200
9.99	99.8001	3.16070	9.99500	.100100
10.00	100.000	3.16228	10.0000	.100000
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	$1/n$

TABLE VIII

COMMON LOGARITHMS

FROM

1 TO 10000

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
0	—	**20**	1. 30 103	**40**	1. 60 206	**60**	1. 77 815	**80**	1. 90 309
1	0. 00 000	21	1. 32 222	41	1. 61 278	61	1. 78 533	81	1. 90 849
2	0. 30 103	22	1. 34 242	42	1. 62 325	62	1. 79 239	82	1. 91 381
3	0. 47 712	23	1. 36 173	43	1. 63 347	63	1. 79 934	83	1. 91 908
4	0. 60 206	24	1. 38 021	44	1. 64 345	64	1. 80 618	84	1. 92 428
5	0. 69 897	25	1. 39 794	45	1. 65 321	65	1. 81 291	85	1. 92 942
6	0. 77 815	26	1. 41 497	46	1. 66 276	66	1. 81 954	86	1. 93 450
7	0. 84 510	27	1. 43 136	47	1. 67 210	67	1. 82 607	87	1. 93 952
8	0. 90 309	28	1. 44 716	48	1. 68 124	68	1. 83 251	88	1. 94 448
9	0. 95 424	29	1. 46 240	49	1. 69 020	69	1. 83 885	89	1. 94 939
10	1. 00 000	**30**	1. 47 712	**50**	1. 69 897	**70**	1. 84 510	**90**	1. 95 424
11	1. 04 139	31	1. 49 136	51	1. 70 757	71	1. 85 126	91	1. 95 904
12	1. 07 918	32	1. 50 515	52	1. 71 600	72	1. 85 733	92	1. 96 379
13	1. 11 394	33	1. 51 851	53	1. 72 428	73	1. 86 332	93	1. 99 848
14	1. 14 613	34	1. 53 148	54	1. 73 139	74	1. 86 923	94	1. 97 313
15	1. 17 609	35	1. 54 407	55	1. 74 036	75	1. 87 506	95	1. 97 772
16	1. 20 412	36	1. 55 630	56	1. 74 819	76	1. 88 081	96	1. 98 227
17	1. 23 045	37	1. 56 820	57	1. 75 587	77	1. 88 649	97	1. 98 677
18	1. 25 527	38	1. 57 978	58	1. 76 343	78	1. 89 209	98	1. 99 123
19	1. 27 875	39	1. 59 106	59	1. 77 085	79	1. 89 763	99	1. 99 564
20	1. 30 103	**40**	1. 60 206	**60**	1. 77 815	**80**	1. 90 309	**100**	2. 00 000

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3	47 712	49 136	50 515	51 851	53 148	54 407	55 630	56 820	57 978	59 106
4	60 206	61 278	62 325	63 347	64 345	65 321	66 276	67 210	68 124	69 020
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9	95 424	95 904	96 379	96 848	97 313	97 772	98 227	98 677	99 123	99 564
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11	04 139	04 532	04 922	05 308	05 690	06 070	06 446	06 819	07 188	07 555
12	07 918	08 279	08 636	08 991	09 342	09 691	10 037	10 380	10 721	11 059
13	11 394	11 727	12 057	12 385	12 710	13 033	13 354	13 672	13 988	14 301
14	14 613	14 922	15 229	15 534	15 836	16 137	16 435	16 732	17 026	17 319
15	17 609	17 898	18 184	18 469	18 752	19 033	19 312	19 590	19 866	20 140
16	20 412	20 683	20 952	21 219	21 484	21 748	22 011	22 272	22 531	22 789
17	23 045	23 300	23 553	23 805	24 055	24 304	24 551	24 797	25 042	25 285
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19	27 875	28 103	28 330	28 556	28 780	29 003	29 226	29 447	29 667	29 885
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21	32 222	32 428	32 634	32 838	33 041	33 244	33 445	33 646	33 846	34 044
22	34 242	34 439	34 635	34 830	35 025	35 218	35 411	35 603	35 793	35 984
23	36 173	36 361	36 549	36 736	36 922	37 107	37 291	37 475	37 658	37 840
24	38 021	38 202	38 382	38 561	38 739	38 917	39 094	39 270	39 445	39 620
25	39 794	39 967	40 140	40 312	40 483	40 654	40 824	40 993	41 162	41 330
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28	44 716	44 871	45 025	45 179	45 332	45 484	45 637	45 788	45 939	46 090
29	46 240	46 389	46 538	46 687	46 835	46 982	47 129	47 276	47 422	47 567
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32	50 515	50 651	50 786	50 920	51 055	51 188	51 322	51 455	51 587	51 720
33	51 851	51 983	52 114	52 244	52 375	52 504	52 634	52 763	52 892	53 020
34	53 148	53 275	53 403	53 529	53 656	53 782	53 908	54 033	54 158	54 283
35	54 407	54 531	54 654	54 777	54 900	55 023	55 145	55 267	55 388	55 509
36	55 630	55 751	55 871	55 991	56 110	56 229	56 348	56 467	56 585	56 703
37	56 820	56 937	57 054	57 171	57 287	57 403	57 519	57 634	57 749	57 864
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39	59 106	59 218	59 329	59 439	59 550	59 660	59 770	59 879	59 988	60 097
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42	62 325	62 428	62 531	62 634	62 737	62 839	62 941	63 043	63 144	63 246
43	63 347	63 448	63 548	63 649	63 749	63 849	63 949	64 048	64 147	64 246
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45	65 321	65 418	65 514	65 610	65 706	65 801	65 896	65 992	66 087	66 181
46	66 276	66 370	66 464	66 558	66 652	66 745	66 839	66 932	67 025	67 117
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49	69 020	69 108	69 197	69 285	69 373	69 461	69 548	69 636	69 723	69 810
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53	72 428	72 509	72 591	72 673	72 754	72 835	72 916	72 997	73 078	73 159
54	73 239	73 320	73 400	73 480	73 560	73 640	73 719	73 799	73 878	73 957
55	74 036	74 115	74 194	74 273	74 351	74 429	74 507	74 586	74 663	74 741
56	74 819	74 896	74 974	75 051	75 128	75 205	75 282	75 358	75 435	75 511
57	75 587	75 664	75 740	75 815	75 891	75 967	76 042	76 118	76 193	76 268
58	76 343	76 418	76 492	76 567	76 641	76 716	76 790	76 864	76 938	77 012
59	77 085	77 159	77 232	77 305	77 379	77 452	77 525	77 597	77 670	77 743
60	77 815	77 887	77 960	78 032	78 104	78 176	78 247	78 319	78 390	78 462
61	78 533	78 604	78 675	78 746	78 817	78 888	78 958	79 029	79 099	79 169
62	79 239	79 309	79 379	79 449	79 518	79 588	79 657	79 727	79 796	79 865
63	79 934	80 003	80 072	80 140	80 209	80 277	80 346	80 414	80 482	80 550
64	80 618	80 686	80 754	80 821	80 889	80 956	81 023	81 090	81 158	81 224
65	81 291	81 358	81 425	81 491	81 558	81 624	81 690	81 757	81 823	81 889
66	81 954	82 020	82 086	82 151	82 217	82 282	82 347	82 413	82 478	82 543
67	82 607	82 672	82 737	82 802	82 866	82 930	82 995	83 059	83 123	83 187
68	83 251	83 315	83 378	83 442	83 506	83 569	83 632	83 696	83 759	83 822
69	83 885	83 948	84 011	84 073	84 136	84 198	84 261	84 323	84 386	84 448
70	84 510	84 572	84 634	84 696	84 757	84 819	84 880	84 942	85 003	85 065
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72	85 733	85 794	85 854	85 914	85 974	86 034	86 094	86 153	86 213	86 273
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75	87 506	87 564	87 622	87 679	87 737	87 795	87 852	87 910	87 967	88 024
76	88 081	88 138	88 195	88 252	88 309	88 366	88 423	88 480	88 536	88 593
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78	89 209	89 265	89 321	89 376	89 432	89 487	89 542	89 597	89 653	89 708
79	89 763	89 818	89 873	89 927	89 982	90 037	90 091	90 146	90 200	90 255
80	90 309	90 363	90 417	90 472	90 526	90 580	90 634	90 687	90 741	90 795
81	90 849	90 902	90 956	91 009	91 062	91 116	91 169	91 222	91 275	91 328
82	91 381	91 434	91 487	91 540	91 593	91 645	91 698	91 751	91 803	91 855
83	91 908	91 960	92 012	92 065	92 117	92 169	92 221	92 273	92 324	92 376
84	92 428	92 480	92 531	92 583	92 634	92 686	92 737	92 788	92 840	92 891
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90	95 424	95 472	95 521	95 569	95 617	95 665	95 713	95 761	95 809	95 856
91	95 904	95 952	95 999	96 047	96 095	96 142	96 190	96 237	96 284	96 332
92	96 379	96 426	96 473	96 520	96 567	96 614	96 661	96 708	96 755	96 802
93	96 848	96 895	96 942	96 988	97 035	97 081	97 128	97 174	97 220	97 267
94	97 313	97 359	97 405	97 451	97 497	97 543	97 589	97 635	97 681	97 727
95	97 772	97 818	97 864	97 909	97 955	98 000	98 046	98 091	98 137	98 182
96	98 227	98 272	98 318	98 363	98 408	98 453	98 498	98 543	98 588	98 632
97	98 677	98 722	98 767	98 811	98 856	98 900	98 945	98 989	99 034	99 078
98	99 123	99 167	99 211	99 255	99 300	99 344	99 388	99 432	99 476	99 520
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102		860	903	945	988	*030	*072	*115	*157	*199	*242
103	01	284	326	368	410	452	494	536	578	620	662
104		703	745	787	828	870	912	953	995	*036	*078
105	02	119	160	202	243	284	325	366	407	449	490
106		531	572	612	653	694	735	776	816	857	898
107		938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03	342	383	423	463	503	543	583	623	663	703
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110	04	139	179	218	258	297	336	376	415	454	493
111		532	571	610	650	689	727	766	805	844	883
112		922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05	308	346	385	423	461	500	538	576	614	652
114		690	729	767	805	843	881	918	956	994	*032
115	06	070	108	145	183	221	258	296	333	371	408
116		446	483	521	558	595	633	670	707	744	781
117		819	856	893	930	967	*004	*041	*078	*115	*151
118	07	188	225	262	298	335	372	408	445	482	518
119		555	591	628	664	700	737	773	809	846	882
120		918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08	279	314	350	386	422	458	493	529	565	600
122		636	672	707	743	778	814	849	884	920	955
123		991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09	342	377	412	447	482	517	552	587	621	656
125		691	726	760	795	830	864	899	934	968	*003
126	10	037	072	106	140	175	209	243	278	312	346
127		380	415	449	483	517	551	585	619	653	687
128		721	755	789	823	857	890	924	958	992	*025
129	11	059	093	126	160	193	227	261	294	327	361
130		394	428	461	494	528	561	594	628	661	694
131		727	760	793	826	860	893	926	959	992	*024
132	12	057	090	123	156	189	222	254	287	320	352
133		385	418	450	483	516	548	581	613	646	678
134		710	743	775	808	840	872	905	937	969	*001
135	13	033	066	098	130	162	194	226	258	290	322
136		354	386	418	450	481	513	545	577	609	640
137		672	704	735	767	799	830	862	893	925	956
138		988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14	301	333	364	395	426	457	489	520	551	582
140		613	644	675	706	737	768	799	829	860	891
141		922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15	229	259	290	320	351	381	412	442	473	503
143		534	564	594	625	655	685	715	746	776	806
144		836	866	897	927	957	987	*017	*047	*077	*107
145	16	137	167	197	227	256	286	316	346	376	406
146		435	465	495	524	554	584	613	643	673	702
147		732	761	791	820	850	879	909	938	967	997
148	17	026	056	085	114	143	173	202	231	260	289
149		319	348	377	406	435	464	493	522	551	580
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153		469	498	526	554	583	611	639	667	696	724
154		752	780	808	837	865	893	921	949	977	*005
155	19	033	061	089	117	145	173	201	229	257	285
156		312	340	368	396	424	451	479	507	535	562
157		590	618	645	673	700	728	756	783	811	838
158		866	893	921	948	976	*003	*030	*058	*085	*112
159	20	140	167	194	222	249	276	303	330	358	385
160		412	439	466	493	520	548	575	602	629	656
161		683	710	737	763	790	817	844	871	898	925
162		952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21	219	245	272	299	325	352	378	405	431	458
164		484	511	537	564	590	617	643	669	696	722
165		748	775	801	827	854	880	906	932	958	985
166	22	011	037	063	089	115	141	167	194	220	246
167		272	298	324	350	376	401	427	453	479	505
168		531	557	583	608	634	660	686	712	737	763
169		789	814	840	866	891	917	943	968	994	*019
170	23	045	070	096	121	147	172	198	223	249	274
171		300	325	350	376	401	426	452	477	502	528
172		553	578	603	629	654	679	704	729	754	779
173		805	830	855	880	905	930	955	980	*005	*030
174	24	055	080	105	130	155	180	204	229	254	279
175		304	329	353	378	403	428	452	477	502	527
176		551	576	601	625	650	674	699	724	748	773
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181		768	792	816	840	864	888	912	935	959	983
182	26	007	031	055	079	102	126	150	174	198	221
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184		482	505	529	553	576	600	623	647	670	694
185		717	741	764	788	811	834	858	881	905	928
186		951	975	998	*021	*045	*068	*091	*114	*138	*161
187	27	184	207	231	254	277	300	323	346	370	393
188		416	439	462	485	508	531	554	577	600	623
189		646	669	692	715	738	761	784	807	830	852
190		875	898	921	944	967	989	*012	*035	*058	*081
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193		556	578	601	623	646	668	691	713	735	758
194		780	803	825	847	870	892	914	937	959	981
195	29	003	026	048	070	092	115	137	159	181	203
196		226	248	270	292	314	336	358	380	403	425
197		447	469	491	513	535	557	579	601	623	645
198		667	688	710	732	754	776	798	820	842	863
199		885	907	929	951	973	994	*016	*038	*060	*081
200	30	103	125	146	168	190	211	233	255	276	298
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P. P.

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4	11.6	11.2
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6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	25
1	2.5
2	5.0
3	7.5
4	10.0
5	12.5
6	15.0
7	17.5
8	20.0
9	22.5

	24	23
1	2.4	2.3
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.6	20.7

	22	21
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2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
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7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

N.	L.	0	1	2	3	4	5	6	7	8	9
200	30	103	125	146	168	190	211	233	255	276	298
201		320	341	363	384	406	428	449	471	492	514
202		535	557	578	600	621	643	664	685	707	728
203		750	771	792	814	835	856	878	899	920	942
204		963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31	175	197	218	239	260	281	302	323	345	366
206		387	408	429	450	471	492	513	534	555	576
207		597	618	639	660	681	702	723	744	765	785
208		806	827	848	869	890	911	931	952	973	994
209	32	015	035	056	077	098	118	139	160	181	201
210		222	243	263	284	305	325	346	366	387	408
211		428	449	469	490	510	531	552	572	593	613
212		634	654	675	695	715	736	756	777	797	818
213		838	858	879	899	919	940	960	980	*001	*021
214	33	041	062	082	102	122	143	163	183	203	224
215		244	264	284	304	325	345	365	385	405	425
216		445	465	486	506	526	546	566	586	606	626
217		646	666	686	706	726	746	766	786	806	826
218		846	866	885	905	925	945	965	985	*005	*025
219	34	044	064	084	104	124	143	163	183	203	223
220		242	262	282	301	321	341	361	380	400	420
221		439	459	479	498	518	537	557	577	596	616
222		635	655	674	694	713	733	753	772	792	811
223		830	850	869	889	908	928	947	967	986	*005
224	35	025	044	064	083	102	122	141	160	180	199
225		218	238	257	276	295	315	334	353	372	392
226		411	430	449	468	488	507	526	545	564	583
227		603	622	641	660	679	698	717	736	755	774
228		793	813	832	851	870	889	908	927	946	965
229		984	*003	*021	*040	*059	*078	*097*	116	*135	*154
230	36	173	192	211	229	248	267	286	305	324	342
231		361	380	399	418	436	455	474	493	511	530
232		549	568	586	605	624	642	661	680	698	717
233		736	754	773	791	810	829	847	866	884	903
234		922	940	959	977	996	*014	*033	*051	*070	*088
235	37	107	125	144	162	181	199	218	236	254	273
236		291	310	328	346	365	383	401	420	438	457
237		475	493	511	530	548	566	585	603	621	639
238		658	676	694	712	731	749	767	785	803	822
239		840	858	876	894	912	931	949	967	985	*003
240	38	021	039	057	075	093	112	130	148	166	184
241		202	220	238	256	274	292	310	328	346	364
242		382	399	417	435	453	471	489	507	525	543
243		561	578	596	614	632	650	668	686	703	721
244		739	757	775	792	810	828	846	863	881	899
245		917	934	952	970	987	*005	*023	*041	*058	*076
246	39	094	111	129	146	164	182	199	217	235	252
247		270	287	305	322	340	358	375	393	410	428
248		445	463	480	498	515	533	550	568	585	602
249		620	637	655	672	690	707	724	742	759	777
250		794	811	829	846	863	881	898	915	933	950
N.	L.	0	1	2	3	4	5	6	7	8	9

P. P.

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9	18.0

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	18
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8	14.4
9	16.2

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9	15.3

N.	L.	0	1	2	3	4	5	6	7	8	9
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251		967	985	*002	*019	*037	*054	*071	*088	*106	*123
252	40	140	157	175	192	209	226	243	261	278	295
253		312	329	346	364	381	398	415	432	449	466
254		483	500	518	535	552	569	586	603	620	637
255		654	671	688	705	722	739	756	773	790	807
256		824	841	858	875	892	909	926	943	960	976
257		993	*010	*027	*044	*061	*078	*095	*111	*128	*145
258	41	162	179	196	212	229	246	263	280	296	313
259		330	347	363	380	397	414	430	447	464	481
260		497	514	531	547	564	581	597	614	631	647
261		664	681	697	714	731	747	764	780	797	814
262		830	847	863	880	896	913	929	946	963	979
263		996	*012	*029	*045	*062	*078	*095	*111	*127	*144
264	42	160	177	193	210	226	243	259	275	292	308
265		325	341	357	374	390	406	423	439	455	472
266		488	504	521	537	553	570	586	602	619	635
267		651	667	684	700	716	732	749	765	781	797
268		813	830	846	862	878	894	911	927	943	959
269		975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43	136	152	169	185	201	217	233	249	265	281
271		297	313	329	345	361	377	393	409	425	441
272		457	473	489	505	521	537	553	569	584	600
273		616	632	648	664	680	696	712	727	743	759
274		775	791	807	823	838	854	870	886	902	917
275		933	949	965	981	996	*012	*028	*044	*059	*075
276	44	091	107	122	138	154	170	185	201	217	232
277		248	264	279	295	311	326	342	358	373	389
278		404	420	436	451	467	483	498	514	529	545
279		560	576	592	607	623	638	654	669	685	700
280		716	731	747	762	778	793	809	824	840	855
281		871	886	902	917	932	948	963	979	994	*010
282	45	025	040	056	071	086	102	117	133	148	163
283		179	194	209	225	240	255	271	286	301	317
284		332	347	362	378	393	408	423	439	454	469
285		484	500	515	530	545	561	576	591	606	621
286		637	652	667	682	697	712	728	743	758	773
287		788	803	818	834	849	864	879	894	909	924
288		939	954	969	984	*000	*015	*030	*045	*060	*075
289	46	090	105	120	135	150	165	180	195	210	225
290		240	255	270	285	300	315	330	345	359	374
291		389	404	419	434	449	464	479	494	509	523
292		538	553	568	583	598	613	627	642	657	672
293		687	702	716	731	746	761	776	790	805	820
294		835	850	864	879	894	909	923	938	953	967
295		982	997	*012	*026	*041	*056	*070	*085	*100	*114
296	47	129	144	159	173	188	202	217	232	246	261
297		276	290	305	319	334	349	363	378	392	407
298		422	436	451	465	480	494	509	524	538	553
299		567	582	596	611	625	640	654	669	683	698
300		712	727	741	756	770	784	799	813	828	842

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P. P.	
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P. P.

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
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301		857	871	885	900	914	929	943	958	972	986	
302	48	001	015	029	044	058	073	087	101	116	130	
303		144	159	173	187	202	216	230	244	259	273	
304		287	302	316	330	344	359	373	387	401	416	**15**
												1 1.5
												2 3.0
305		430	444	458	473	487	501	515	530	544	558	3 4.5
306		572	586	601	615	629	643	657	671	686	700	4 6.0
307		714	728	742	756	770	785	799	813	827	841	5 7.5
308		855	869	883	897	911	926	940	954	968	982	6 9.0
309		996	*010	*024	*038	*052	*066	*080	*094	*108	*122	7 10.5
												8 12.0
												9 13.5
310	49	136	150	164	178	192	206	220	234	248	262	
311		276	290	304	318	332	346	360	374	388	402	
312		415	429	443	457	471	485	499	513	527	541	
313		554	568	582	596	610	624	638	651	665	679	
314		693	707	721	734	748	762	776	790	803	817	
315		831	845	859	872	886	900	914	927	941	955	
316		969	982	996	*010	*024	*037	*051	*065	*079	*092	**14**
317	50	106	120	133	147	161	174	188	202	215	229	1 1.4
318		243	256	270	284	297	311	325	338	352	365	2 2.8
319		379	393	406	420	433	447	461	474	488	501	3 4.2
												4 5.6
												5 7.0
320		515	529	542	556	569	583	596	610	623	637	6 8.4
321		651	664	678	691	705	718	732	745	759	772	7 9.8
322		786	799	813	826	840	853	866	880	893	907	8 11.2
323		920	934	947	961	974	987	*001	*014	*028	*041	9 12.6
324	51	055	068	081	095	108	121	135	148	162	175	
325		188	202	215	228	242	255	268	282	295	308	
326		322	335	348	362	375	388	402	415	428	441	
327		455	468	481	495	508	521	534	548	561	574	
328		587	601	614	627	640	654	667	680	693	706	
329		720	733	746	759	772	786	799	812	825	838	
												13
												1 1.3
330		851	865	878	891	904	917	930	943	957	970	2 2.6
331		983	996	*009	*022	*035	*048	*061	*075	*088	*101	3 3.9
332	52	114	127	140	153	166	179	192	205	218	231	4 5.2
333		244	257	270	284	297	310	323	336	349	362	5 6.5
334		375	388	401	414	427	440	453	466	479	492	6 7.8
												7 9.1
												8 10.4
												9 11.7
335		504	517	530	543	556	569	582	595	608	621	
336		634	647	660	673	686	699	711	724	737	750	
337		763	776	789	802	815	827	840	853	866	879	
338		892	905	917	930	943	956	969	982	994	*007	
339	53	020	033	046	058	071	084	097	110	122	135	
340		148	161	173	186	199	212	224	237	250	263	
341		275	288	301	314	326	339	352	364	377	390	**12**
342		403	415	428	441	453	466	479	491	504	517	1 1.2
343		529	542	555	567	580	593	605	618	631	643	2 2.4
344		656	668	681	694	706	719	732	744	757	769	3 3.6
												4 4.8
345		782	794	807	820	832	845	857	870	882	895	5 6.0
346		908	920	933	945	958	970	983	995	*008	*020	6 7.2
347	54	033	045	058	070	083	095	108	120	133	145	7 8.4
348		158	170	183	195	208	220	233	245	258	270	8 9.6
349		283	295	307	320	332	345	357	370	382	394	9 10.8
350		407	419	432	444	456	469	481	494	506	518	
N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
350	54	407	419	432	444	456	469	481	494	506	518	
351		531	543	555	568	580	593	605	617	630	642	
352		654	667	679	691	704	716	728	741	753	765	
353		777	790	802	814	827	839	851	864	876	888	
354		900	913	925	937	949	962	974	986	998	*011	**13**
355	55	023	035	047	060	072	084	096	108	121	133	1 1.3
356		145	157	169	182	194	206	218	230	242	255	2 2.6
357		267	279	291	303	315	328	340	352	364	376	3 3.9
358		388	400	413	425	437	449	461	473	485	497	4 5.2
359		509	522	534	546	558	570	582	594	606	618	5 6.5
												6 7.8
												7 9.1
												8 10.4
												9 11.7
360		630	642	654	666	678	691	703	715	727	739	
361		751	763	775	787	799	811	823	835	847	859	
362		871	883	895	907	919	931	943	955	967	979	
363		991	*003	*015	*027	*038	*050	*062	*074	*086	*098	
364	56	110	122	134	146	158	170	182	194	205	217	
365		229	241	253	265	277	289	301	312	324	336	
366		348	360	372	384	396	407	419	431	443	455	**12**
367		467	478	490	502	514	526	538	549	561	573	1 1.2
368		585	597	608	620	632	644	656	667	679	691	2 2.4
369		703	714	726	738	750	761	773	785	797	808	3 3.6
												4 4.8
370		820	832	844	855	867	879	891	902	914	926	5 6.0
371		937	949	961	972	984	996	*008	*019	*031	*043	6 7.2
372	57	054	066	078	089	101	113	124	136	148	159	7 8.4
373		171	183	194	206	217	229	241	252	264	276	8 9.6
374		287	299	310	322	334	345	357	368	380	392	9 10.8
375		403	415	426	438	449	461	473	484	496	507	
376		519	530	542	553	565	576	588	600	611	623	
377		634	646	657	669	680	692	703	715	726	738	
378		749	761	772	784	795	807	818	830	841	852	
379		864	875	887	898	910	921	933	944	955	967	**11**
380		978	990	*001	*013	*024	*035	*047	*058	*070	*081	1 1.1
381	58	092	104	115	127	138	149	161	172	184	195	2 2.2
382		206	218	229	240	252	263	274	286	297	309	3 3.3
383		320	331	343	354	365	377	388	399	410	422	4 4.4
384		433	444	456	467	478	490	501	512	524	535	5 5.5
												6 6.6
												7 7.7
												8 8.8
												9 9.9
385		546	557	569	580	591	602	614	625	636	647	
386		659	670	681	692	704	715	726	737	749	760	
387		771	782	794	805	816	827	838	850	861	872	
388		883	894	906	917	928	939	950	961	973	984	
389		995	*006	*017	*028	*040	*051	*062	*073	*084	*095	
390	59	106	118	129	140	151	162	173	184	195	207	
391		218	229	240	251	262	273	284	295	306	318	**10**
392		329	340	351	362	373	384	395	406	417	428	1 1.0
393		439	450	461	472	483	494	506	517	528	539	2 2.0
394		550	561	572	583	594	605	616	627	638	649	3 3.0
												4 4.0
												5 5.0
395		660	671	682	693	704	715	726	737	748	759	6 6.0
396		770	780	791	802	813	824	835	846	857	868	7 7.0
397		879	890	901	912	923	934	945	956	966	977	8 8.0
398		988	999	*010	*021	*032	*043	*054	*065	*076	*086	9 9.0
399	60	097	108	119	130	141	152	163	173	184	195	
400		206	217	228	239	249	260	271	282	293	304	
N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.

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400	60	206	217	228	239	249	260	271	282	293	304	
401		314	325	336	347	358	369	379	390	401	412	
402		423	433	444	455	466	477	487	498	509	520	
403		531	541	552	563	574	584	595	606	617	627	
404		638	649	660	670	681	692	703	713	724	735	
405		746	756	767	778	788	799	810	821	831	842	
406		853	863	874	885	895	906	917	927	938	949	
407		959	970	981	991	*002	*013	*023	*034	*045	*055	
408	61	066	077	087	098	109	119	130	140	151	162	**11**
409		172	183	194	204	215	225	236	247	257	268	1 1.1
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410		278	289	300	310	321	331	342	352	363	374	3 3.3
411		384	395	405	416	426	437	448	458	469	479	4 4.4
412		490	500	511	521	532	542	553	563	574	584	5 5.5
413		595	606	616	627	637	648	658	669	679	690	6 6.6
414		700	711	721	731	742	752	763	773	784	794	7 7.7
												8 8.8
415		805	815	826	836	847	857	868	878	888	899	9 9.9
416		909	920	930	941	951	962	972	982	993	*003	
417	62	014	024	034	045	055	066	076	086	097	107	
418		118	128	138	149	159	170	180	190	201	211	
419		221	232	242	252	263	273	284	294	304	315	
420		325	335	346	356	366	377	387	397	408	418	
421		428	439	449	459	469	480	490	500	511	521	
422		531	542	552	562	572	583	593	603	613	624	
423		634	644	655	665	675	685	696	706	716	726	**10**
424		737	747	757	767	778	788	798	808	818	829	1 1.0
												2 2.0
425		839	849	859	870	880	890	900	910	921	931	3 3.0
426		941	951	961	972	982	992	*002	*012	*022	*033	4 4.0
427	63	043	053	063	073	083	094	104	114	124	134	5 5.0
428		144	155	165	175	185	195	205	215	225	236	6 6.0
429		246	256	266	276	286	296	306	317	327	337	7 7.0
												8 8.0
430		347	357	367	377	387	397	407	417	428	438	9 9.0
431		448	458	468	478	488	498	508	518	528	538	
432		548	558	568	579	589	599	609	619	629	639	
433		649	659	669	679	689	699	709	719	729	739	
434		749	759	769	779	789	799	809	819	829	839	
435		849	859	869	879	889	899	909	919	929	939	
436		949	959	969	979	988	998	*008	*018	*028	*038	
437	64	048	058	068	078	088	098	108	118	128	137	**9**
438		147	157	167	177	187	197	207	217	227	237	1 0.9
439		246	256	266	276	286	296	306	316	326	335	2 1.8
												3 2.7
440		345	355	365	375	385	395	404	414	424	434	4 3.6
441		444	454	464	473	483	493	503	513	523	532	5 4.5
442		542	552	562	572	582	591	601	611	621	631	6 5.4
443		640	650	660	670	680	689	699	709	719	729	7 6.3
444		738	748	758	768	777	787	797	807	816	826	8 7.2
												9 8.1
445		836	846	856	865	875	885	895	904	914	924	
446		933	943	953	963	972	982	992	*002	*011	*021	
447	65	031	040	050	060	070	079	089	099	108	118	
448		128	137	147	157	167	176	186	196	205	215	
449		225	234	244	254	263	273	283	292	302	312	
450		321	331	341	350	360	369	379	389	398	408	
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N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
450	65	321	331	341	350	360	369	379	389	398	408	
451		418	427	437	447	456	466	475	485	495	504	
452		514	523	533	543	552	562	571	581	591	600	
453		610	619	629	639	648	658	667	677	686	696	
454		706	715	725	734	744	753	763	772	782	792	
455		801	811	820	830	839	849	858	868	877	887	
456		896	906	916	925	935	944	954	963	973	982	
457		992	*001	*011	*020	*030	*039	*049	*058	*068	*077	
458	66	087	096	106	115	124	134	143	153	162	172	**10**
459		181	191	200	210	219	229	238	247	257	266	1 1.0
460		276	285	295	304	314	323	332	342	351	361	2 2.0 3 3.0
461		370	380	389	398	408	417	427	436	445	455	4 4.0
462		464	474	483	492	502	511	521	530	539	549	5 5.0
463		558	567	577	586	596	605	614	624	633	642	6 6.0
464		652	661	671	680	689	699	708	717	727	736	7 7.0 8 8.0
465		745	755	764	773	783	792	801	811	820	829	9 9.0
466		839	848	857	867	876	885	894	904	913	922	
467		932	941	950	960	969	978	987	997	*006	*015	
468	67	025	034	043	052	062	071	080	089	099	108	
469		117	127	136	145	154	164	173	182	191	201	
470		210	219	228	237	247	256	265	274	284	293	
471		302	311	321	330	339	348	357	367	376	385	
472		394	403	413	422	431	440	449	459	468	477	
473		486	495	504	514	523	532	541	550	560	569	**9**
474		578	587	596	605	614	624	633	642	651	660	1 0.9
475		669	679	688	697	706	715	724	733	742	752	2 1.8 3 2.7
476		761	770	779	788	797	806	815	825	834	843	4 3.6
477		852	861	870	879	888	897	906	916	925	934	5 4.5
478		943	952	961	970	979	988	997	*006	*015	*024	6 5.4 7 6.3
479	68	034	043	052	061	070	079	088	097	106	115	8 7.2 9 8.1
480		124	133	142	151	160	169	178	187	196	205	
481		215	224	233	242	251	260	269	278	287	296	
482		305	314	323	332	341	350	359	368	377	386	
483		395	404	413	422	431	440	449	458	467	476	
484		485	494	502	511	520	529	538	547	556	565	
485		574	583	592	601	610	619	628	637	646	655	
486		664	673	681	690	699	708	717	726	735	744	
487		753	762	771	780	789	797	806	815	824	833	**8**
488		842	851	860	869	878	886	895	904	913	922	1 0.8
489		931	940	949	958	966	975	984	993	*002	*011	2 1.6 3 2.4
490	69	020	028	037	046	055	064	073	082	090	099	4 3.2 5 4.0
491		108	117	126	135	144	152	161	170	179	188	6 4.8
492		197	205	214	223	232	241	249	258	267	276	7 5.6
493		285	294	302	311	320	329	338	346	355	364	8 6.4 9 7.2
494		373	381	390	399	408	417	425	434	443	452	
495		461	469	478	487	496	504	513	522	531	539	
496		548	557	556	574	583	592	601	609	618	627	
497		636	644	653	662	671	679	688	697	705	714	
498		723	732	740	749	758	767	775	784	793	801	
499		810	819	827	836	845	854	862	871	880	888	
500		897	906	914	923	932	940	949	958	966	975	
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N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
500	69	897	906	914	923	932	940	949	958	966	975	
501		984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70	070	079	088	096	105	114	122	131	140	148	
503		157	165	174	183	191	200	209	217	226	234	
504		243	252	260	269	278	286	295	303	312	321	
505		329	338	346	355	364	372	381	389	398	406	
506		415	424	432	441	449	458	467	475	484	492	
507		501	509	518	526	535	544	552	561	569	578	
508		586	595	603	612	621	629	638	646	655	663	
509		672	680	689	697	706	714	723	731	740	749	**9**
510		757	766	774	783	791	800	808	817	825	834	1 0.9 2 1.8
511		842	851	859	868	876	885	893	902	910	919	3 2.7
512		927	935	944	952	961	969	978	986	995	*003	4 3.6
513	71	012	020	029	037	046	054	063	071	079	088	5 4.5
514		096	105	113	122	130	139	147	155	164	172	6 5.4 7 6.3
515		181	189	198	206	214	223	231	240	248	257	8 7.2 9 8.1
516		265	273	282	290	299	307	315	324	332	341	
517		349	357	366	374	383	391	399	408	416	425	
518		433	441	450	458	466	475	483	492	500	508	
519		517	525	533	542	550	559	567	575	584	592	
520		600	609	617	625	634	642	650	659	667	675	
521		684	692	700	709	717	725	734	742	750	759	
522		767	775	784	792	800	809	817	825	834	842	
523		850	858	867	875	883	892	900	908	917	925	**8**
524		933	941	950	958	966	975	983	991	999	*008	1 0.8 2 1.6
525	72	016	024	032	041	049	057	066	074	082	090	3 2.4
526		099	107	115	123	132	140	148	156	165	173	4 3.2 5 4.0
527		181	189	198	206	214	222	230	239	247	255	6 4.8
528		263	272	280	288	296	304	313	321	329	337	7 5.6
529		346	354	362	370	378	387	395	403	411	419	8 6.4 9 7.2
530		428	436	444	452	460	469	477	485	493	501	
531		509	518	526	534	542	550	558	567	575	583	
532		591	599	607	616	624	632	640	648	656	665	
533		673	681	689	697	705	713	722	730	738	746	
534		754	762	770	779	787	795	803	811	819	827	
535		835	843	852	860	868	876	884	892	900	908	
536		916	925	933	941	949	957	965	973	981	989	
537		997	*006	*014	*022	*030	*038	*046	*054	*062	*070	**7**
538	73	078	086	094	102	111	119	127	135	143	151	1 0.7 2 1.4
539		159	167	175	183	191	199	207	215	223	231	3 2.1
540		239	247	255	263	272	280	288	296	304	312	4 2.8 5 3.5
541		320	328	336	344	352	360	368	376	384	392	6 4.2
542		400	408	416	424	432	440	448	456	464	472	7 4.9 8 5.6
543		480	488	496	504	512	520	528	536	544	552	9 6.3
544		560	568	576	584	592	600	608	616	624	632	
545		640	648	656	664	672	679	687	695	703	711	
546		719	727	735	743	751	759	767	775	783	791	
547		799	807	815	823	830	838	846	854	862	870	
548		878	886	894	902	910	918	926	933	941	949	
549		957	965	973	981	989	997	*005	*013	*020	*028	
550	74	036	044	052	060	068	076	084	092	099	107	
N.	L.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	P. P.

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
550	74	036	044	052	060	068	076	084	092	099	107	
551		115	123	131	139	147	155	162	170	178	186	
552		194	202	210	218	225	233	241	249	257	265	
553		273	280	288	296	304	312	320	327	335	343	
554		351	359	367	374	382	390	398	406	414	421	
555		429	437	445	453	461	468	476	484	492	500	
556		507	515	523	531	539	547	554	562	570	578	
557		586	593	601	609	617	624	632	640	648	656	
558		663	671	679	687	695	702	710	718	726	733	
559		741	749	757	764	772	780	788	796	803	811	
560		819	827	834	842	850	858	865	873	881	889	
561		896	904	912	920	927	935	943	950	958	966	
562		974	981	989	997	*005	*012	*020	*028	*035	*043	
563	75	051	059	066	074	082	089	097	105	113	120	8
564		128	136	143	151	159	166	174	182	189	197	1 0.8
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565		205	213	220	228	236	243	251	259	266	274	3 2.4
566		282	289	297	305	312	320	328	335	343	351	4 3.2
567		358	366	374	381	389	397	404	412	420	427	5 4.0
568		435	442	450	458	465	473	481	488	496	504	6 4.8
569		511	519	526	534	542	549	557	565	572	580	7 5.6
												8 6.4
												9 7.2
570		587	595	603	610	618	626	633	641	648	656	
571		664	671	679	686	694	702	709	717	724	732	
572		740	747	755	762	770	778	785	793	800	808	
573		815	823	831	838	846	853	861	868	876	884	
574		891	899	906	914	921	929	937	944	952	959	
575		967	974	982	989	997	*005	*012	*020	*027	*035	
576	76	042	050	057	065	072	080	087	095	103	110	
577		118	125	133	140	148	155	163	170	178	185	
578		193	200	208	215	223	230	238	245	253	260	
579		268	275	283	290	298	305	313	320	328	335	
580		343	350	358	365	373	380	388	395	403	410	
581		418	425	433	440	448	455	462	470	477	485	
582		492	500	507	515	522	530	537	545	552	559	
583		567	574	582	589	597	604	612	619	626	634	7
584		641	649	656	664	671	678	686	693	701	708	1 0.7
												2 1.4
585		716	723	730	738	745	753	760	768	775	782	3 2.1
586		790	797	805	812	819	827	834	842	849	856	4 2.8
587		864	871	879	886	893	901	908	916	923	930	5 3.5
588		938	945	953	960	867	975	982	989	997	*004	6 4.2
589	77	012	019	026	034	041	048	056	063	070	078	7 4.9
												8 5.6
												9 6.3
590		085	093	100	107	115	122	129	137	144	151	
591		159	166	173	181	188	195	203	210	217	225	
592		232	240	247	254	262	269	276	283	291	298	
593		305	313	320	327	335	342	349	357	364	371	
594		379	386	393	401	408	415	422	430	437	444	
595		452	459	466	474	481	488	495	503	510	517	
596		525	532	539	546	554	561	568	576	583	590	
597		597	605	612	619	627	634	641	648	656	663	
598		670	677	685	692	699	706	714	721	728	735	
599		743	750	757	764	772	779	786	793	801	808	
600		815	822	830	837	844	851	859	866	873	880	
N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
600	77	815	822	830	837	844	851	859	866	873	880	
601		887	895	902	909	916	924	931	938	945	952	
602		960	967	974	981	988	996	*003	*010	*017	*025	
603	78	032	039	046	053	061	068	075	082	089	097	
604		104	111	118	125	132	140	147	154	161	168	
605		176	183	190	197	204	211	219	226	233	240	
606		247	254	262	269	276	283	290	297	305	312	
607		319	326	333	340	347	355	362	369	376	383	
608		390	398	405	412	419	426	433	440	447	455	**8**
609		462	469	476	483	490	497	504	512	519	526	1 0.8
610		533	540	547	554	561	569	576	583	590	597	2 1.6 3 2.4 4 3.2
611		604	611	618	625	633	640	647	654	661	668	5 4.0
612		675	682	689	696	704	711	718	725	732	739	6 4.8
613		746	753	760	767	774	781	789	796	803	810	7 5.6
614		817	824	831	838	845	852	859	866	873	880	8 6.4 9 7.2
615		888	895	902	909	916	923	930	937	944	951	
616		958	965	972	979	986	993	*000	*007	*014	*021	
617	79	029	036	043	050	057	064	071	078	085	092	
618		099	106	113	120	127	134	141	148	155	162	
619		169	176	183	190	197	204	211	218	225	232	
620		239	246	253	260	267	274	281	288	295	302	
621		309	316	323	330	337	344	351	358	365	372	
622		379	386	393	400	407	414	421	428	435	442	**7**
623		449	456	463	470	477	484	491	498	505	511	1 0.7
624		518	525	532	539	546	553	560	567	574	581	2 1.4 3 2.1 4 2.8
625		588	595	602	609	616	623	630	637	644	650	5 3.5
626		657	664	671	678	685	692	699	706	713	720	6 4.2
627		727	734	741	748	754	761	768	775	782	789	7 4.9
628		796	803	810	817	824	831	837	844	851	858	8 5.6
629		865	872	879	886	893	900	906	913	920	927	9 6.3
630		934	941	948	955	962	969	975	982	989	996	
631	80	003	010	017	024	030	037	044	051	058	065	
632		072	079	085	092	099	106	113	120	127	134	
633		140	147	154	161	168	175	182	188	195	202	
634		209	216	223	229	236	243	250	257	264	271	
635		277	284	291	298	305	312	318	325	332	339	
636		346	353	359	366	373	380	387	393	400	407	
637		414	421	428	434	441	448	455	462	468	475	
638		482	489	496	502	509	516	523	530	536	543	**6**
639		550	557	564	570	577	584	591	598	604	611	1 0.6
640		618	625	632	638	645	652	659	665	672	679	2 1.2 3 1.8 4 2.4
641		686	693	699	706	713	720	726	733	740	747	5 3.0
642		754	760	767	774	781	787	794	801	808	814	6 3.6
643		821	828	835	841	848	855	862	868	875	882	7 4.2
644		889	895	902	909	916	922	929	936	943	949	8 4.8 9 5.4
645		956	963	969	976	983	990	996	*003	*010	*017	
646	81	023	030	037	043	050	057	064	070	077	084	
647		090	097	104	111	117	124	131	137	144	151	
648		158	164	171	178	184	191	198	204	211	218	
649		224	231	238	245	251	258	265	271	278	285	
650		291	298	305	311	318	325	331	338	345	351	
N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
650	81	291	298	305	311	318	325	331	338	345	351	
651		358	365	371	378	385	391	398	405	411	418	
652		425	431	438	445	451	458	465	471	478	485	
653		491	498	505	511	518	525	531	538	544	551	
654		558	564	571	578	584	591	598	604	611	617	
655		624	631	637	644	651	657	664	671	677	684	
656		690	697	704	710	717	723	730	737	743	750	
657		757	763	770	776	783	790	796	803	809	816	
658		823	829	836	842	849	856	862	869	875	882	
659		889	895	902	908	915	921	928	935	941	948	
660		954	961	968	974	981	987	994	*000	*007	*014	
661	82	020	027	033	040	046	053	060	066	073	079	
662		086	092	099	105	112	119	125	132	138	145	
663		151	158	164	171	178	184	191	197	204	210	**7**
664		217	223	230	236	243	249	256	263	269	276	1 0.7
												2 1.4
665		282	289	295	302	308	315	321	328	334	341	3 2.1
666		347	354	360	367	373	380	387	393	400	406	4 2.8
667		413	419	426	432	439	445	452	458	465	471	5 3.5
668		478	484	491	497	504	510	517	523	530	536	6 4.2
669		543	549	556	562	569	575	582	588	595	601	7 4.9
												8 5.6
670		607	614	620	627	633	640	646	653	659	666	9 6.3
671		672	679	685	692	698	705	711	718	724	730	
672		737	743	750	756	763	769	776	782	789	795	
673		802	808	814	821	827	834	840	847	853	860	
674		866	872	879	885	892	898	905	911	918	924	
675		930	937	943	950	956	963	969	975	982	988	
676		995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83	059	065	072	078	085	091	097	104	110	117	
678		123	129	136	142	149	155	161	168	174	181	
679		187	193	200	206	213	219	225	232	238	245	
680		251	257	264	270	276	283	289	296	302	308	
681		315	321	327	334	340	347	353	359	366	372	
682		378	385	391	398	404	410	417	423	429	436	**6**
683		442	448	455	461	467	474	480	487	493	499	1 0.6
684		506	512	518	525	531	537	544	550	556	563	2 1.2
												3 1.8
685		569	575	582	588	594	601	607	613	620	626	4 2.4
686		632	639	645	651	658	664	670	677	683	689	5 3.0
687		696	702	708	715	721	727	734	740	746	753	6 3.6
688		759	765	771	778	784	790	797	803	809	816	7 4.2
689		822	828	835	841	847	853	860	866	872	879	8 4.8
												9 5.4
690		885	891	897	904	910	916	923	929	935	942	
691		948	954	960	967	973	979	985	992	998	*004	
692	84	011	017	023	029	036	042	048	055	061	067	
693		073	080	086	092	098	105	111	117	123	130	
694		136	142	148	155	161	167	173	180	186	192	
695		198	205	211	217	223	230	236	242	248	255	
696		261	267	273	280	286	292	298	305	311	317	
697		323	330	336	342	348	354	361	367	373	379	
698		386	392	398	404	410	417	423	429	435	442	
699		448	454	460	466	473	479	485	491	497	504	
700		510	516	522	528	535	541	547	553	559	566	
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N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
700	84	510	516	522	528	535	541	547	553	559	566	
701		572	578	584	590	597	603	609	615	621	628	
702		634	640	646	652	658	665	671	677	683	689	
703		696	702	708	714	720	726	733	739	745	751	
704		757	763	770	776	782	788	794	800	807	813	
705		819	825	831	837	844	850	856	862	868	874	
706		880	887	893	899	905	911	917	924	930	936	
707		942	948	954	960	967	973	979	985	991	997	
708	85	003	009	016	022	028	034	040	046	052	058	**7**
709		065	071	077	083	089	095	101	107	114	120	1 0.7 2 1.4 3 2.1
710		126	132	138	144	150	156	163	169	175	181	4 2.8
711		187	193	199	205	211	217	224	230	236	242	5 3.5
712		248	254	260	266	272	278	285	291	297	303	6 4.2
713		309	315	321	327	333	339	345	352	358	364	7 4.9 8 5.6
714		370	376	382	388	394	400	406	412	418	425	9 6.3
715		431	437	443	449	455	461	467	473	479	485	
716		491	497	503	509	516	522	528	534	540	546	
717		552	558	564	570	576	582	588	594	600	606	
718		612	618	625	631	637	643	649	655	661	667	
719		673	679	685	691	697	703	709	715	721	727	
720		733	739	745	751	757	763	769	775	781	788	
721		794	800	806	812	818	824	830	836	842	848	
722		854	860	866	872	878	884	890	896	902	908	
723		914	920	926	932	938	944	950	956	962	968	**6**
724		974	980	986	992	998	*004	*010	*016	*022	*028	1 0.6 2 1.2 3 1.8
725	86	034	040	046	052	058	064	070	076	082	088	4 2.4 5 3.0
726		094	100	106	112	118	124	130	136	141	147	6 3.6
727		153	159	165	171	177	183	189	195	201	207	7 4.2
728		213	219	225	231	237	243	249	255	261	267	8 4.8
729		273	279	285	291	297	303	308	314	320	326	9 5.4
730		332	338	344	350	356	362	368	374	380	386	
731		392	398	404	410	415	421	427	433	439	445	
732		451	457	463	469	475	481	487	493	499	504	
733		510	516	522	528	534	540	546	552	558	564	
734		570	576	581	587	593	599	605	611	617	623	
735		629	635	641	646	652	658	664	670	676	682	
736		688	694	700	705	711	717	723	729	735	741	
737		747	753	759	764	770	776	782	788	794	800	**5**
738		806	812	817	823	829	835	841	847	853	859	1 0.5 2 1.0
739		864	870	876	882	888	894	900	906	911	917	3 1.5
740		923	929	935	941	947	953	958	964	970	976	4 2.0 5 2.5
741		982	988	994	999	*005	*011	*017	*023	*029	*035	6 3.0
742	87	040	046	052	058	064	070	075	081	087	093	7 3.5 8 4.0
743		099	105	111	116	122	128	134	140	146	151	9 4.5
744		157	163	169	175	181	186	192	198	204	210	
745		216	221	227	233	239	245	251	256	262	268	
746		274	280	286	291	297	303	309	315	320	326	
747		332	338	344	349	355	361	367	373	379	384	
748		390	396	402	408	413	419	425	431	437	442	
749		448	454	460	466	471	477	483	489	495	500	
750		506	512	518	523	529	535	541	547	552	558	
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750	87	506	512	518	523	529	535	541	547	552	558	
751		564	570	576	581	587	593	599	604	610	616	
752		622	628	633	639	645	651	656	662	668	674	
753		679	685	691	697	703	708	714	720	726	731	
754		737	743	749	754	760	766	772	777	783	789	
755		795	800	806	812	818	823	829	835	841	846	
756		852	858	864	869	875	881	887	892	898	904	
757		910	915	921	927	933	938	944	950	955	961	
758		967	973	978	984	990	996	*001	*007	*013	*018	
759	88	024	030	036	041	047	053	058	064	070	076	
760		081	087	093	098	104	110	116	121	127	133	
761		138	144	150	156	161	167	173	178	184	190	
762		195	201	207	213	218	224	230	235	241	247	
763		252	258	264	270	275	281	287	292	298	304	**6**
764		309	315	321	326	332	338	343	349	355	360	1 0.6
765		366	372	377	383	389	395	400	406	412	417	2 1.2 3 1.8 4 2.4
766		423	429	434	440	446	451	457	463	468	474	5 3.0
767		480	485	491	497	502	508	513	519	525	530	6 3.6
768		536	542	547	553	559	564	570	576	581	587	7 4.2
769		593	598	604	610	615	621	627	632	638	643	8 4.8 9 5.4
770		649	655	660	666	672	677	683	689	694	700	
771		705	711	717	722	728	734	739	745	750	756	
772		762	767	773	779	784	790	795	801	807	812	
773		818	824	829	835	840	846	852	857	863	868	
774		874	880	885	891	897	902	908	913	919	925	
775		930	936	941	947	953	958	964	969	975	981	
776		986	992	997	*003	*009	*014	*020	*025	*031	*037	
777	89	042	048	053	059	064	070	076	081	087	092	
778		098	104	109	115	120	126	131	137	143	148	
779		154	159	165	170	176	182	187	193	198	204	
780		209	215	221	226	232	237	243	248	254	260	
781		265	271	276	282	287	293	298	304	310	315	
782		321	326	332	337	343	348	354	360	365	371	**5**
783		376	382	387	393	398	404	409	415	421	426	1 0.5
784		432	437	443	448	454	459	465	470	476	481	2 1.0 3 1.5 4 2.0
785		487	492	498	504	509	515	520	526	531	537	5 2.5
786		542	548	553	559	564	570	575	581	586	592	6 3.0
787		597	603	609	614	620	625	631	636	642	647	7 3.5
788		653	658	664	669	675	680	686	691	697	702	8 4.0
789		708	713	719	724	730	735	741	746	752	757	9 4.5
790		763	768	774	779	785	790	796	801	807	812	
791		818	823	829	834	840	845	851	856	862	867	
792		873	878	883	889	894	900	905	911	916	922	
793		927	933	938	944	949	955	960	966	971	977	
794		982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90	037	042	048	053	059	064	069	075	080	086	
796		091	097	102	108	113	119	124	129	135	140	
797		146	151	157	162	168	173	179	184	189	195	
798		200	206	211	217	222	227	233	238	244	249	
799		255	260	266	271	276	282	287	293	298	304	
800		309	314	320	325	331	336	342	347	352	358	
N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
800	90	309	314	320	325	331	336	342	347	352	358	
801		363	369	374	380	385	390	396	401	407	412	
802		417	423	428	434	439	445	450	455	461	466	
803		472	477	482	488	493	499	504	509	515	520	
804		526	531	536	542	547	553	558	563	569	574	
805		580	585	590	596	601	607	612	617	623	628	
806		634	639	644	650	655	660	666	671	677	682	
807		687	693	698	703	709	714	720	725	730	736	
808		741	747	752	757	763	768	773	779	784	789	
809		795	800	806	811	816	822	827	832	838	843	
810		849	854	859	865	870	875	881	886	891	897	
811		902	907	913	918	924	929	934	940	945	950	
812		956	961	966	972	977	982	988	993	998	*004	
813	91	009	014	020	025	030	036	041	046	052	057	6
814		062	068	073	078	084	089	094	100	105	110	1 0.6
815		116	121	126	132	137	142	148	153	158	164	2 1.2 3 1.8 4 2.4
816		169	174	180	185	190	196	201	206	212	217	5 3.0
817		222	228	233	238	243	249	254	259	265	270	6 3.6
818		275	281	286	291	297	302	307	312	318	323	7 4.2
819		328	334	339	344	350	355	360	365	371	376	8 4.8 9 5.4
820		381	387	392	397	403	408	413	418	424	429	
821		434	440	445	450	455	461	466	471	477	482	
822		487	492	498	503	508	514	519	524	529	535	
823		540	545	551	556	561	566	572	577	582	587	
824		593	598	603	609	614	619	624	630	635	640	
825		645	651	656	661	666	672	677	682	687	693	
826		698	703	709	714	719	724	730	735	740	745	
827		751	756	761	766	772	777	782	787	793	798	
828		803	808	814	819	824	829	834	840	845	850	
829		855	861	866	871	876	882	887	892	897	903	
830		908	913	918	924	929	934	939	944	950	955	
831		960	965	971	976	981	986	991	997	*002	*007	
832	92	012	018	023	028	033	038	044	049	054	059	5
833		065	070	075	080	085	091	096	101	106	111	1 0.5
834		117	122	127	132	137	143	148	153	158	163	2 1.0 3 1.5 4 2.0
835		169	174	179	184	189	195	200	205	210	215	5 2.5
836		221	226	231	236	241	247	252	257	262	267	6 3.0
837		273	278	283	288	293	298	304	309	314	319	7 3.5
838		324	330	335	340	345	350	355	361	366	371	8 4.0 9 4.5
839		376	381	387	392	397	402	407	412	418	423	
840		428	433	438	443	449	454	459	464	469	474	
841		480	485	490	495	500	505	511	516	521	526	
842		531	536	542	547	552	557	562	567	572	578	
843		583	588	593	598	603	609	614	619	624	629	
844		634	639	645	650	655	660	665	670	675	681	
845		686	691	696	701	706	711	716	722	727	732	
846		737	742	747	752	758	763	768	773	778	783	
847		788	793	799	804	809	814	819	824	829	834	
848		840	845	850	855	860	865	870	875	881	886	
849		891	896	901	906	911	916	921	927	932	937	
850		942	947	952	957	962	967	973	978	983	988	
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N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
850	92	942	947	952	957	962	967	973	978	983	988	
851		993	998	*003	*008	*013	*018	*024	*029	*034	*039	
852	93	044	049	054	059	064	069	075	080	085	090	
853		095	100	105	110	115	120	125	131	136	141	
854		146	151	156	161	166	171	176	181	186	192	
855		197	202	207	212	217	222	227	232	237	242	
856		247	252	258	263	268	273	278	283	288	293	
857		298	303	308	313	318	323	328	334	339	344	
858		349	354	359	364	369	374	379	384	389	394	**6**
859		399	404	409	414	420	425	430	435	440	445	1 0.6
												2 1.2
860		450	455	460	465	470	475	480	485	490	495	3 1.8
861		500	505	510	515	520	526	531	536	541	546	4 2.4
862		551	556	561	566	571	576	581	586	591	596	5 3.0
863		601	606	611	616	621	626	631	636	641	646	6 3.6
864		651	656	661	666	671	676	682	687	692	697	7 4.2
												8 4.8
865		702	707	712	717	722	727	732	737	742	747	9 5.4
866		752	757	762	767	772	777	782	787	792	797	
867		802	807	812	817	822	827	832	837	842	847	
868		852	857	862	867	872	877	882	887	892	897	
869		902	907	912	917	922	927	932	937	942	947	
870		952	957	962	967	972	977	982	987	992	997	
871	94	002	007	012	017	022	027	032	037	042	047	
872		052	057	062	067	072	077	082	086	091	096	**5**
873		101	106	111	116	121	126	131	136	141	146	1 0.5
874		151	156	161	166	171	176	181	186	191	196	2 1.0
												3 1.5
875		201	206	211	216	221	226	231	236	240	245	4 2.0
876		250	255	260	265	270	275	280	285	290	295	5 2.5
877		300	305	310	315	320	325	330	335	340	345	6 3.0
878		349	354	359	364	369	374	379	384	389	394	7 3.5
879		399	404	409	414	419	424	429	433	438	443	8 4.0
												9 4.5
880		448	453	458	463	468	473	478	483	488	493	
881		498	503	507	512	517	522	527	532	537	542	
882		547	552	557	562	567	571	576	581	586	591	
883		596	601	606	611	616	621	626	630	635	640	
884		645	650	655	660	665	670	675	680	685	689	
885		694	699	704	709	714	719	724	729	734	738	
886		743	748	753	758	763	768	773	778	783	787	
887		792	797	802	807	812	817	822	827	832	836	**4**
888		841	846	851	856	861	866	871	876	880	885	1 0.4
889		890	895	900	905	910	915	919	924	929	934	2 0.8
												3 1.2
890		939	944	949	954	959	963	968	973	978	983	4 1.6
891		988	993	998	*002	*007	*012	*017	*022	*027	*032	5 2.0
892	95	036	041	046	051	056	061	066	071	075	080	6 2.4
893		085	090	095	100	105	109	114	119	124	129	7 2.8
894		134	139	143	148	153	158	163	168	173	177	8 3.2
												9 3.6
895		182	187	192	197	202	207	211	216	221	226	
896		231	236	240	245	250	255	260	265	270	274	
897		279	284	289	294	299	303	308	313	318	323	
898		328	332	337	342	347	352	357	361	366	371	
899		376	381	386	390	395	400	405	410	415	419	
900		424	429	434	439	444	448	453	458	463	468	
N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.

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900	95	424	429	434	439	444	448	453	458	463	468	
901		472	477	482	487	492	497	501	506	511	516	
902		521	525	530	535	540	545	550	554	559	564	
903		569	574	578	583	588	593	598	602	607	612	
904		617	622	626	631	636	641	646	650	655	660	
905		665	670	674	679	684	689	694	698	703	708	
906		713	718	722	727	732	737	742	746	751	756	
907		761	766	770	775	780	785	789	794	799	804	
908		809	813	818	823	828	832	837	842	847	852	
909		856	861	866	871	875	880	885	890	895	899	
910		904	909	914	018	923	928	933	938	942	947	
911		952	957	961	966	971	976	980	985	990	995	
912		999	*004	*009	*014	*019	*023	*028	*033	*038	*042	
913	96	047	052	057	061	066	071	076	080	085	090	**5**
914		095	099	104	109	114	118	123	128	133	137	1 0.5
915		142	147	152	156	161	166	171	175	180	185	2 1.0
916		190	194	199	204	209	213	218	223	227	232	3 1.5
917		237	242	246	251	256	261	265	270	275	280	4 2.0
918		284	289	294	298	303	308	313	317	322	327	5 2.5
919		332	336	341	346	350	355	360	365	369	374	6 3.0
920		379	384	388	393	398	402	407	412	417	421	7 3.5
921		426	431	435	440	445	450	454	459	464	468	8 4.0
922		473	478	483	487	492	497	501	506	511	515	9 4.5
923		520	525	530	534	539	544	548	553	558	562	
924		567	572	577	581	586	591	595	600	605	609	
925		614	619	624	628	633	638	642	647	652	656	
926		661	666	670	675	680	685	689	694	699	703	
927		708	713	717	722	727	731	736	741	745	750	
928		755	759	764	769	774	778	783	788	792	797	
929		802	806	811	816	820	825	830	834	839	844	
930		848	853	858	862	876	872	876	881	886	890	
931		895	900	904	909	914	918	923	928	932	937	
932		942	946	951	956	960	965	970	974	979	984	
933		988	993	997	*002	*007	*011	*016	*021	*025	*030	**4**
934	97	035	039	044	049	053	058	063	067	072	077	1 0.4
935		081	086	090	095	100	104	109	114	118	123	2 0.8
936		128	132	137	142	146	151	155	160	165	169	3 1.2
937		174	179	183	188	192	197	202	206	211	216	4 1.6
938		220	225	230	234	239	243	248	253	257	262	5 2.0
939		267	271	276	280	285	290	294	299	304	308	6 2.4
940		313	317	322	327	331	336	340	345	350	354	7 2.8
941		359	364	368	373	377	382	387	391	396	400	8 3.2
942		405	410	414	419	424	428	433	437	442	447	9 3.6
943		451	456	460	465	470	474	479	483	488	493	
944		497	502	506	511	516	520	525	529	534	539	
945		543	548	552	557	562	566	571	575	580	585	
946		589	594	598	603	607	612	617	621	626	630	
947		635	640	644	649	653	658	663	667	672	676	
948		681	685	690	695	699	704	708	713	717	722	
949		727	731	736	740	745	749	754	759	763	768	
950		772	777	782	786	791	795	800	804	809	813	
N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
950	97	772	777	782	786	791	795	800	804	809	813	
951		818	823	827	832	836	841	845	850	855	859	
952		864	868	873	877	882	886	891	896	900	905	
953		909	914	918	923	928	932	937	941	946	950	
954		955	959	964	968	973	978	982	987	991	996	
955	98	000	005	009	014	019	023	028	032	037	041	
956		046	050	055	059	064	068	073	078	082	087	
957		091	096	100	105	109	114	118	123	127	132	
958		137	141	146	150	155	159	164	168	173	177	
959		182	186	191	195	200	204	209	214	218	223	
960		227	232	236	241	245	250	254	259	263	268	
961		272	277	281	286	290	295	299	304	308	313	
962		318	322	327	331	336	340	345	349	354	358	
963		363	367	372	376	381	385	390	394	399	403	**5**
964		408	412	417	421	426	430	435	439	444	448	1 0.5
965		453	457	462	466	471	475	480	484	489	493	2 1.0 3 1.5 4 2.0
966		498	502	507	511	516	520	525	529	534	538	5 2.5
967		543	547	552	556	561	565	570	574	579	583	6 3.0
968		588	592	597	601	605	610	614	619	623	628	7 3.5
969		632	637	641	646	650	655	659	664	668	673	8 4.0 9 4.5
970		677	682	686	691	695	700	704	709	713	717	
971		722	726	731	735	740	744	749	753	758	762	
972		767	771	776	780	784	789	793	798	802	807	
973		811	816	820	825	829	834	838	843	847	851	
974		856	860	865	869	874	878	883	887	892	896	
975		900	905	909	914	918	923	927	932	936	941	
976		945	949	954	958	963	967	972	976	981	985	
977		989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	99	034	038	043	047	052	056	061	065	069	074	
979		078	083	087	092	096	100	105	109	114	118	
980		123	127	131	136	140	145	149	154	158	162	
981		167	171	176	180	185	189	193	198	202	207	
982		211	216	220	224	229	233	238	242	247	251	
983		255	260	264	269	273	277	282	286	291	295	**4**
984		300	304	308	313	317	322	326	330	335	339	1 0.4
985		344	348	352	357	361	366	370	374	379	383	2 0.8 3 1.2
986		388	392	396	401	405	410	414	419	423	427	4 1.6
987		432	436	441	445	449	454	458	463	467	471	5 2.0
988		476	480	484	489	493	498	502	506	511	515	6 2.4
989		520	524	528	533	537	542	546	550	555	559	7 2.8 8 3.2 9 3.6
990		564	568	572	577	581	585	590	594	599	603	
991		607	612	616	621	625	629	634	638	642	647	
992		651	656	660	664	669	673	677	682	686	691	
993		695	699	704	708	712	717	721	726	730	734	
994		739	743	747	752	756	760	765	769	774	778	
995		782	787	791	795	800	804	808	813	817	822	
996		826	830	835	939	843	848	852	856	861	865	
997		870	874	878	883	887	891	896	900	904	909	
998		913	917	922	926	930	935	939	944	948	952	
999		957	961	965	970	974	978	983	987	991	996	
1000	00	000	004	009	013	017	022	026	030	035	039	
N.	L.	0	1	2	3	4	5	6	7	8	9	

	Number.	Logarithm
Base of Naperian logarithms	$e = 2.71828183$	0.4342945
Modulus of common logarithms	$u = 0.43429448$	9.6377843–10
Circumference of a circle in degrees . . .	$= 360$	2.5563025
Circumference of a circle in minutes . .	$= 21600$	4.3344538
Circumference of a circle in seconds . .	$= 1296000$	6.1126050
Radian expressed in degrees	$= 57.29578$	1.7581226
Radian expressed in minutes	$= 3437.7468$	3.5362739
Radian expressed in seconds	$= 206264.806$	5.3144251
Ratio of a circumference to diameter . .	$\pi = 3.14159265$	0.4971499

$\pi = 3.14159265358979323846264338328$

Number	Logarithm		
$2\pi = 6.28318531$	0.7981799	$\pi^2 = 9.86960440$	0.9942997
$4\pi = 12.56637061$	1.0992099	$\frac{1}{\pi^2} = 0.10132118$	9.0057003–10
$\frac{\pi}{2} = 1.57079633$	0.1961199	$\sqrt{\pi} = 1.77245385$	0.2485749
$\frac{\pi}{3} = 1.04719755$	0.0200286	$\frac{1}{\sqrt{\pi}} = 0.56418958$	9.7514251–10
$\frac{4\pi}{3} = 4.18879020$	0.6220886	$\sqrt{\frac{3}{\pi}} = 0.97720502$	9.9899857–10
$\frac{\pi}{4} = 0.78539816$	9.8950899–10	$\sqrt{\frac{4}{\pi}} = 1.12837917$	0.0524551
$\frac{\pi}{6} = 0.52359878$	9.7189986–10	$\sqrt[3]{\pi} = 1.46459189$	0.1657166
$\frac{1}{\pi} = 0.31830989$	9.5028501–10	$\frac{1}{\sqrt[3]{\pi}} = 0.68278406$	9.8342834–10
$\frac{1}{2\pi} = 0.15915494$	9.2018201–10	$\sqrt[3]{\pi^2} = 2.14502940$	0.3314332
$\frac{3}{\pi} = 0.95492966$	9.9799714–10	$\sqrt[3]{\frac{3}{4\pi}} = 0.62035049$	9.7926371–10
$\frac{4}{\pi} = 1.27323954$	0.1049101	$\sqrt[3]{\frac{\pi}{6}} = 0.80599598$	9.9063329–10

If the radius $r = 1$, the length of the arc is:

for 1 degree $= \frac{\pi}{180}$	$= 0.01745329$	8.2418774–10
for 1 minute $= \frac{\pi}{10800}$	$= 0.00029089$	6.4637261–10
for 1 second $= \frac{\pi}{648000}$	$= 0.00000485$	4.6855749–10
	$\sin 1'' = 0.00000485$	4.6855749–10

TABLE IX

THE LOGARITHMS

OF THE

SINE, COSINE, TANGENT, AND COTANGENT

FROM 0° TO 90°

TO FIVE PLACES

L. Cos.				Log Sin.				0°				Log Tan.		
0.00	′	″		0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	
000	**0**	0	4.	—	68557	98660	*16270	*28763	*38454	*46373	*53067	*58866	*63982	*68557	50
000		10	5.	68557	72697	76476	79952	83170	86167	88969	91602	94085	96433	98660	40
000		20		98660	*00779	*02800	*04730	*06579	*08351	*10055	*11694	*13273	*14797	*16270	30
000		30	6.	16270	17694	19072	20409	21705	22964	24188	25378	26536	27664	28763	20
000		40		28763	29836	30882	31904	32903	33879	34833	35767	36682	37577	38454	10
000		50		38454	39315	40158	40985	41797	42594	43376	44145	44900	45643	46373	0 **59**
000	**1**	0	6.4	6373	7090	7797	8492	9175	9849	*0512	*1165	*1808	*2442	*3067	50
000		10	6.5	3067	3683	4291	4890	5481	6064	6639	7207	7767	8320	8866	40
000		20		8866	9406	9939	*0465	*0985	*1499	*2007	*2509	*3006	*3496	*3982	30
000		30	6.6	3982	4462	4936	5406	5870	6330	6785	7235	7680	8121	8557	20
000		40		8557	8990	9418	9841	*0261	*0676	*1088	*1496	*1900	*2300	*2697	10
000		50	6.7	2697	3090	3479	3865	4248	4627	5003	5376	5746	6112	6476	0 **58**
000	**2**	0		6476	6836	7193	7548	7900	8248	8595	8938	9278	9616	9952	50
000		10		9952	*0285	*0615	*0943	*1268	*1591	*1911	*2230	*2545	*2859	*3170	40
000		20	6.8	3170	3479	3786	4091	4394	4694	4993	5289	5584	5876	6167	30
000		30		6167	6455	6742	7027	7310	7591	7870	8147	8423	8697	8969	20
000		40		8969	9240	9509	9776	*0042	*0306	*0568	*0829	*1088	*1346	*1602	10
000		50	6.9	1602	1857	2110	2362	2612	2861	3109	3355	3599	3843	4085	0 **57**
000	**3**	0		4085	4325	4565	4803	5039	5275	5509	5742	5973	6204	6433	50
000		10		6433	6661	6888	7113	7338	7561	7783	8004	8224	8443	8660	40
000		20		8660	8877	9093	9307	9520	9733	9944	*0155	*0364	*0572	*0779	30
000		30	7.0	0779	0986	1191	1395	1599	1801	2003	2203	2403	2602	2800	20
000		40		2800	2997	3193	3388	3582	3776	3968	4160	4351	4541	4730	10
000		50		4730	4919	5106	5293	5479	5664	5849	6032	6215	6397	6579	0 **56**
000	**4**	0		6579	6759	6939	7118	7296	7474	7651	7827	8003	8177	8351	50
000		10		8351	8525	8698	8870	9041	9211	9381	9551	9719	9887	*0055	40
000		20	7.1	0055	0222	0388	0553	0718	0882	1046	1209	1371	1533	1694	30
000		30		1694	1854	2014	2174	2333	2491	2648	2805	2962	3118	3273	20
000		40		3273	3428	3582	3736	3889	4042	4194	4346	4497	4647	4797	10
000		50		4797	4947	5096	5244	5392	5540	5687	5833	5979	6125	6270	0 **55**
0.00				10″	9″	8″	7″	6″	5″	4″	3″	2″	1″	0″	″ ′

L Sin.	Log Cos.	89°	Log Cot.

L. Cos. Log Sin. 0°

	144	143	142	141	140	139		138	137	136	135	134	133	
1	14′4	14′3	14′2	14′1	14′0	13′9	1	13′8	13′7	13′6	13′5	13′4	13′3	1
2	28′8	28′6	28′4	28′2	28′0	27′8	2	27′6	27′4	27′2	27′0	26′8	26′6	2
3	43′2	42′9	42′6	42′3	42′0	41′7	3	41′4	41′1	40′8	40′5	40′2	39′9	3
4	57′6	57′2	56′8	56′4	56′0	55′6	4	55′2	54′8	54′4	54′0	53′6	53′2	4
5	72′0	71′5	71′0	70′5	70′0	69′5	5	69′0	68′5	68′0	67′5	67′0	66′5	5
6	86′4	85′8	85′2	84′6	84′0	83′4	6	82′8	82′2	81′8	81′0	80′4	79′8	6
7	100′8	100′1	99′4	98′7	98′0	97′3	7	96′6	95′9	95′2	94′5	93′8	93′1	7
8	115′2	114′4	113′6	112′8	112′0	111′2	8	110′4	109′6	108′8	108′0	107′2	106′4	8
9	129′6	128′7	127′8	126′9	126′0	125′1	9	124′2	123′3	122′4	121′5	120′6	119′7	9

	132	131	130	129	128	127		126	125	124	123	122	121	
1	13′2	13′1	13′0	12′9	12′8	12′7	1	12′6	12′5	12′4	12′3	12′2	12′1	1
2	26′4	26′2	26′0	25′8	25′6	25′4	2	25′2	25′0	24′8	24′6	24′4	24′2	2
3	39′6	39′3	39′0	38′7	38′4	38′1	3	37′8	37′5	37′2	36′9	36′6	36′3	3
4	52′8	52′4	52′0	51′6	51′2	50′8	4	50′4	50′0	49′6	49′2	48′8	48′4	4
5	66′0	65′5	65′0	64′5	64′0	63′5	5	63′0	62′5	62′0	61′5	61′0	60′5	5
6	79′2	78′6	78′0	77′4	76′8	76′2	6	75′6	75′0	74′4	73′8	73′2	72′6	6
7	92′4	91′7	91′0	90′3	89′6	88′9	7	88′2	87′5	86′8	86′1	85′4	84′7	7
8	105′6	104′8	104′0	103′2	102′4	101′6	8	100′8	100′0	99′2	98′4	97′6	96′8	8
9	118′8	117′9	117′0	116′1	115′2	114′3	9	113′4	112′5	111′6	110′7	109′8	108′9	9

	120	119	118	117	116	115		114	113	112	111	110	109	
1	12′0	11′9	11′8	11′7	11′6	11′5	1	11′4	11′3	11′2	11′1	11′0	10′9	1
2	24′0	23′8	23′6	23′4	23′2	23′0	2	22′8	22′6	22′4	22′2	22′0	21′8	2
3	36′0	35′7	35′4	35′1	34′8	34′5	3	34′2	33′9	33′6	33′3	33′0	32′7	3
4	48′0	47′6	47′2	46′8	46′4	46′0	4	45′6	45′2	44′8	44′4	44′0	43′6	4
5	60′0	59′5	59′0	58′5	58′0	57′5	5	57′0	56′5	56′0	55′5	55′0	54′5	5
6	72′0	71′4	70′8	70′2	69′6	69′0	6	68′4	67′8	67′2	66′6	66′0	65′4	6
7	84′0	83′3	82′6	81′9	81′2	80′5	7	79′8	79′1	78′4	77′7	77′0	76′3	7
8	96′0	95′2	94′4	93′6	92′8	92′0	8	91′2	90′4	89′6	88′8	88′0	87′2	8
9	108′0	107′1	106′2	105′3	104′4	103′5	9	102′6	101′7	100′8	99′9	99′0	98′1	9

0.00	′	″	0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″		
000	**5**	0	7.1 6270	6414	6558	6702	6845	6987	7130	7271	7413	7553	7694	50	
000		10	7694	7834	7973	8112	8250	8389	8526	8663	8800	8937	9072	40	
000		20	9072	9208	9343	9478	9612	9746	9879	*0012	*0145	*0277	*0409	30	
000		30	7.2 0409	0540	0671	0802	0932	1062	1191	1320	1449	1577	1705	20	
000		40	1705	1833	1960	2087	2213	2339	2465	2590	2715	2840	2964	10	
000		50	2964	3088	3212	3335	3458	3580	3702	3824	3946	4067	4188	0	**54**
000	**6**	0	4188	4308	4428	4548	4668	4787	4906	5024	5142	5260	5378	50	
000		10	5378	5495	5612	5728	5845	5961	6076	6192	6307	6421	6536	40	
000		20	6536	6650	6764	6877	6991	7104	7216	7329	7441	7552	7664	30	
000		30	7664	7775	7886	7997	8107	8217	8327	8437	8546	8655	8763	20	
000		40	8763	8872	8980	9088	9196	9303	9410	9517	9623	9730	9836	10	
000		50	9836	9942	*0047	*0152	*0257	*0362	*0467	*0571	*0675	*0779	*0882	0	**53**
000	**7**	0	7.3 0882	0986	1089	1191	1294	1396	1498	1600	1702	1803	1904	50	
000		10	1904	2005	2106	2206	2306	2406	2506	2606	2705	2804	2903	40	
000		20	2903	3001	3100	3198	3296	3393	3491	3588	3685	3782	3879	30	
000		30	3879	3975	4071	4167	4263	4359	4454	4549	4644	4739	4833	20	
000		40	4833	4928	5022	5116	5209	5303	5396	5489	5582	5675	5767	10	
000		50	5767	5860	5952	6044	6135	6227	6318	6409	6500	6591	6682	0	**52**
000	**8**	0	6682	6772	6862	6952	7042	7132	7221	7310	7399	7488	7577	50	
000		10	7577	7666	7754	7842	7930	8018	8106	8193	8280	8367	8454	40	
000		20	8454	8541	8628	8714	8800	8887	8972	9058	9144	9229	9314	30	
000		30	9314	9400	9484	9569	9654	9738	9822	9906	9990	*0074	*0158	20	
000		40	7.4 0158	0241	0324	0408	0491	0573	0656	0739	0821	0903	0985	10	
000		50	0985	1067	1149	1230	1312	1393	1474	1555	1636	1716	1797	0	**51**
000	**9**	0	1797	1877	1957	2037	2117	2197	2277	2356	2435	2515	2594	50	
000		10	2594	2673	2751	2830	2908	2987	3065	3143	3221	3299	3376	40	
000		20	3376	3454	3531	3608	3685	3762	3839	3916	3992	4069	4145	30	
000		30	4145	4221	4297	4373	4449	4524	4600	4675	4750	4825	4900	20	
000		40	4900	4975	5050	5124	5199	5273	5347	5421	5495	5569	5643	10	
000		50	5643	5716	5790	5863	5936	6009	6082	6155	6228	6300	6373	0	**50**
0.00			10″	9″	8″	7″	6″	5″	4″	3″	2″	1″	0″	″	′

L. Sin. 89° Log Cos.

Log Tan. 0°

	108	107	106	105	104	103		102	101	99	98	97	96	
1	10'8	10'7	10'6	10'5	10'4	10'3	1	10'2	10'1	9'9	9'8	9'7	9'6	1
2	21'6	21'4	21'2	21'0	20'8	20'6	2	20'4	20'2	19'8	19'6	19'4	19'2	2
3	32'4	32'1	31'8	31'5	31'2	30'9	3	30'6	30'3	29'7	29'4	29'1	28'8	3
4	43'2	42'8	42'4	42'0	41'6	41'2	4	40'8	40'4	39'6	39'2	38'8	38'4	4
5	54'0	53'5	53'0	52'5	52'0	51'5	5	51'0	50'5	49'5	49'0	48'5	48'0	5
6	64'8	64'2	63'6	63'0	62'4	61'8	6	61'2	60'6	59'4	58'8	58'2	57'6	6
7	75'6	74'9	74'2	73'5	72'8	72'1	7	71'4	70'7	69'3	68'6	67'9	67'2	7
8	86'4	85'6	84'8	84'0	83'2	82'4	8	81'6	80'8	79'2	78'4	77'6	76'8	8
9	97'2	96'3	95'4	94'5	93'6	92'7	9	91'8	90'9	89'1	88'2	87'3	86'4	9
	95	**94**	**93**	**92**	**91**	**90**		**89**	**88**	**87**	**86**	**85**	**84**	
1	9'5	9'4	9'3	9'2	9'1	9'0	1	8'9	8'8	8'7	8'6	8'5	8'4	1
2	19'0	18'8	18'6	18'4	18'2	18'0	2	17'8	17'6	17'4	17'2	17'0	16'8	2
3	28'5	28'2	27'9	27'6	27'3	27'0	3	26'7	26'4	26'1	25'8	25'5	25'2	3
4	38'0	37'6	37'2	36'8	36'4	36'0	4	35'6	35'2	34'8	34'4	34'0	33'6	4
5	47'5	47'0	46'5	46'0	45'5	45'0	5	44'5	44'0	43'5	43'0	42'5	42'0	5
6	57'0	56'4	55'8	55'2	54'6	54'0	6	53'4	52'8	52'2	51'6	51'0	50'4	6
7	66'5	65'8	65'1	64'4	63'7	63'0	7	62'3	61'6	60'9	60'2	59'5	58'8	7
8	76'0	75'2	74'4	73'6	72'8	72'0	8	71'2	70'4	69'6	68'8	68'0	67'2	8
9	85'5	84'6	83'7	82'8	81'9	81'0	9	80'1	79'2	78'3	77'4	76'5	75'6	9
	83	**82**	**81**	**80**	**79**	**78**		**77**	**76**	**75**	**74**	**73**	**72**	
1	8'3	8'2	8'1	8'0	7'9	7'8	1	7'7	7'6	7'5	7'4	7'3	7'2	1
2	16'6	16'4	16'2	16'0	15'8	15'6	2	15'4	15'2	15'0	14'8	14'6	14'4	2
3	24'9	24'6	24'3	24'0	23'7	23'4	3	23'1	22'8	22'5	22'2	21'9	21'6	3
4	33'2	32'8	32'4	32'0	31'6	31'2	4	30'8	30'4	30'0	29'6	29'2	28'8	4
5	41'5	41'0	40'5	40'0	39'5	39'0	5	38'5	38'0	37'5	37'0	36'5	36'0	5
6	49'8	49'2	48'6	48'0	47'4	46'8	6	46'2	45'6	45'0	44'4	43'8	43'2	6
7	58'1	57'4	56'7	56'0	55'3	54'6	7	53'9	53'2	52'5	51'8	51'1	50'4	7
8	66'4	65'6	64'8	64'0	63'2	62'4	8	61'6	60'8	60'0	59'2	58'4	57'6	8
9	74'7	73'8	72'9	72'0	71'1	70'2	9	69'3	68'4	67'5	66'6	65'7	64'8	9

′	″		0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″		
5	0	7.1	6270	6414	6558	6702	6845	6988	7130	7271	7413	7553	7694	50	
	10		7694	7834	7973	8112	8250	8389	8526	8663	8800	8937	9073	40	
	20		9073	9208	9343	9478	9612	9746	9879	*0012	*0145	*0277	*0409	30	
	30	7.2	0409	0540	0671	0802	0932	1062	1191	1321	1449	1577	1705	20	
	40		1705	1833	1960	2087	2213	2339	2465	2590	2715	2840	2964	10	
	50		2964	3088	3212	3335	3458	3580	3703	3824	3946	4067	4188	0	**54**
6	0		4188	4308	4428	4548	4668	4787	4906	5024	5142	5260	5378	50	
	10		5378	5495	5612	5728	5845	5961	6076	6192	6307	6421	6536	40	
	20		6536	6650	6764	6877	6991	7104	7216	7329	7441	7552	7664	30	
	30		7664	7775	7886	7997	8107	8217	8327	8437	8546	8655	8764	20	
	40		8764	8872	8980	9088	9196	9303	9410	9517	9624	9730	9836	10	
	50		9836	9942	*0047	*0153	*0258	*0362	*0467	*0571	*0675	*0779	*0882	0	**53**
7	0	7.3	0882	0986	1089	1192	1294	1396	1499	1600	1702	1803	1904	50	
	10		1904	2005	2106	2206	2307	2406	2506	2606	2705	2804	2903	40	
	20		2903	3001	3100	3198	3296	3394	3491	3588	3685	3782	3879	30	
	30		3879	3975	4071	4167	4263	4359	4454	4549	4644	4739	4833	20	
	40		4833	4928	5022	5116	5209	5303	5396	5489	5582	5675	5767	10	
	50		5767	5860	5952	6044	6135	6227	6318	6409	6500	6591	6682	0	**52**
8	0		6682	6772	6862	6952	7042	7132	7221	7310	7400	7488	7577	50	
	10		7577	7666	7754	7842	7930	8018	8106	8193	8281	8368	8455	40	
	20		8455	8541	8628	8714	8801	8887	8973	9058	9144	9229	9315	30	
	30		9315	9400	9485	9569	9654	9738	9823	9907	9991	*0074	*0158	20	
	40	7.4	0158	0241	0325	0408	0491	0574	0656	0739	0821	0903	0985	10	
	50		0985	1067	1149	1230	1312	1393	1474	1555	1636	1716	1797	0	**51**
9	0		1797	1877	1958	2038	2117	2197	2277	2356	2436	2515	2594	50	
	10		2594	2673	2751	2830	2909	2987	3065	3143	3221	3299	3376	40	
	20		3376	3454	3531	3608	3686	3762	3839	3916	3992	4069	4145	30	
	30		4145	4221	4297	4373	4449	4524	4600	4675	4750	4825	4900	20	
	40		4900	4975	5050	5124	5199	5273	5347	5421	5495	5569	5643	10	
	50		5643	5716	5790	5863	5936	6009	6082	6155	6228	6300	6373	0	**50**
			10″	9″	8″	7″	6″	5″	4″	3″	2″	1″	0″	″	′

89° Log Cot.

0.00	′	″	0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″		d.		P. P.
000	**10**	0	7.46 373	445	517	589	661	733	805	876	948	*019	*090	50	72		**72**
		10	7.47 090	162	233	303	374	445	515	586	656	726	797	40	71	1	7.2
000		20	797	867	936	*006	*076	*145	*215	*284	*353	*422	*491	30	69	2	14.4
000		30	7.48 491	560	629	698	766	835	903	971	*039	*108	*175	20	68	3	21.6
000		40	7.49 175	243	311	379	446	513	581	648	715	782	849	10	67	4	28.8
000		50	849	916	982	*049	*115	*182	*248	*314	*380	*446	*512	0 **49**	66	5	36.0
																6	43.2
																7	50.4
000	**11**	0	7.50 512	578	643	709	774	840	905	970	*035	*100	*165	50	65	8	57.6
000		10	7.51 165	230	294	359	423	488	552	616	680	744	808	40	64	9	64.8
000		20	808	872	936	999	*063	*126	*190	*253	*316	*379	*442	30	63		**70**
000		30	7.52 442	505	568	631	693	756	818	881	943	*005	*067	20	62	1	7.0
000		40	7.53 067	129	191	253	315	376	438	499	561	622	683	10	62	2	14.0
000		50	683	744	805	866	927	988	*049	*109	*170	*230	*291	0 **48**	61	3	21.0
																4	28.0
																5	35.0
000	**12**	0	7.54 291	351	411	471	531	591	651	711	771	830	890	50	60	6	42.0
000		10	890	949	*009	*068	*127	*186	*245	*304	*363	*422	*481	40	59	7	49.0
000		20	7.55 481	539	598	656	715	773	831	889	948	*006	*064	30	58	8	56.0
000		30	7.56 064	121	179	237	295	352	410	467	524	582	639	20	58	9	63.0
000		40	639	696	753	810	867	924	980	*037	*094	*150	*206	10	57		**68**
000		50	7.57 206	263	319	375	431	488	544	599	655	711	767	0 **47**	56	1	6.8
																2	13.6
																3	20.4
000	**13**	0	767	822	878	934	989	*044	*100	*155	*210	*265	*320	50	55	4	27.2
000		10	7.58 320	375	430	485	539	594	649	703	758	812	866	40	55	5	34.0
000		20	866	921	975	*029	*083	*137	*191	*245	*299	*352	*406	30	54	6	40.8
000		30	7.59 406	459	513	566	620	673	726	780	833	886	939	20	53	7	47.6
000		40	939	992	*045	*097	*150	*203	*255	*308	*360	*413	*465	10	53	8	54.4
000		50	7.60 465	517	570	622	674	726	778	830	882	934	985	0 **46**	52	9	61.2
																	66
																1	6.6
000	**14**	0	985	*037	*089	*140	*192	*243	*294	*346	*397	*448	*499	50	51	2	13.2
000		10	7.61 499	550	601	652	703	754	805	855	906	957	*007	40	51	3	19.8
000		20	7.62 007	058	108	158	209	259	309	359	409	459	509	30	50	4	26.4
000		30	509	559	609	659	708	758	808	857	907	956	*006	20	50	5	33.0
000		40	7.63 006	055	104	153	203	252	301	350	399	448	496	10	49	6	39.6
000		50	496	545	594	642	691	740	788	837	885	933	982	0 **45**	49	7	46.2
																8	52.8
																9	59.4
000	**15**	0	982	*030	*078	*126	*174	*222	*270	*318	*366	*414	*461	50	48		**64**
000		10	7.64 461	509	557	604	652	699	747	794	842	889	936	40	48	1	6.4
000		20	936	983	*030	*078	*125	*172	*218	*265	*312	*359	*406	30	47	2	12.8
000		30	7.65 406	452	499	546	592	638	685	731	778	824	870	20	46	3	19.2
000		40	870	916	962	*009	*055	*101	*146	*192	*238	*284	*330	10	46	4	25.6
000		50	7.66 330	375	421	467	512	558	603	649	694	739	784	0 **44**	45	5	32.0
																6	38.4
																7	44.8
000	**16**	0	784	830	875	920	965	*010	*055	*100	*145	*190	*235	50	45	8	51.2
000		10	7.67 235	279	324	369	413	458	502	547	591	636	680	40	44	9	57.6
*000		20	680	724	768	813	857	901	945	989	*033	*077	*121	30	44		**62**
*999		30	7.68 121	165	208	252	296	340	383	427	470	514	557	20	44	1	6.2
999		40	557	601	644	687	731	774	817	860	903	946	989	10	43	2	12.4
999		50	989	*032	*075	*118	*161	*204	*247	*289	*332	*375	*417	0 **43**	43	3	18.6
																4	24.8
																5	31.0
999	**17**	0	7.69 417	460	502	545	587	630	672	714	757	799	841	50	42	6	37.2
999		10	841	883	925	967	*009	*051	*093	*135	*177	*219	*261	40	42	7	43.4
999		20	7.70 261	302	344	386	427	469	510	552	593	635	676	30	42	8	49.6
999		30	676	718	759	800	841	883	924	965	*006	*047	*088	20	41	9	55.8
999		40	7.71 088	129	170	211	251	292	333	374	414	455	496	10	41		**61**
999		50	496	536	577	617	658	698	739	779	819	859	900	0 **42**	40	1	6.1
																2	12.2
																3	18.3
999	**18**	0	900	940	980	*020	*060	*100	*140	*180	*220	*260	*300	50	40	4	24.4
999		10	7.72 300	340	380	419	459	499	538	578	618	657	697	40	40	5	30.5
999		20	697	736	775	815	854	894	933	972	*011	*050	*090	30	39	6	36.6
999		30	7.73 090	129	168	207	246	285	324	363	401	440	479	20	39	7	42.7
999		40	479	518	557	595	634	673	711	750	788	827	865	10	39	8	48.8
999		50	865	904	942	980	*019	*057	*095	*133	*171	*210	*248	0 **41**	38	9	54.9
																	60
																1	6.0
999	**19**	0	7.74 248	286	324	362	400	438	476	514	551	589	627	50	38	2	12.0
999		10	627	665	703	740	778	815	853	891	928	966	*003	40	38	3	18.0
999		20	7.75 003	040	078	115	153	190	227	264	302	339	376	30	37	4	24.0
999		30	376	413	450	487	524	561	598	635	672	709	745	20	37	5	30.0
999		40	745	782	819	856	892	929	966	*002	*039	*075	*112	10	37	6	36.0
999		50	7.76 112	148	185	221	258	294	330	367	403	439	475	0 **40**	36	7	42.0
																8	48.0
																9	54.0
9.99			10″	9″	8″	7″	6″	5″	4″	3″	2″	1″	0″	″	′	d.	P. P.

Log Tan. 0°

′	″		0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″		d.
10	0	7.46	373	445	517	589	661	733	805	876	948	*019	*091	50	72
	10	7.47	091	162	233	304	374	445	516	586	656	727	797	40	71
	20		797	867	937	*006	*076	*146	*215	*284	*354	*423	*492	30	70
	30	7.48	492	561	629	698	767	835	903	972	*040	*108	*176	20	68
	40	7.49	176	243	311	379	446	514	581	648	715	782	849	10	67
	50		849	916	982	*049	*115	*182	*248	*314	*380	*446	*512	0 49	66
11	0	7.50	512	578	643	709	774	840	905	970	*035	*100	*165	50	65
	10	7.51	165	230	295	359	424	488	552	617	681	745	809	40	64
	20		809	872	936	*000	*063	*127	*190	*253	*316	*380	*443	30	63
	30	7.52	443	505	568	631	694	756	819	881	943	*005	*067	20	62
	40	7.53	067	129	191	253	315	377	438	500	561	622	683	10	62
	50		683	745	806	867	927	988	*049	*110	*170	*231	*291	0 48	61
12	0	7.54	291	351	411	471	532	591	651	711	771	830	890	50	60
	10		890	949	*009	*068	*127	*186	*245	*304	*363	*422	*481	40	59
	20	7.55	481	539	598	657	715	773	832	890	948	*006	*064	30	58
	30	7.56	064	122	179	237	295	352	410	467	525	582	639	20	58
	40		639	696	753	810	867	924	981	*037	*094	*150	*207	10	57
	50	7.57	207	263	319	376	432	488	544	600	656	711	767	0 47	56
13	0		767	823	878	934	989	*045	*100	*155	*210	*265	*320	50	55
	10	7.58	320	375	430	485	540	594	649	704	758	812	867	40	55
	20		867	921	975	*029	*083	*137	*191	*245	*299	*353	*406	30	54
	30	7.59	406	460	513	567	620	673	727	780	833	886	939	20	53
	40		939	992	*045	*098	*150	*203	*256	*308	*361	*413	*466	10	53
	50	7.60	466	518	570	622	674	726	778	830	882	934	986	0 46	52
14	0		986	*037	*089	*140	*192	*243	*295	*346	*397	*449	*500	50	51
	10	7.61	500	551	602	653	704	754	805	856	906	957	*008	40	51
	20	7.62	008	058	108	159	209	259	310	360	410	460	510	30	50
	30		510	560	609	659	709	759	808	858	907	957	*006	20	50
	40	7.63	006	055	105	154	203	252	301	350	399	448	497	10	49
	50		497	546	594	643	692	740	789	837	885	934	982	0 45	48
15	0		982	*030	*078	*127	*175	*223	*271	*318	*366	*414	*462	50	48
	10	7.64	462	510	557	605	652	700	747	795	842	889	937	40	48
	20		937	984	*031	*078	*125	*172	*219	*266	*313	*359	*406	30	47
	30	7.65	406	453	499	546	592	639	685	732	778	824	871	20	46
	40		871	917	963	*009	*055	*101	*147	*193	*239	*284	*330	10	46
	50	7.66	330	376	421	467	513	558	604	649	694	740	785	0 44	46
16	0		785	830	875	920	966	*011	*056	*100	*145	*190	*235	50	45
	10	7.67	235	280	324	369	414	458	503	547	592	636	680	40	44
	20		680	725	769	813	857	901	946	990	*034	*077	*121	30	44
	30	7.68	121	165	209	253	296	340	384	427	471	514	558	20	44
	40		558	601	645	688	731	774	818	861	904	947	990	10	43
	50		990	*033	*076	*119	*162	*204	*247	*290	*333	*375	*418	0 43	43
17	0	7.69	418	460	503	545	588	630	673	715	757	799	842	50	42
	10		842	884	926	968	*010	*052	*094	*136	*178	*219	*261	40	42
	20	7.70	261	303	345	386	428	469	511	553	594	635	677	30	42
	30		677	718	759	801	842	883	924	965	*006	*047	*088	20	41
	40	7.71	088	129	170	211	252	293	334	374	415	456	496	10	41
	50		496	537	577	618	658	699	739	779	820	860	900	0 42	40
18	0		900	940	981	*021	*061	*101	*141	*181	*221	*261	*301	50	40
	10	7.72	301	340	380	420	460	499	539	579	618	658	697	40	40
	20		697	737	776	815	855	894	933	973	*012	*051	*090	30	39
	30	7.73	090	129	168	207	246	285	324	363	402	441	480	20	39
	40		480	518	557	596	635	673	712	750	789	827	866	10	39
	50		866	904	943	981	*019	*058	*096	*134	*172	*210	*248	0 41	38
19	0	7.74	248	286	325	363	401	438	476	514	552	590	628	50	38
	10		628	665	703	741	779	816	854	891	929	966	*004	40	38
	20	7.75	004	041	079	116	153	191	228	265	302	339	377	30	37
	30		377	414	451	488	525	562	599	636	672	709	746	20	37
	40		746	783	820	856	893	930	966	*003	*040	*076	*113	10	37
	50	7.76	113	149	186	222	258	295	331	367	404	440	476	0 40	36
			10″	9″	8″	7″	6″	5″	4″	3″	2″	1″	0″	″ ′	d.

P. P.			
	59	**58**	**57**
1	5.9	5.8	5.7
2	11.8	11.6	11.4
3	17.7	17.4	17.1
4	23.6	23.2	22.8
5	29.5	29.0	28.5
6	35.4	34.8	34.2
7	41.3	40.6	39.9
8	47.2	46.4	45.6
9	53.1	52.2	51.3
	56	**55**	**54**
1	5.6	5.5	5.4
2	11.2	11.0	10.8
3	16.8	16.5	16.2
4	22.4	22.0	21.6
5	28.0	27.5	27.0
6	33.6	33.0	32.4
7	39.2	38.5	37.8
8	44.8	44.0	43.2
9	50.4	49.5	48.6
	53	**52**	**51**
1	5.3	5.2	5.1
2	10.6	10.4	10.2
3	15.9	15.6	15.3
4	21.2	20.8	20.4
5	26.5	26.0	25.5
6	31.8	31.2	30.6
7	37.1	36.4	35.7
8	42.4	41.6	40.8
9	47.7	46.8	45.9
	50	**49**	**48**
1	5.0	4.9	4.8
2	10.0	9.8	9.6
3	15.0	14.7	14.4
4	20.0	19.6	19.2
5	25.0	24.5	24.0
6	30.0	29.4	28.8
7	35.0	34.3	33.6
8	40.0	39.2	38.4
9	45.0	44.1	43.2
	47	**46**	**45**
1	4.7	4.6	4.5
2	9.4	9.2	9.0
3	14.1	13.8	13.5
4	18.8	18.4	18.0
5	23.5	23.0	22.5
6	28.2	27.6	27.0
7	32.9	32.2	31.5
8	37.6	36.8	36.0
9	42.3	41.4	40.5
	44	**43**	**42**
1	4.4	4.4	3.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8
	41	**40**	**39**
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1
	38	**37**	**36**
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

P. P.

9.99	′	″		0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″			d.
999	**20**	0	7.76	475	512	548	584	620	656	692	728	764	800	836	50		36
999		10		836	872	907	943	979	*015	*051	*086	*122	*158	*193	40		36
999		20	7.77	193	229	264	300	335	371	406	442	477	512	548	30		36
999		30		548	583	618	654	689	724	759	794	829	864	899	20		35
999		40		899	934	969	*004	*039	*074	*109	*144	*179	*213	*248	10		35
999		50	7.78	248	283	318	352	387	422	456	491	525	560	594	0	**39**	35
999	**21**	0		594	629	663	698	732	766	801	835	869	903	938	50		34
999		10		938	972	*006	*040	*074	*108	*142	*176	*210	*244	*278	40		34
999		20	7.79	278	312	346	380	414	448	481	515	549	582	616	30		34
999		30		616	650	683	717	751	784	818	851	885	918	952	20		34
999		40		952	985	*018	*052	*085	*118	*152	*185	*218	*251	*284	10		33
999		50	7.80	284	317	351	384	417	450	483	516	549	582	615	0	**38**	33
999	**22**	0		615	647	680	713	746	779	812	844	877	910	942	50		33
999		10		942	975	*008	*040	*073	*105	*138	*170	*203	*235	*268	40		33
999		20	7.81	268	300	332	365	397	429	462	494	526	558	591	30		32
999		30		591	623	655	687	719	751	783	815	847	879	911	20		32
999		40		911	943	975	*007	*039	*070	*102	*134	*166	*198	*229	10		32
999		50	7.82	229	261	293	324	356	387	419	451	482	514	545	0	**37**	32
999	**23**	0		545	577	608	639	671	702	733	765	796	827	859	50		31
999		10		859	890	921	952	983	*015	*046	*077	*108	*139	*170	40		31
999		20	7.83	170	201	232	263	294	325	356	387	417	448	479	30		31
999		30		479	510	541	571	602	633	663	694	725	755	786	20		31
999		40		786	817	847	878	908	939	969	*000	*030	*060	*091	10		30
999		50	7.84	091	121	151	182	212	242	273	303	333	363	393	0	**36**	30
999	**24**	0		393	424	454	484	514	544	574	604	634	664	694	50		30
999		10		694	724	754	784	814	843	873	903	933	963	992	40		30
999		20		992	*022	*052	*082	*111	*141	*171	*200	*230	*259	*289	30		30
999		30	7.85	289	318	348	377	407	436	466	495	525	554	583	20		29
999		40		583	613	642	671	701	730	759	788	817	847	876	10		29
999		50		876	905	934	963	992	*021	*050	*079	*108	*137	*166	0	**35**	29
999	**25**	0	7.86	166	195	224	253	282	311	340	368	397	426	455	50		29
999		10		455	484	512	541	570	598	627	656	684	713	741	40		29
999		20		741	770	799	827	856	884	913	941	969	998	*026	30		28
999		30	7.87	026	055	083	111	140	168	196	224	253	281	309	20		28
999		40		309	337	366	394	422	450	478	506	534	562	590	10		28
999		50		590	618	646	674	702	730	758	786	814	842	870	0	**34**	28
999	**26**	0		870	897	925	953	981	*009	*036	*064	*092	*119	*147	50		28
999		10	7.88	147	175	202	230	258	285	313	340	368	395	423	40		28
999		20		423	450	478	505	533	560	587	615	642	669	697	30		27
999		30		697	724	751	779	806	833	860	888	915	942	969	20		27
999		40		969	996	*023	*050	*077	*105	*132	159	*186	*213	*240	10		27
999		50	7.89	240	267	294	320	347	374	401	428	455	482	509	0	**33**	27
999	**27**	0		509	535	562	589	616	642	669	696	722	749	776	50		27
999		10		776	802	829	856	882	909	935	962	988	*015	*041	40		26
999		20	7.90	041	068	094	121	147	174	200	226	253	279	305	30		26
999		30		305	332	358	384	411	437	463	489	515	542	568	20		26
999		40		568	594	620	646	672	698	725	751	777	803	829	10		26
999		50		829	855	881	907	933	958	984	*010	*036	*062	*088	0	**32**	26
999	**28**	0	7.91	088	114	140	165	191	217	243	269	294	320	346	50		26
999		10		346	371	397	423	448	474	500	525	551	576	602	40		26
999		20		602	627	653	678	704	729	755	780	806	831	857	30		26
999		30		857	882	907	933	958	983	*009	*034	*059	*085	*110	20		25
998		40	7.92	110	135	160	186	211	236	261	286	311	336	362	10		25
998		50		362	387	412	437	462	487	512	537	562	587	612	0	**31**	25
998	**29**	0		612	637	662	687	712	737	761	786	811	836	861	50		25
998		10		861	886	910	935	960	985	*009	*034	*059	*084	*108	40		25
998		20	7.93	108	133	158	182	207	231	256	281	305	330	354	30		25
998		30		354	379	403	428	452	477	501	526	550	575	599	20		24
998		40		599	623	648	672	696	721	745	769	794	818	842	10		24
998		50		842	866	891	915	939	963	988	*012	*036	*060	*084	0	**30**	24
9.99				10″	9″	8″	7″	6″	5″	4″	3″	2″	1″	0″	″	′	d.

Log Tan. 0°

′	″		0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″		d.		P. P.	
20	0	7.76	476	512	548	585	621	657	693	729	765	801	837	50	36		**37**	**36**
	10		837	872	908	944	980	*016	*051	*087	*123	*158	*194	40	36	1	3.7	3.6
	20	7.77	194	230	265	301	336	372	407	442	478	513	549	30	36	2	7.4	7.2
	30		549	584	619	654	690	725	760	795	830	865	900	20	35	3	11.1	10.8
	40		900	935	970	*005	*040	*075	*110	*145	*179	*214	*249	10	35	4	14.8	14.4
	50	7.78	249	284	318	353	388	422	457	492	526	561	595	0 **39**	35	5	18.5	18.0
																6	22.2	21.6
																7	25.9	25.2
21	0		595	630	664	698	733	767	801	836	870	904	938	50	34	8	29.6	28.8
	10		938	973	*007	*041	*075	*109	*143	*177	*211	*245	*279	40	34	9	33.3	32.4
	20	7.79	279	313	347	381	415	448	482	516	550	583	617	30	34			
	30		617	651	684	718	751	785	819	852	886	919	952	20	34		**35**	**34**
	40		952	986	*019	*053	*086	*119	*152	*186	*219	*252	*285	10	33	1	3.5	3.4
	50	7.80	285	318	351	385	418	451	484	517	550	583	615	0 **38**	33	2	7.0	6.8
																3	10.5	10.2
																4	14.0	13.6
22	0		615	648	681	714	747	780	812	845	878	911	943	50	33	5	17.5	17.0
	10		943	976	*009	*041	*074	*106	*139	*171	*204	*236	*269	40	33	6	21.0	20.4
	20	7.81	269	301	333	366	398	430	463	495	527	559	591	30	32	7	24.5	23.8
	30		591	624	656	688	720	752	784	816	848	880	912	20	32	8	28.0	27.2
	40		912	944	976	*008	*040	*071	*103	*135	*167	*198	*230	10	32	9	31.5	30.6
	50	7.82	230	262	294	325	357	388	420	452	483	515	546	0 **37**	32			
																	33	**32**
23	0		546	578	609	640	672	703	734	766	797	828	860	50	31	1	3.3	3.2
	10		860	891	922	953	984	*016	*047	*078	*109	*140	*171	40	31	2	6.6	6.4
	20	7.83	171	202	233	264	295	326	357	388	418	449	480	30	31	3	9.9	9.6
	30		480	511	542	572	603	634	664	695	726	756	787	20	31	4	13.2	12.8
	40		787	818	848	879	909	940	970	*001	*031	*061	*092	10	30	5	16.5	16.0
	50	7.84	092	122	152	183	213	243	274	304	334	364	394	0 **36**	30	6	19.8	19.2
																7	23.1	22.4
																8	26.4	25.6
24	0		394	425	455	485	515	545	575	605	635	665	695	50	30	9	29.7	28.8
	10		695	725	755	785	815	845	874	904	934	964	993	40	30			
	20		993	*023	*053	*083	*112	*142	*172	*201	*231	*260	*290	30	30		**31**	**30**
	30	7.85	290	319	349	378	408	437	467	496	526	555	584	20	29	1	3.1	3.0
	40		584	614	643	672	702	731	760	789	819	848	877	10	29	2	6.2	6.0
	50		877	906	935	964	993	*022	*051	*080	*109	*138	*167	0 **35**	29	3	9.3	9.0
																4	12.4	12.0
25	0	7.86	167	196	225	254	283	312	341	370	398	427	456	50	29	5	15.5	15.0
	10		456	485	513	542	571	600	628	657	685	714	743	40	29	6	18.6	18.0
	20		743	771	800	828	857	885	914	942	971	999	*027	30	28	7	21.7	21.0
	30	7.87	027	056	084	113	141	169	197	226	254	282	310	20	28	8	24.8	24.0
	40		310	339	367	395	423	451	479	507	535	563	591	10	28	9	27.9	27.0
	50		591	619	647	675	703	731	759	787	815	843	871	0 **34**	28			
																	29	**28**
																1	2.9	2.8
26	0		871	899	926	954	982	*010	*037	*065	*093	*121	*148	50	28	2	5.8	5.6
	10	7.88	148	176	204	231	259	286	314	342	369	397	424	40	28	3	8.7	8.4
	20		424	452	479	506	534	561	589	616	643	671	698	30	27	4	11.6	11.2
	30		698	725	753	780	807	834	862	889	916	943	970	20	27	5	14.5	14.0
	40		970	997	*025	*052	*079	*106	*133	*160	*187	*214	*241	10	27	6	17.4	16.8
	50	7.89	241	268	295	322	349	376	403	429	456	483	510	0 **33**	27	7	20.3	19.6
																8	23.2	22.4
																9	26.1	25.2
27	0		510	537	563	590	617	644	670	697	724	750	777	50	27			
	10		777	804	830	857	884	910	937	963	990	*016	*043	40	27		**27**	**26**
	20	7.90	043	069	096	122	149	175	201	228	254	280	307	30	26	1	2.7	2.6
	30		307	333	359	386	412	438	464	491	517	543	569	20	26	2	5.4	5.2
	40		569	595	622	648	674	700	726	752	778	804	830	10	26	3	8.1	7.8
	50		830	856	882	908	934	960	986	*012	*038	*064	*089	0 **32**	26	4	10.8	10.4
																5	13.5	13.0
																6	16.2	15.6
28	0	7.91	089	115	141	167	193	218	244	270	296	321	347	50	26	7	18.9	18.2
	10		347	373	398	424	450	475	501	527	552	578	603	40	26	8	21.6	20.8
	20		603	629	654	680	705	731	756	782	807	833	858	30	26	9	24.3	23.4
	30		858	883	909	934	960	985	*010	*036	*061	*086	*111	20	25			
	40	7.92	111	137	162	187	212	237	263	288	313	338	363	10	25		**25**	**24**
	50		363	388	413	438	463	488	513	538	563	588	613	0 **31**	25	1	2.5	2.4
																2	5.0	4.8
29	0		613	638	663	688	713	738	763	788	813	838	862	50	25	3	7.5	7.2
	10		862	887	912	937	961	986	*011	*036	*060	*085	*110	40	25	4	10.0	9.6
	20	7.93	110	134	159	184	208	233	258	282	307	331	356	30	25	5	12.5	12.0
	30		356	380	405	429	454	478	503	527	552	576	601	20	24	6	15.0	14.4
	40		601	625	649	674	698	722	747	771	795	820	844	10	24	7	17.5	16.8
	50		844	868	892	917	941	965	989	*013	*038	*062	*086	0 **30**	24	8	20.0	19.2
																9	22.5	21.6
			10″	9″	8″	7″	6″	5″	4″	3″	2″	1″	0″	″	′	d.	P. P.	

9.99	′	″		0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″			d.
998	**30**	0	7.94	084	108	132	157	181	205	229	253	277	301	325	50		24
998		10		325	349	373	397	421	445	469	492	516	540	564	40		24
998		20		564	588	612	636	659	683	707	731	755	778	802	30		24
998		30		802	826	849	873	897	921	944	968	991	*015	*039	20		24
998		40	7.95	039	062	086	109	133	157	180	204	227	251	274	10		24
998		50		274	298	321	**344**	**368**	**391**	415	438	461	485	508	0	**29**	**23**
998	**31**	0		508	532	555	578	601	625	648	671	695	718	741	50		23
998		10		741	764	787	811	834	857	880	903	926	950	973	40		23
998		20		973	996	*019	*042	*065	*088	*111	*134	*157	*180	*203	30		23
998		30	7.96	203	226	249	272	295	318	341	364	386	409	432	20		23
998		40		432	455	478	501	524	546	569	592	615	637	660	10		23
998		50		660	683	706	728	751	774	796	819	842	864	887	0	**28**	23
998	**32**	0		887	910	932	955	977	*000	*022	*045	*068	*090	*113	50		23
998		10	7.97	113	135	158	180	202	225	247	270	292	315	337	40		22
998		20		337	359	382	404	426	449	471	493	516	538	560	30		22
998		30		560	583	605	627	649	672	694	716	738	760	782	20		22
998		40		782	805	827	849	871	893	915	937	959	981	*003	10		22
998		50	7.98	003	025	048	070	092	114	136	157	179	201	223	0	**27**	22
998	**33**	0		223	245	267	289	311	333	355	377	398	420	442	50		22
998		10		442	464	486	508	529	551	573	595	616	638	660	40		22
998		20		660	682	703	725	747	768	790	812	833	855	876	30		22
998		30		876	898	920	941	963	984	*006	*027	*049	*070	*092	20		22
998		40	7.99	092	113	135	156	178	199	221	242	264	285	306	10		21
998		50		306	328	349	371	392	413	435	456	477	499	520	0	**26**	21
998	**34**	0		520	541	562	584	605	626	647	669	690	711	732	50		21
998		10		732	753	775	796	817	838	859	880	901	922	943	40		21
998		20		943	965	986	*007	*028	*049	*070	*091	*112	*133	*154	30		21
998		30	8.00	154	175	196	217	238	259	279	300	321	342	363	20		21
998		40		363	384	405	426	447	467	488	509	530	551	571	10		21
998		50		571	592	613	634	654	675	696	717	737	758	779	0	**25**	**21**
998	**35**	0		779	799	820	841	861	882	903	923	944	964	985	50		21
998		10		985	*006	*026	*047	*067	*088	*108	*129	*149	*170	*190	40		20
998		20	8.01	190	211	231	252	272	293	313	333	354	374	395	30		20
998		30		395	415	435	456	476	496	517	537	557	578	598	20		20
998		40		598	618	639	659	679	699	720	740	760	780	801	10		20
998		50		801	821	841	861	881	901	922	942	962	982	*002	0	**24**	20
998	**36**	0	8.02	002	022	042	062	082	102	123	143	163	183	203	50		20
998		10		203	223	243	263	283	303	323	343	362	382	402	40		20
998		20		402	422	442	462	482	502	522	542	561	581	601	30		20
998		30		601	621	641	661	680	700	720	740	759	779	799	20		20
998		40		799	819	838	858	878	898	917	937	957	976	996	10		20
998		50		996	*016	*035	*055	*074	*094	*114	*133	*153	*172	*192	0	**23**	20
997	**37**	0	8.03	192	212	231	251	270	290	309	329	348	368	387	50		20
997		10		387	407	426	446	465	484	504	523	543	562	581	40		19
997		20		581	601	620	640	659	678	698	717	736	756	775	30		19
997		30		775	794	813	833	852	871	891	910	929	948	967	20		19
997		40		967	987	*006	*025	*044	*063	*083	*102	*121	*140	*159	10		19
997		50	8.04	159	178	197	217	236	255	274	293	312	331	350	0	**22**	19
997	**38**	0		350	369	388	407	426	445	464	483	502	521	540	50		19
997		10		540	559	578	597	616	635	654	673	692	710	729	40		19
997		20		729	748	767	786	805	824	843	861	880	899	918	30		19
997		30		918	937	955	974	993	*012	*030	*049	*068	*087	*105	20		19
997		40	8.05	105	124	143	161	180	199	218	236	255	274	292	10		19
997		50		292	311	329	348	367	385	404	422	441	460	478	0	**21**	19
997	**39**	0		478	497	515	534	552	571	589	608	626	645	663	50		18
997		10		663	682	700	719	737	756	774	792	811	829	848	40		18
997		20		848	866	885	903	921	940	958	976	995	*013	*031	30		18
997		30	8.06	031	050	068	086	105	123	141	159	178	196	214	20		18
997		40		214	232	251	269	287	305	324	342	360	378	396	10		18
997		50		396	414	433	451	469	487	505	523	541	560	578	0	**20**	18
9.99				10″	9″	8″	7″	6″	5″	4″	3″	2″	1″	0″	″	′	d.

Log. Tan. 0°

′	″		0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″		d.
30	0	7.94	086	110	134	158	182	206	230	254	278	302	326	50	24
	10		326	350	374	398	422	446	470	494	518	542	566	40	24
	20		566	590	613	637	661	685	709	732	756	780	804	30	24
	30		804	827	851	875	899	922	946	970	993*017	*040	20		24
	40	7.95	040	064	088	111	135	158	182	205	229	252	276	10	24
	50		276	299	323	346	370	393	416	440	463	487	510	0 **29**	23
31	0		510	533	557	580	603	627	650	673	696	720	743	50	23
	10		743	766	789	812	836	859	882	905	928	951	974	40	23
	20		974	998*021	*044	*067	*090	*113	*136	*159	*182	*205	30		23
	30	7.96	205	228	251	274	297	320	343	365	388	411	434	20	23
	40		434	457	480	503	525	548	571	594	617	639	662	10	23
	50		662	685	708	730	753	776	798	821	844	866	889	0 **28**	23
32	0		889	911	934	957	979	*002	*024	*047	*069	*092	*114	50	22
	10	7.97	114	137	159	182	204	227	249	272	294	317	339	40	22
	20		339	361	384	406	428	451	473	495	518	540	562	30	22
	30		562	585	607	629	651	673	696	718	740	762	784	20	22
	40		784	807	829	851	873	895	917	939	961	983*005	10		22
	50	7.98	005	027	050	072	094	116	138	159	181	203	225	0 **27**	22
33	0		225	247	269	291	313	335	357	379	400	422	444	50	22
	10		444	466	488	510	531	553	575	597	618	640	662	40	22
	20		662	684	705	727	749	770	792	814	835	857	878	30	22
	30		878	900	922	943	965	986	*008	*029	*051	*073	*094	20	22
	40	7.99	094	116	137	158	180	201	223	244	266	287	308	10	21
	50		308	330	351	373	394	415	437	458	479	501	522	0 **26**	21
34	0		522	543	564	586	607	628	649	671	692	713	734	50	21
	10		734	755	777	798	819	840	861	882	903	925	946	40	21
	20		946	967	988*009	*030	*051	*072	*093	*114	*135	*156	30		21
	30	8.00	156	177	198	219	240	261	282	303	324	344	365	20	21
	40		365	386	407	428	449	470	490	511	532	553	574	10	21
	50		574	594	615	636	657	677	698	719	740	760	781	0 **25**	21
35	0		781	802	822	843	864	884	905	925	946	967	987	50	21
	10		987*008	*028	*049	*070	*090	*111	*131	*152	*172	*193	40		21
	20	8.01	193	213	234	254	274	295	315	336	356	377	397	30	20
	30		397	417	438	458	478	499	519	539	560	580	600	20	20
	40		600	621	641	661	682	702	722	742	762	783	803	10	20
	50		803	823	843	863	884	904	924	944	964	984*004	0 **24**		20
36	0	8.02	004	025	045	065	085	105	125	145	165	185	205	50	20
	10		205	225	245	265	285	305	325	345	365	385	405	40	20
	20		405	425	445	464	484	504	524	544	564	584	604	30	20
	30		604	623	643	663	683	703	722	742	762	782	801	20	20
	40		801	821	841	861	880	900	920	939	959	979	998	10	20
	50		998*018	*038	*057	*077	*097	*116	*136	*155	*175	*194	0 **23**		20
37	0	8.03	194	214	234	253	273	292	312	331	351	370	390	50	20
	10		390	409	429	448	468	487	506	526	545	565	584	40	19
	20		584	603	623	642	661	681	700	720	739	758	777	30	19
	30		777	797	816	835	855	874	893	912	932	951	970	20	19
	40		970	989*008	*028	*047	*066	*085	*104	*124	*143	*162	10		19
	50	8.04	162	181	200	219	238	257	276	296	315	334	353	0 **22**	19
38	0		353	372	391	410	429	448	467	486	505	524	543	50	19
	10		543	562	581	600	619	638	656	675	694	713	732	40	19
	20		732	751	770	789	808	826	845	864	883	902	921	30	19
	30		921	939	958	977	996	*014	*033	*052	*071	*089	*108	20	19
	40	8.05	108	127	146	164	183	202	220	239	258	276	295	10	19
	50		295	314	332	351	369	388	407	425	444	462	481	0 **21**	19
39	0		481	499	518	537	555	574	592	611	629	648	666	50	18
	10		666	685	703	722	740	758	777	795	814	832	851	40	18
	20		851	869	887	906	924	943	961	979	998*016	*034	30		18
	30	8.06	034	053	071	089	107	126	144	162	181	199	217	20	18
	40		217	235	254	272	290	308	326	345	363	381	399	10	18
	50		399	417	436	454	472	490	508	526	544	562	581	0 **20**	18
			10″	9″	8″	7″	6″	5″	4″	3″	2″	1″	0″	″ ′	d.

P. P.

	25	
1	2.5	
2	5.0	
3	7.5	
4	10.0	
5	12.5	
6	15.0	
7	17.5	
8	20.0	
9	22.5	

	24	**23**
1	2.4	2.3
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.6	20.7

	22
1	2.2
2	4.4
3	6.6
4	8.8
5	11.0
6	13.2
7	15.4
8	17.6
9	19.8

	21
1	2.1
2	4.2
3	6.3
4	8.4
5	10.5
6	12.6
7	14.7
8	16.8
9	18.9

	20	**19**
1	2.0	1.9
2	4.0	3.8
3	6.0	5.7
4	8.0	7.6
5	10.0	9.5
6	12.9	11.4
7	14.0	13.3
8	16.0	15.2
9	18.0	17.1

	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

9.99	′	″		0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″			d.
997	**40**	0	8.06	578	596	614	632	650	668	686	704	722	740	758	50		18
997		10		758	776	794	812	830	848	866	884	902	920	938	40		18
997		20		938	956	974	992	*010	*028	*046	*063	*081	*099	*117	30		18
997		30	8.07	117	135	153	171	189	206	224	242	260	278	295	20		18
997		40		295	313	331	349	367	384	402	420	438	455	473	10		18
997		50		473	491	509	526	544	562	579	597	615	632	650	0	**19**	18
997	**41**	0		650	668	685	703	721	738	756	773	791	809	826	50		18
997		10		626	844	861	879	896	914	932	949	967	984	*002	40		18
997		20	8.08	002	019	037	054	072	089	107	124	141	159	176	30		17
997		30		176	194	211	229	246	263	281	298	316	333	350	20		17
997		40		350	368	385	403	420	437	455	472	489	506	524	10		17
997		50		524	541	558	576	593	610	627	645	662	679	696	0	**18**	17
997	**42**	0		696	714	731	748	765	783	800	817	834	851	868	50		17
997		10		868	886	903	920	937	954	971	988	*006	*023	*040	40		17
997		20	8.09	040	057	074	091	108	125	142	159	176	193	210	30		17
997		30		210	227	244	261	278	295	312	329	346	363	380	20		17
997		40		380	397	414	431	448	465	482	499	516	533	550	10		17
997		50		550	567	583	600	617	634	651	668	685	701	718	0	**17**	17
997	**43**	0		718	735	752	769	786	802	819	836	853	870	886	50		17
997		10		886	903	920	937	953	970	987	*004	*020	*037	*054	40		17
997		20	8.10	054	070	087	104	120	137	154	170	187	204	220	30		17
997		30		220	237	254	270	287	303	320	337	353	370	386	20		17
996		40		386	403	420	436	453	469	486	502	519	535	552	10		17
996		50		552	568	585	601	618	634	651	667	684	700	717	0	**16**	16
996	**44**	0		717	733	750	766	782	799	815	832	848	864	881	50		16
996		10		881	897	914	930	946	963	979	995	*012	*028	*044	40		16
996		20	8.11	044	061	077	093	110	126	142	159	175	191	207	30		16
996		30		207	224	240	256	272	289	305	321	337	354	370	20		16
996		40		370	386	402	418	435	451	467	483	499	515	531	10		16
996		50		531	548	564	580	596	612	628	644	660	677	693	0	**15**	16
996	**45**	0		693	709	725	741	757	773	789	805	821	837	853	50		16
996		10		853	869	885	901	917	933	949	965	981	997	*013	40		16
996		20	8.12	013	029	045	061	077	093	109	125	141	157	172	30		16
996		30		172	188	204	220	236	252	268	284	300	315	331	20		16
996		40		331	347	363	379	395	410	426	442	458	474	489	10		16
996		50		489	505	521	537	553	568	584	600	616	631	647	0	**14**	16
996	**46**	0		647	663	679	694	710	726	741	757	773	788	804	50		16
996		10		804	820	836	851	867	882	898	914	929	945	961	40		16
996		20		961	976	992	*007	*023	*039	*054	*070	*085	*101	*117	30		16
996		30	8.13	117	132	148	163	179	194	210	225	241	256	272	20		16
996		40		272	287	303	318	334	349	365	380	396	411	427	10		16
996		50		427	442	458	473	489	504	519	535	550	566	581	0	**13**	15
996	**47**	0		581	596	612	627	643	658	673	689	704	719	735	50		15
996		10		735	750	765	781	796	811	827	842	857	873	888	40		15
996		20		888	903	919	934	949	964	980	995	*010	*025	*041	30		15
996		30	8.14	041	056	071	086	101	117	132	147	162	178	193	20		15
996		40		193	208	223	238	253	269	284	299	314	329	344	10		15
996		50		344	359	375	390	405	420	435	450	465	480	495	0	**12**	15
996	**48**	0		495	510	525	541	556	571	586	601	616	631	646	50		15
996		10		646	661	676	691	706	721	736	751	766	781	796	40		15
996		20		796	811	826	841	856	871	886	901	915	930	945	30		15
996		30		945	960	975	990	*005	*020	*035	*050	*065	*079	*094	20		15
996		40	8.15	094	109	124	139	154	169	183	198	213	228	243	10		15
996		50		243	258	272	287	302	317	332	346	361	376	391	0	**11**	15
996	**49**	0		391	406	420	435	450	465	479	494	509	523	538	50		15
996		10		538	553	568	582	597	612	626	641	656	670	685	40		15
996		20		685	700	714	729	744	758	773	788	802	817	832	30		15
995		30		832	846	861	875	890	905	919	934	948	963	978	20		15
995		40		978	992	*007	*021	*036	*050	*065	*079	*094	*109	*123	10		14
995		50	8.16	123	138	152	167	181	196	210	225	239	254	268	0	**10**	14
9.99				10″	9″	8″	7″	6″	5″	4″	3″	2″	1″	0″	″	′	d.

Log Tan. 0°

′	″		0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″		d.	P. P.
40	0	8.06	581	599	617	635	653	671	689	707	725	743	761	50	18	
	10		761	779	797	815	833	851	869	887	905	923	941	40	18	
	20		941	959	977	995	*013	*031	*049	*066	*084	*102	*120	30	18	
	30	8.07	120	138	156	174	192	209	227	245	263	281	298	20	18	
	40		298	316	334	352	370	387	405	423	441	458	476	10	18	**18**
	50		476	494	512	529	547	565	582	600	618	635	653	0	**19** 18	1 1.8 2 3.6
41	0		653	671	688	706	724	741	759	776	794	812	829	50	18	3 5.4 4 7.2
	10		829	847	864	882	900	917	935	952	970	987	*005	40	18	5 9.0
	20	8.08	005	022	040	057	075	092	110	127	145	162	180	30	18	6 10.8
	30		180	197	214	232	249	267	284	301	319	336	354	20	17	7 12.6
	40		354	371	388	406	423	440	458	475	492	510	527	10	17	8 14.4
	50		527	544	562	579	596	613	631	648	665	682	700	0	**18** 17	9 16.2
42	0		700	717	734	751	769	786	803	820	837	855	872	50	17	
	10		872	889	906	923	940	957	975	992	*009	*026	*043	40	17	
	20	8.09	043	060	077	094	111	128	146	163	180	197	214	30	17	
	30		214	231	248	265	282	299	316	333	350	367	384	20	17	**17**
	40		384	401	418	435	452	468	485	502	519	536	553	10	17	1 1.7
	50		553	570	587	604	621	637	654	671	688	705	722	0	**17** 17	2 3.4 3 5.1
43	0		722	739	755	772	789	806	823	839	856	873	890	50	17	4 6.8 5 8.5
	10		890	907	923	940	957	974	990	*007	*024	*040	*057	40	17	6 10.2
	20	8.10	057	074	091	107	124	141	157	174	191	207	224	30	17	7 11.9
	30		224	240	257	274	290	307	324	340	357	373	390	20	17	7 13.6
	40		390	407	423	440	456	473	489	506	522	539	555	10	16	9 15.3
	50		555	572	588	605	621	638	654	671	687	704	720	0	**16** 16	
44	0		720	737	753	770	786	802	819	835	852	868	884	50	16	
	10		884	901	917	934	950	966	983	999	*015	*032	*048	40	16	
	20	8.11	048	064	081	097	113	130	146	162	178	195	211	30	16	**16**
	30		211	227	244	260	276	292	309	325	341	357	373	20	16	1 1.6
	40		373	390	406	422	438	454	471	487	503	519	535	10	16	2 3.2
	50		535	551	567	584	600	616	632	648	664	680	696	0	**15** 16	3 4.8 4 6.4
45	0		696	712	729	745	761	777	793	809	825	841	857	50	16	5 8.0
	10		857	873	889	905	921	937	953	969	985	*001	*017	40	16	6 9.6 7 11.2
	20	8.12	017	033	049	065	081	097	113	129	144	160	176	30	16	8 12.8
	30		176	192	208	224	240	256	272	288	303	319	335	20	16	9 14.4
	40		335	351	367	383	398	414	430	446	462	478	493	10	16	
	50		493	509	525	541	556	572	588	604	620	635	651	0	**14** 16	
46	0		651	667	682	698	714	730	745	761	777	792	808	50	16	
	10		808	824	839	855	871	886	902	918	933	949	965	40	16	**15**
	20		965	980	996	*011	*027	*043	*058	*074	*089	*105	*121	30	16	1 1.5
	30	8.13	121	136	152	167	183	198	214	229	245	260	276	20	16	2 3.0
	40		276	291	307	322	338	353	369	384	400	415	431	10	16	3 4.5 4 6.0
	50		431	446	462	477	493	508	523	539	554	570	585	0	**13** 15	5 7.5
47	0		585	601	616	631	647	662	677	693	708	724	739	50	15	6 9.0 7 10.5
	10		739	754	770	785	800	816	831	846	861	877	892	40	15	8 12.0
	20		892	907	923	938	953	968	984	999	*014	*029	*045	30	15	9 13.5
	30	8.14	045	060	075	090	106	121	136	151	166	182	197	20	15	
	40		197	212	227	242	258	273	288	303	318	333	348	10	15	
	50		348	364	379	394	409	424	439	454	469	484	500	0	**12** 15	
48	0		500	515	530	545	560	575	590	605	620	635	650	50	15	**14**
	10		650	665	680	695	710	725	740	755	770	785	800	40	15	1 1.4
	20		800	815	830	845	860	875	890	905	920	935	950	30	15	2 2.8
	30		950	965	980	994	*009	*024	*039	*054	*069	*084	*099	20	15	3 4.2
	40	8.15	099	114	128	143	158	173	188	203	218	232	247	10	15	4 5.6 5 7.0
	50		247	262	277	292	306	321	336	351	366	380	395	0	**11** 15	6 8.4 7 9.8
49	0		395	410	425	439	454	469	484	498	513	528	543	50	15	8 11.2
	10		543	557	572	587	602	616	631	646	660	675	690	40	15	9 12.6
	20		690	704	719	734	748	763	778	792	807	822	836	30	15	
	30		836	851	865	880	895	909	924	938	953	968	982	20	15	
	40		982	997	*011	*026	*040	*055	*070	*084	*099	*113	*128	10	15	
	50	8.16	128	142	157	171	186	200	215	229	244	258	273	0	**10** 14	
			10″	9″	8″	7″	6″	5″	4″	3″	2″	1″	0″	″	′ d.	P. P.

89° Log Cot.

9.99	′	″		0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″			d.
995	**50**	0	8.16	268	283	297	311	326	340	355	369	384	398	413	50		14
995		10		413	427	441	456	470	485	499	513	528	542	557	40		14
995		20		557	571	585	600	614	628	643	657	672	686	700	30		14
995		30		700	715	729	743	757	772	786	800	815	829	843	20		14
995		40		843	858	872	886	900	915	929	943	957	972	986	10		14
995		50		986	*000	*014	*029	*043	*057	*071	*085	*100	*114	*128	0	**9**	14
995	**51**	0	8.17	128	142	156	171	185	199	213	227	241	256	270	50		14
995		10		270	284	298	312	326	340	355	369	383	397	411	40		14
995		20		411	425	439	453	467	481	495	510	524	538	552	30		14
995		30		552	566	580	594	608	622	636	650	664	678	692	20		14
995		40		692	706	720	734	748	762	776	790	804	818	832	10		14
995		50		832	846	86	874	888	902	916	930	943	957	971	0	**8**	14
995	**52**	0		971	985	99	*013	*027	*041	*055	*069	*082	*096	*110	50		14
995		10	8.18	110	124	13	152	166	180	193	207	221	235	249	40		14
995		20		249	263	27	290	304	318	332	345	359	373	387	30		14
995		30		387	401	414	428	442	456	469	483	497	511	524	20		14
995		40		524	538	552	566	579	593	607	621	634	648	662	10		14
995		50		662	675	689	703	716	730	744	757	771	785	798	0	**7**	14
995	**53**	0		798	812	826	839	853	867	880	894	908	921	935	50		14
995		10		935	948	962	976	989	*003	*016	*030	*044	*057	*071	40		14
995		20	8.19	071	084	098	111	125	139	152	166	179	193	206	30		14
995		30		206	220	233	247	260	274	287	301	314	328	341	20		14
995		40		341	355	368	382	395	409	422	436	449	463	476	10		14
995		50		476	489	503	516	530	543	557	570	583	597	610	0	**6**	13
995	**54**	0		610	624	637	650	664	677	691	704	717	731	744	50		13
995		10		744	757	771	784	797	811	824	837	851	864	877	40		13
995		20		877	891	904	917	931	944	957	971	984	997	*010	30		13
995		30	8.20	010	024	037	050	064	077	090	103	117	130	143	20		13
995		40		143	156	170	183	196	209	222	236	249	262	275	10		13
994		50		275	288	302	315	328	341	354	368	381	394	407	0	**5**	13
994	**55**	0		407	420	433	446	460	473	486	499	512	525	538	50		13
994		10		538	552	565	578	591	604	617	630	643	656	669	40		13
994		20		669	682	696	709	722	735	748	761	774	787	800	30		13
994		30		800	813	826	839	852	865	878	891	904	917	930	20		13
994		40		930	943	956	969	982	995	*008	*021	*034	*047	*060	10		13
994		50	8.21	060	073	086	099	112	125	138	151	164	177	189	0	**4**	13
994	**56**	0		189	202	215	228	241	254	267	280	293	306	319	50		13
994		10		319	331	344	357	370	383	396	409	422	434	447	40		13
994		20		447	460	473	486	499	511	524	537	550	563	576	30		13
994		30		576	588	601	614	627	640	652	665	678	691	703	20		13
994		40		703	716	729	742	754	767	780	793	805	818	831	10		13
994		50		831	844	856	869	882	895	907	920	933	945	958	0	**3**	13
994	**57**	0		958	971	983	996	*009	*022	*034	*047	*060	*072	*085	50		13
994		10	8.22	085	098	110	123	136	148	161	173	186	199	211	40		13
994		20		211	224	237	249	262	274	287	300	312	325	337	30		13
994		30		337	350	363	375	388	400	413	425	438	451	463	20		13
994		40		463	476	488	501	513	526	538	551	563	576	588	10		12
994		50		588	601	613	626	638	651	663	676	688	701	713	0	**2**	12
994	**58**	0		713	726	738	751	763	776	788	801	813	826	838	50		12
994		10		838	850	863	875	888	900	913	925	937	950	962	40		12
994		20		962	975	987	999	*012	*024	*037	*049	*061	*074	*086	30		12
994		30	8.23	086	098	111	123	136	148	160	173	185	197	210	20		12
994		40		210	222	234	247	259	271	284	296	308	321	333	10		12
994		50		333	345	357	370	382	394	407	419	431	443	456	0	**1**	12
994	**59**	0		456	468	480	492	505	517	529	541	554	566	578	50		12
994		10		578	590	603	615	627	639	652	664	676	688	700	40		12
994		20		700	713	725	737	749	761	773	786	798	810	822	30		12
993		30		822	834	846	859	871	883	895	907	919	931	944	20		12
993		40		944	956	968	980	992	*004	*016	*028	*041	*053	*065	10		12
993		50	8.24	065	077	089	101	113	125	137	149	161	173	186	0	**0**	12
9.99				10″	9″	8″	7″	6″	5″	4″	3″	2″	1″	0″	″	′	d.

Log Tan. 0°

′	″		0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″			d.	P. P.
50	0	8.16	273	287	302	316	331	345	359	374	388	403	417	50		14	
	10		417	432	446	460	475	489	504	518	533	547	561	40		14	
	20		561	576	590	604	619	633	647	662	676	691	705	30		14	
	30		705	719	734	748	762	776	791	805	819	834	848	20		14	
	40		848	862	877	891	905	919	934	948	962	976	991	10		14	
	50		991	*005	*019	*033	*048	*062	*076	*090	*104	*119	*133	0	**9**	14	
51	0	8.17	133	147	161	175	190	204	218	232	246	260	275	50		14	**15**
	10		275	289	303	317	331	345	359	373	388	402	416	40		14	1 1.5
	20		416	430	444	458	472	486	500	514	528	543	557	30		14	2 3.0
	30		557	571	585	599	613	627	641	655	669	683	697	20		14	3 4.5
	40		697	711	725	739	753	767	781	795	809	823	837	10		14	4 6.0
	50		837	851	865	879	893	907	921	934	948	962	976	0	**8**	14	5 7.5
52	0		976	990	*004	*018	*032	*046	*060	*074	*087	*101	*115	50		14	6 9.0
	10	8.18	115	129	143	157	171	185	198	212	226	240	254	40		14	7 10.5
	20		254	268	281	295	309	323	337	351	364	378	392	30		14	8 12.0
	30		392	406	419	433	447	461	475	488	502	516	530	20		14	9 13.5
	40		530	543	557	571	585	598	612	626	639	653	667	10		14	
	50		667	681	694	708	722	735	749	763	776	790	804	0	**7**	14	
53	0		804	817	831	845	858	872	886	899	913	926	940	50		14	
	10		940	954	967	981	994	*008	*022	*035	*049	*062	*076	40		14	
	20	8.19	076	090	103	117	130	144	157	171	184	198	211	30		14	**14**
	30		211	225	239	252	266	279	293	306	320	333	347	20		14	1 1.4
	40		347	360	374	387	401	414	427	441	454	468	481	10		13	2 2.8
	50		481	495	508	522	535	548	562	575	589	602	616	0	**6**	13	3 4.2
54	0		616	629	642	656	669	683	696	709	723	736	749	50		13	4 5.6
	10		749	763	776	789	803	816	830	843	856	870	883	40		13	5 7.0
	20		883	896	910	923	936	949	963	976	989	*003	*016	30		13	6 8.4
	30	8.20	016	029	042	056	069	082	096	109	122	135	149	20		13	7 9.8
	40		149	162	175	188	201	215	228	241	254	268	281	10		13	8 11.2
	50		281	294	307	320	334	347	360	373	386	399	413	0	**5**	13	9 12.6
55	0		413	426	439	452	465	478	491	505	518	531	544	50		13	
	10		544	557	570	583	596	610	623	636	649	662	675	40		13	
	20		675	688	701	714	727	740	753	767	780	793	806	30		13	
	30		806	819	832	845	858	871	884	897	910	923	936	20		13	**13**
	40		936	949	962	975	988	*001	*014	*027	*040	*053	*066	10		13	1 1.3
	50	8.21	066	079	092	105	118	131	144	156	169	182	195	0	**4**	13	2 2.6
56	0		195	208	221	234	247	260	273	286	299	311	324	50		13	3 3.9
	10		324	337	350	363	376	389	402	414	427	440	453	40		13	4 5.2
	20		453	466	479	492	504	517	530	543	556	569	581	30		13	5 6.5
	30		581	594	607	620	633	645	658	671	684	697	709	20		13	6 7.8
	40		709	722	735	748	760	773	786	799	811	824	837	10		13	7 9.1
	50		837	850	862	875	888	901	913	926	939	951	964	0	**3**	13	8 10.4
57	0		964	977	989	*002	*015	*028	*040	*053	*066	*078	*091	50		13	9 11.7
	10	8.22	091	104	116	129	142	154	167	179	192	205	217	40		13	
	20		217	230	243	255	268	280	293	306	318	331	343	30		13	
	30		343	356	369	381	394	406	419	431	444	457	469	20		13	
	40		469	482	494	507	519	532	544	557	569	582	595	10		13	**12**
	50		595	607	620	632	645	657	670	682	695	707	720	0	**2**	12	1 1.2
58	0		720	732	744	757	769	782	794	807	819	832	844	50		12	2 2.4
	10		844	857	869	881	894	906	919	931	944	956	968	40		12	3 3.6
	20		968	981	993	*006	*018	*030	*043	*055	*068	*080	*092	30		12	4 4.8
	30	8.23	092	105	117	130	142	154	167	179	191	204	216	20		12	5 6.0
	40		216	228	241	253	265	278	290	302	315	327	339	10		12	6 7.2
	50		339	352	364	376	388	401	413	425	438	450	462	0	**1**	12	7 8.4
59	0		462	474	487	499	511	523	536	548	560	572	585	50		12	8 9.6
	10		585	597	609	621	634	646	658	670	682	695	707	40		12	9 10.8
	20		707	719	731	743	756	768	780	792	804	816	829	30		12	
	30		829	841	853	865	877	889	902	914	926	938	950	20		12	
	40		950	962	974	987	999	*011	*023	*035	*047	*059	*071	10		12	
	50	8.24	071	083	096	108	120	132	144	156	168	180	192	0	**0**	12	
			10″	9″	8″	7″	6″	5″	4″	3″	2″	1″	0″	″	′	d.	P. P.

9.99	′		0″	10″	20″	30″	40″	50″	60″		d.
993	**0**	8.24	186	306	426	546	665	785	903	59	120
993	1		903	*022	*140	*258	*375	*493	*609	58	118
993	2	8.25	609	726	842	958	*074	*189	*304	57	116
993	3	8.26	304	419	533	648	761	875	988	56	114
992	4		988	*101	*214	*326	*438	*550	*661	55	112
992	5	8.27	661	773	883	994	*104	*215	*324	54	110
992	6	8.28	324	434	543	652	761	869	977	53	109
992	7		977	*085	*193	*300	*407	*514	*621	52	107
992	8	8.29	621	727	833	939	*044	*150	*255	51	106
991	9	8.30	255	359	464	568	672	776	879	**50**	104
991	**10**		879	983	*086	*188	*291	*393	*495	49	103
991	11	8.31	495	597	699	800	901	*002	*103	48	101
990	12	8.32	103	203	303	403	503	602	702	47	100
990	13		702	801	899	998	*096	*195	*292	46	98
990	14	8.33	292	390	488	585	682	779	875	45	97
990	15		875	972	*068	*164	*260	*355	*450	44	96
989	16	8.34	450	546	640	735	830	924	*018	43	95
989	17	8.35	018	112	206	299	392	485	578	42	93
989	18		578	671	764	856	948	*040	*131	41	92
989	19	8.36	131	223	314	405	496	587	678	**40**	91
988	**20**		678	768	858	948	*038	*128	*217	39	90
988	21	8.37	217	306	395	484	573	662	750	38	89
988	22		750	838	926	*014	*101	*189	*276	37	88
987	23	8.38	276	363	450	537	624	710	796	36	87
987	24		796	882	968	*054	*139	*225	*310	35	86
987	25	8.39	310	395	480	565	649	734	818	34	85
986	26		818	902	986	*070	*153	*237	*320	33	84
986	27	8.40	320	403	486	569	651	734	816	32	83
986	28		816	898	980	*062	*144	*225	*307	31	82
985	29	8.41	307	388	469	550	631	711	792	**30**	81
985	**30**		792	872	952	*032	*112	*192	*272	29	80
985	31	8.42	272	351	430	510	589	667	746	28	79
984	32		746	825	903	982	*060	*138	*216	27	78
984	33	8.43	216	293	371	448	526	603	680	26	77
984	34		680	757	834	910	987	*063	*139	25	76
983	35	8.44	139	216	292	367	443	519	594	24	76
983	36		594	669	745	820	895	969	*044	23	75
983	37	8.45	044	119	193	267	341	415	489	22	74
982	38		489	563	637	710	784	857	930	21	74
982	39		930	*003	*076	*149	*222	*294	*366	**20**	73
982	**40**	8.46	366	439	511	583	655	727	799	19	72
981	41		799	870	942	*013	*084	*155	*226	18	71
981	42	8.47	226	297	368	439	509	580	650	17	71
981	43		650	720	790	860	930	*000	*069	16	70
980	44	8.48	069	139	208	278	347	416	485	15	69
980	45		485	554	622	691	760	828	896	14	68
979	46		896	965	*033	*101	*169	*236	*304	13	68
979	47	8.49	304	372	439	506	574	641	708	12	67
979	48		708	775	842	908	975	*042	*108	11	67
978	49	8.50	108	174	241	307	373	439	504	**10**	66
978	**50**		504	570	636	701	767	832	897	9	66
977	51		897	963	*028	*092	*157	*222	*287	8	65
977	52	8.51	287	351	416	480	544	609	673	7	64
977	53		673	737	801	864	928	992	*055	6	64
976	54	8.52	055	119	182	245	308	371	434	5	63
976	55		434	497	560	623	685	748	810	4	63
975	56		810	872	935	997	*059	*121	*183	3	62
975	57	8.53	183	245	306	368	429	491	552	2	62
974	58		552	614	675	736	797	858	919	1	61
974	59		919	979	*040	*101	*161	*222	*282	**0**	60
9.99			60″	50″	40″	30″	20″	10″	0″	′	d.

P. P.

	120	**119**	**118**
1	12.0	11.9	11.8
2	24.0	23.8	23.6
3	36.0	35.7	35.4
4	48.0	47.6	47.2
5	60.0	59.5	59.0
6	72.0	71.4	70.8
7	84.0	83.3	82.6
8	96.0	95.2	94.4
9	108.0	107.1	106.2

	117	**116**	**115**
1	11.7	11.6	11.5
2	23.4	23.2	23.0
3	35.1	34.8	34.5
4	46.8	46.4	46.0
5	58.5	58.0	57.5
6	70.2	69.6	69.0
7	81.9	81.2	80.5
8	93.6	92.8	92.0
9	105.3	104.4	103.5

	114	**113**	**112**	**111**
1	11.4	11.3	11.2	11.1
2	22.8	22.6	22.4	22.2
3	34.2	33.9	33.6	33.3
4	45.6	45.2	44.8	44.4
5	57.0	56.5	56.0	55.5
6	68.4	67.8	67.2	66.6
7	79.8	79.1	78.4	77.7
8	91.2	90.4	89.6	88.8
9	102.6	101.7	100.8	99.9

	110	**109**	**108**	**107**
1	11.0	10.9	10.8	10.7
2	22.0	21.8	21.6	21.4
3	33.0	32.7	32.4	32.1
4	44.0	43.6	43.2	42.8
5	55.0	54.5	54.0	53.5
6	66.0	65.4	64.8	64.2
7	77.0	76.3	75.6	74.9
8	88.0	87.2	86.4	85.6
9	99.0	98.1	97.2	96.3

	106	**105**	**104**	**103**
1	10.6	10.5	10.4	10.3
2	21.2	21.0	20.8	20.6
3	31.8	31.5	31.2	30.9
4	42.4	42.0	41.6	41.2
5	53.0	52.5	52.0	51.5
6	63.6	63.0	62.4	61.8
7	74.2	73.5	72.8	72.1
8	84.8	84.0	83.2	82.4
9	95.4	94.5	93.6	92.7

	102	**101**	**100**	**99**
1	10.2	10.1	10.0	9.9
2	20.4	20.2	20.0	19.8
3	30.6	30.3	30.0	29.7
4	40.8	40.4	40.0	39.6
5	51.0	50.5	50.0	49.5
6	61.2	60.6	60.0	59.4
7	71.4	70.7	70.0	69.3
8	81.6	80.8	80.0	79.2
9	91.8	90.9	90.0	89.1

	98	**97**	**96**	**95**
1	9.8	9.7	9.6	9.5
2	19.6	19.4	19.2	19.0
3	29.4	29.1	28.8	28.5
4	39.2	38.8	38.4	38.0
5	49.0	48.5	48.0	47.5
6	58.8	58.2	57.6	57.0
7	68.6	67.9	67.2	66.5
8	78.4	77.6	76.8	76.0
9	88.2	87.3	86.4	85.5

P. P.

Log Tan. 1°

′	0″	10″	20″	30″	40″	50″	60″		d.
0	8.24 192	313	433	553	672	791	910	59	120
1	910	*029	*147	*265	*382	*500	*616	58	118
2	8.25 616	733	849	965	*081	*196	*312	57	116
3	8.26 312	426	541	655	769	882	996	56	114
4	996	*109	*221	*334	*446	*558	*669	55	112
5	8.27 669	780	891	*002	*112	*223	*332	54	110
6	8.28 332	442	551	660	769	877	986	53	109
7	986	*094	*201	*309	*416	*523	*629	52	107
8	8.29 629	736	842	947	*053	*158	*263	51	106
9	8.30 263	368	473	577	681	785	888	**50**	104
10	888	992	*095	*198	*300	*403	*505	49	103
11	8.31 505	606	708	809	911	*012	*112	48	101
12	8.32 112	213	313	413	513	612	711	47	100
13	711	810	909	*008	*106	*205	*302	46	98
14	8.33 302	400	498	595	692	789	886	45	97
15	886	982	*078	*174	*270	*366	*461	44	96
16	8.34 461	556	651	746	840	935	*029	43	95
17	8.35 029	123	217	310	403	497	590	42	94
18	590	682	775	867	959	*051	*143	41	92
19	8.36 143	235	326	417	508	599	689	**40**	91
20	689	780	870	960	*050	*140	*229	39	90
21	8.37 229	318	408	497	585	674	762	38	89
22	762	850	938	*026	*114	*202	*289	37	88
23	8.38 289	376	463	550	636	723	809	36	87
24	809	895	981	*067	*153	*238	*323	35	86
25	8.39 323	408	493	578	663	747	832	34	85
26	832	916	*000	*083	*167	*250	*334	33	84
27	8.40 334	417	500	583	665	748	830	32	83
28	830	913	995	*077	*158	*240	*321	31	82
29	8.41 321	403	484	565	646	726	807	**30**	81
30	807	887	967	*048	*127	*207	*287	29	80
31	8.42 287	366	446	525	604	683	762	28	79
32	762	840	919	997	*075	*154	*232	27	78
33	8.43 232	309	387	464	542	619	696	26	77
34	696	773	850	927	*003	*080	*156	25	77
35	8.44 156	232	308	384	460	536	611	24	76
36	611	686	762	837	912	987	*061	23	75
37	8.45 061	136	210	285	359	433	507	22	74
38	507	581	655	728	802	875	948	21	74
39	948	*021	*094	*167	*240	*312	*385	**20**	73
40	8.46 385	457	529	602	674	745	817	19	72
41	817	889	960	*032	*103	*174	*245	18	71
42	8.47 245	316	387	458	528	599	669	17	71
43	669	740	810	880	950	*020	*089	16	70
44	8.48 089	159	228	298	367	436	505	15	69
45	505	574	643	711	780	849	917	14	69
46	917	985	*053	*121	*189	*257	*325	13	68
47	8.49 325	393	460	528	595	662	729	12	67
48	729	796	863	930	997	*063	*130	11	67
49	8.50 130	196	263	329	395	461	527	**10**	66
50	527	593	658	724	789	855	920	9	66
51	920	985	*050	*115	*180	*245	*310	8	65
52	8.51 310	374	439	503	568	632	696	7	64
53	696	760	824	888	952	*015	*079	6	64
54	8.52 079	143	206	269	332	396	459	5	63
55	459	522	584	647	710	772	835	4	63
56	835	897	960	*022	*084	*146	*208	3	62
57	8.53 208	270	332	393	455	516	578	2	62
58	578	639	700	762	823	884	945	1	61
59	945	*005	*066	*127	*187	*248	*308	**0**	60
	60″	50″	40″	30″	20″	10″	0″	′	d.

P. P.

	94	93	92	91	90
1	9.4	9.3	9.2	9.1	9.0
2	18.8	18.6	18.4	18.2	18.0
3	28.2	27.9	27.6	27.3	27.0
4	37.6	37.2	36.8	36.4	36.0
5	47.0	46.5	46.0	45.5	45.0
6	56.4	55.8	55.2	54.6	54.0
7	65.8	65.1	64.4	63.7	63.0
8	75.2	74.4	73.6	72.8	72.0
9	84.6	83.7	82.8	81.9	81.0

	89	88	87	86	85
1	8.9	8.8	8.7	8.6	8.5
2	17.8	17.6	17.4	17.2	17.0
3	26.7	26.4	26.1	25.8	25.5
4	35.6	35.2	34.8	34.4	34.0
5	44.5	44.0	43.5	43.0	42.5
6	53.4	52.8	52.2	51.6	51.0
7	62.3	61.6	60.9	60.2	59.5
8	71.2	70.4	69.6	68.8	68.0
9	80.1	79.2	78.3	77.4	76.5

	84	83	82	81	80
1	8.4	8.3	8.2	8.1	8.0
2	16.8	16.6	16.4	16.2	16.0
3	25.2	24.9	24.6	24.3	24.0
4	33.6	33.2	32.8	32.4	32.0
5	42.0	41.5	41.0	40.5	40.0
6	50.4	49.8	49.2	48.6	48.0
7	58.8	58.1	57.4	56.7	56.0
8	67.2	66.4	65.6	64.8	64.0
9	75.6	74.7	73.8	72.9	72.0

	79	78	77	76	75
1	7.9	7.8	7.7	7.6	7.5
2	15.8	15.6	15.4	15.2	15.0
3	23.7	23.4	23.1	22.8	22.5
4	31.6	31.2	30.8	30.4	30.0
5	39.5	39.0	38.5	38.0	37.5
6	47.4	46.8	46.2	45.6	45.0
7	55.3	54.6	53.9	53.2	52.5
8	63.2	62.4	61.6	60.8	60.0
9	71.1	70.2	69.3	68.4	67.5

	74	73	72	71	70
1	7.4	7.3	7.2	7.1	7.0
2	14.8	14.6	14.4	14.2	14.0
3	22.2	21.9	21.6	21.3	21.0
4	29.6	29.2	28.8	28.4	28.0
5	37.0	36.5	36.0	35.5	35.0
6	44.4	43.8	43.2	42.6	42.0
7	51.8	51.1	50.4	49.7	49.0
8	59.2	58.4	57.6	56.8	56.0
9	66.6	65.7	64.8	63.9	63.0

	69	68	67	66	65
1	6.9	6.8	6.7	6.6	6.5
2	13.8	13.6	13.4	13.2	13.0
3	20.7	20.4	20.1	19.8	19.5
4	27.6	27.2	26.8	26.4	26.0
5	34.5	34.0	33.5	33.0	32.5
6	41.4	40.8	40.2	39.6	39.0
7	48.3	47.6	46.9	46.2	45.5
8	55.2	54.4	53.6	52.8	52.0
9	62.1	61.2	60.3	59.4	58.5

	64	63	62	61	60
1	6.4	6.3	6.2	6.1	6.0
2	12.8	12.6	12.4	12.2	12.0
3	19.2	18.9	18.6	18.3	18.0
4	25.6	25.2	24.8	24.4	24.0
5	32.0	31.5	31.0	30.5	30.0
6	38.4	37.8	37.2	36.6	36.0
7	44.8	44.1	43.4	42.7	42.0
8	51.2	50.4	49.6	48.8	48.0
9	57.6	56.7	55.8	54.9	54.0

P. P.

9.99	′	0″	10″	20″	30″	40″	50″	60″			d.	P. P.
974	**0**	8.54 282	342	402	462	522	582	642	59	973	60	
973	1	642	702	762	821	881	940	999	58	973	60	
973	2	999	*059	*118	*177	*236	*295	*354	57	972	59	
972	3	8.55 354	413	471	530	589	647	705	56	972	58	**61**
972	4	705	764	822	880	938	996	*054	55	971	58	1 6.1
971	5	8.56 054	112	170	227	285	342	400	54	971	58	2 12.2 3 18.3 4 24.4
971	6	400	457	515	572	629	686	743	53	970	57	5 30.5 6 36.6
970	7	743	800	857	914	970	*027	*084	52	970	57	7 42.7 8 48.8
970	8	8.57 084	140	196	253	309	365	421	51	969	56	9 54.9
969	9	421	477	533	589	645	701	757	**50**	969	56	
969	**10**	757	812	868	923	979	*034	*089	49	968	55	
968	11	8.58 089	144	200	255	310	364	419	48	968	55	
968	12	419	474	529	583	638	693	747	47	967	55	**60**
967	13	747	801	856	910	964	*018	*072	46	967	54	1 6.0
967	14	8.59 072	126	180	234	288	341	395	45	967	54	2 12.0 3 18.0
967	15	395	448	502	555	609	662	715	44	966	53	4 24.0 5 30.0
966	16	715	768	821	874	927	980	*033	43	966	53	6 36.0 7 42.0
966	17	8.60 033	086	139	191	244	296	349	42	965	53	8 48.0 9 54.0
965	18	349	401	454	506	558	610	662	41	964	52	
964	19	662	714	766	818	870	922	973	**40**	964	52	
964	**20**	973	*025	*077	*128	*180	*231	*282	39	963	52	
963	21	8.61 282	334	385	436	487	538	589	38	963	51	**59**
963	22	589	640	691	742	792	843	894	37	962	51	1 5.9
962	23	894	944	995	*045	*096	*146	*196	36	962	50	2 11.8 3 17.7
962	24	8.62 196	246	297	347	397	447	497	35	961	50	4 23.6 5 29.5
961	25	497	546	596	646	696	745	795	34	961	50	6 35.4 7 41.3
961	26	795	844	894	943	993	*042	*091	33	960	49	8 47.2 9 53.1
960	27	8.63 091	140	189	238	288	336	385	32	960	49	
960	28	385	434	483	532	580	629	678	31	959	49	
959	29	678	726	775	823	871	920	968	**30**	959	48	
959	**30**	968	*016	*064	*112	*160	*208	*256	29	958	48	**58**
958	31	8.64 256	304	352	400	448	495	543	28	958	48	1 5.8
958	32	543	590	638	685	733	780	827	27	957	47	2 11.6 3 17.4
957	33	827	875	922	969	*016	*063	*110	26	956	47	4 23.2 5 29.0
956	34	8.65 110	157	204	251	298	344	391	25	956	47	6 34.8 7 40.6
956	35	391	438	484	531	577	624	670	24	955	46	8 46.4 9 52.2
955	36	670	717	763	809	855	901	947	23	955	46	
955	37	947	994	*040	*085	*131	*177	*223	22	954	46	
954	38	8.66 223	269	314	360	406	451	497	21	954	46	
954	39	497	542	588	633	678	724	769	**20**	953	45	
953	**40**	769	814	859	904	949	994	*039	19	952	45	**57**
952	41	8.67 039	084	129	174	219	263	308	18	952	45	1 5.7
952	42	308	353	397	442	486	531	575	17	951	44	2 11.4 3 17.1
951	43	575	619	664	708	752	796	841	16	951	44	4 22.8 5 28.5
951	44	841	885	929	973	*017	*060	*104	15	950	44	6 34.2 7 39.9
950	45	8.68 104	148	192	236	279	323	367	14	949	44	8 45.6 9 51.3
949	46	367	410	454	497	540	584	627	13	949	43	
949	47	627	670	714	757	800	843	886	12	948	43	
948	48	886	929	972	*015	*058	*101	*144	11	948	43	
948	49	8.69 144	187	229	272	315	357	400	**10**	947	43	**56**
947	**50**	400	442	485	527	570	612	654	9	946	42	1 5.6
946	51	654	697	739	781	823	865	907	8	946	42	2 11.2 3 16.8
946	52	907	949	991	*033	*075	*117	*159	7	945	42	4 22.4 5 28.0
945	53	8.70 159	201	242	284	326	367	409	6	944	42	6 33.6 7 39.2
944	54	409	451	492	534	575	616	658	5	944	42	8 44.8 9 50.4
944	55	658	699	740	781	823	864	905	4	943	41	
943	56	905	946	987	*028	*069	*110	*151	3	942	41	
942	57	8.71 151	192	232	273	314	355	395	2	942	41	
942	58	395	436	476	517	557	598	638	1	941	40	
941	59	638	679	719	759	800	840	880	**0**	940	40	
		60″	50″	40″	30″	20″	10″	0″	′	9.99	d.	P. P.

Log Tan. 2°

′	0″	10″	20″	30″	40″	50″	60″		d.
0	8.54 308	369	429	489	549	609	669	59	60
1	669	729	789	848	908	967	*027	58	60
2	8.55 027	086	145	205	264	323	382	57	59
3	382	441	499	558	617	675	734	56	59
4	734	792	850	909	967	*025	*083	55	58
5	8.56 083	141	199	256	314	372	429	54	58
6	429	487	544	601	659	716	773	53	57
7	773	830	887	944	*000	*057	*114	52	57
8	8.57 114	170	227	283	340	396	452	51	56
9	452	508	564	620	676	732	788	**50**	56
10	788	843	899	955	*010	*065	*121	49	56
11	8.58 121	176	231	286	341	396	451	48	55
12	451	506	561	616	670	725	779	47	55
13	779	834	888	943	997	*051	*105	46	54
14	8.59 105	159	213	267	321	375	428	45	54
15	428	482	536	589	642	696	749	44	54
16	749	802	856	909	962	*015	*068	43	53
17	8.60 068	121	173	226	279	331	384	42	53
18	384	436	489	541	593	646	698	41	52
19	698	750	802	854	906	958	*009	**40**	52
20	8.61 009	061	113	164	216	267	319	39	52
21	319	370	422	473	524	575	626	38	51
22	626	677	728	779	830	881	931	37	51
23	931	982	*033	*083	*134	*184	*234	36	50
24	8.62 234	285	335	385	435	485	535	35	50
25	535	585	635	685	735	784	834	34	50
26	834	884	933	983	*032	*081	*131	33	50
27	8.63 131	180	229	278	328	377	426	32	49
28	426	475	523	572	621	670	718	31	49
29	718	767	816	864	913	961	*009	**30**	48
30	8.64 009	058	106	154	202	250	298	29	48
31	298	346	394	442	490	538	585	28	48
32	585	633	681	728	776	823	870	27	48
33	870	918	965	*012	*060	*107	*154	26	47
34	8.65 154	201	248	295	342	388	435	25	47
35	435	482	529	575	622	668	715	24	47
36	715	761	808	854	900	947	993	23	46
37	993	*039	*085	*131	*177	*223	*269	22	46
38	8.66 269	315	361	406	452	498	543	21	46
39	543	589	634	680	725	771	816	**20**	46
40	816	861	906	952	997	*042	*087	19	45
41	8.67 087	132	177	222	267	312	356	18	45
42	356	401	446	490	535	579	624	17	45
43	624	668	713	757	801	846	890	16	44
44	890	934	978	*022	*066	*110	*154	15	44
45	8.68 154	198	242	286	330	373	417	14	44
46	417	461	504	548	592	635	678	13	44
47	678	722	765	808	852	895	938	12	43
48	938	981	*024	*067	*110	*153	*196	11	43
49	8.69 196	239	282	325	368	410	453	**10**	43
50	453	496	538	581	623	666	708	9	42
51	708	750	793	835	877	920	962	8	42
52	962	*004	*046	*088	*130	*172	*214	7	42
53	8.70 214	256	298	339	381	423	465	6	42
54	465	506	548	589	631	673	714	5	42
55	714	755	797	838	879	921	962	4	41
56	962	*003	*044	*085	*126	*167	*208	3	41
57	8.71 208	249	290	331	372	413	453	2	41
58	453	494	535	575	616	657	697	1	41
59	697	738	778	819	859	899	940	**0**	40
	60″	50″	40″	30″	20″	10″	0″	′	d.

P. P.

	55	54	53
1	5.5	5.4	5.3
2	11.0	10.8	10.6
3	16.5	16.2	15.9
4	22.0	21.6	21.2
5	27.5	27.0	26.5
6	33.0	32.4	31.8
7	38.5	37.8	37.1
8	44.0	43.2	42.4
9	49.5	48.6	47.7

	52	51
1	5.2	5.1
2	10.4	10.2
3	15.6	15.3
4	20.8	20.4
5	26.0	25.5
6	31.2	30.6
7	36.4	35.7
8	41.6	40.8
9	46.8	45.9

	50	49	48
1	5.0	4.9	4.8
2	10.0	9.8	9.6
3	15.0	14.7	14.4
4	20.0	19.6	19.2
5	25.0	24.5	24.0
6	30.0	29.4	28.8
7	35.0	34.3	33.6
8	40.0	39.2	38.4
9	45.0	44.1	43.2

	47	46	45
1	4.7	4.6	4.5
2	9.4	9.2	9.0
3	14.1	13.8	13.5
4	18.8	18.4	18.0
5	23.5	23.0	22.5
6	28.2	27.6	27.0
7	32.9	32.2	31.5
8	37.6	36.8	36.0
9	42.3	41.4	40.5

	44	43
1	4.4	4.3
2	8.8	8.6
3	13.2	12.9
4	17.6	17.2
5	22.0	21.5
6	26.4	25.8
7	30.8	30.1
8	35.2	34.4
9	39.6	38.7

	42	41	40
1	4.2	4.1	4.0
2	8.4	8.2	8.0
3	12.6	12.3	12.0
4	16.8	16.4	16.0
5	21.0	20.5	20.0
6	25.2	24.6	24.0
7	29.4	28.7	28.0
8	33.6	32.8	32.0
9	37.8	36.9	36.0

P. P.

9.99	′	0″	10″	20″	30″	40″	50″	60″			d.
940	**0**	8.71 880	920	960	*000	*040	*080	*120	59	940	40
940	1	8.72 120	160	200	240	280	320	359	58	939	40
939	2	359	399	439	478	518	558	597	57	938	40
938	3	597	637	676	716	755	794	834	56	938	40
938	4	834	873	912	951	991	*030	*069	55	937	39
937	5	8.73 069	108	147	186	225	264	303	54	936	39
936	6	303	342	380	419	458	497	535	53	936	39
936	7	535	574	613	651	690	728	767	52	935	39
935	8	767	805	844	882	920	959	997	51	934	38
934	9	997	*035	*073	*112	*150	*188	*226	**50**	934	38
934	**10**	8.74 226	264	302	340	378	416	454	49	933	38
933	11	454	491	529	567	605	642	680	48	932	38
932	12	680	718	755	793	831	868	906	47	932	38
932	13	906	943	980	*018	*055	*092	*130	46	931	37
931	14	8.75 130	167	204	241	279	316	353	45	930	37
930	15	353	390	427	464	501	538	575	44	929	37
929	16	575	612	648	685	722	759	795	43	929	37
929	17	795	832	869	905	942	979	*015	42	928	37
928	18	8.76 015	052	088	125	161	197	234	41	927	36
927	19	234	270	306	343	379	415	451	**40**	926	36
926	**20**	451	487	523	559	595	631	667	39	926	36
926	31	667	703	739	775	811	847	883	38	925	36
925	22	883	919	954	990	*026	*061	*097	37	924	36
924	23	8.77 097	133	168	204	239	275	310	36	923	36
923	24	310	346	381	416	452	487	522	35	923	35
923	25	522	558	593	628	663	698	733	34	922	35
922	26	733	768	803	838	873	908	943	33	921	35
921	27	943	978	*013	*048	*083	*118	*152	32	920	35
920	28	8.78 152	187	222	257	291	326	360	31	920	35
920	29	360	395	430	464	499	533	568	**30**	919	35
919	**30**	568	602	636	671	705	739	774	29	918	34
918	21	774	808	842	876	910	945	979	28	917	34
917	32	979	*013	*047	*081	*115	*149	*183	27	917	34
917	33	8.79 183	217	251	284	318	352	386	26	916	34
916	34	386	420	453	487	521	555	588	25	915	34
915	35	588	622	655	689	722	756	789	24	914	34
914	36	789	823	856	890	923	956	990	23	913	34
913	37	990	*023	*056	*090	*123	*156	*189	22	913	33
913	38	8.80 189	222	255	289	322	355	388	21	912	33
912	39	388	421	454	487	519	552	585	**20**	911	33
911	**40**	585	618	651	684	716	749	782	19	910	33
910	41	782	815	847	880	913	945	978	18	909	33
909	42	978	*010	*043	*075	*108	*140	*173	17	909	32
909	43	8.81 173	205	237	270	302	334	367	16	908	32
908	44	367	399	431	463	496	528	560	15	907	32
907	45	560	592	624	656	688	720	752	14	906	32
906	46	752	784	816	848	880	912	944	13	905	32
905	47	944	975	*007	*039	*071	*103	*134	12	904	32
904	48	8.82 134	166	198	229	261	292	324	11	904	32
904	49	324	356	387	419	450	482	513	**10**	903	32
903	**50**	513	544	576	607	639	670	701	9	902	31
902	51	701	732	764	795	826	857	888	8	901	31
901	52	888	920	951	982	*013	*044	*075	7	900	31
900	53	8.83 075	106	137	168	199	230	261	6	899	31
899	54	261	292	322	353	384	415	446	5	898	31
898	55	446	476	507	538	568	599	630	4	898	31
898	56	630	660	691	721	752	783	813	3	897	30
897	57	813	844	874	904	935	965	996	2	896	30
896	58	996	*026	*056	*087	*117	*147	*177	1	895′	30
895	59	8.84 177	208	238	268	298	328	358	**0**	894	30
		60″	50″	40″	30″	20″	10″	0″	′	9.99	d.

P. P.

	40	39
1	4.0	3.9
2	8.0	7.8
3	12.0	11.7
4	16.0	15.6
5	20.0	19.5
6	24.0	23.4
7	28.0	27.3
8	32.0	31.2
9	36.0	35.1

	38	37
1	3.8	3.7
2	7.6	7.4
3	11.4	11.1
4	15.2	14.8
5	19.0	18.5
6	22.8	22.2
7	26.6	25.9
8	30.4	29.6
9	34.2	33.3

	36
1	3.6
2	7.2
3	10.8
4	14.4
5	18.0
6	21.6
7	25.2
8	28.8
9	32.4

	35	34
1	3.5	3.4
2	7.0	6.8
3	10.5	10.2
4	14.0	13.6
5	17.5	17.0
6	21.0	20.4
7	24.5	23.8
8	28.0	27.2
9	31.5	30.6

	33	32
1	3.3	3.2
2	6.6	6.4
3	9.9	9.6
4	13.2	12.8
5	16.5	16.0
6	19.8	19.2
7	23.1	22.4
8	26.4	25.6
9	29.7	28.8

	31	30
1	3.1	3.0
2	6.2	6.0
3	9.3	9.0
4	12.4	12.0
5	15.5	15.0
6	18.6	18.0
7	21.7	21.0
8	24.8	24.0
9	27.9	27.0

Log Tan. 3°

′	0″	10″	20″	30″	40″	50″	60″		d.
0	8.71 940	980	*020	*060	*100	*141	*181	59	40
1	8.72 181	221	261	301	341	380	420	58	40
2	420	460	500	540	579	619	659	57	40
3	659	698	738	777	817	856	896	56	40
4	896	935	975	*014	*053	*093	*132	55	39
5	8.73 132	171	210	249	288	327	366	54	39
6	366	405	444	483	522	561	600	53	39
7	600	638	677	716	754	793	832	52	39
8	832	870	909	947	986	*024	*063	51	38
9	8.74 063	101	139	178	216	254	292	**50**	38
10	292	330	369	407	445	483	521	49	38
11	521	559	597	634	672	710	748	48	38
12	748	786	823	861	899	936	974	47	38
13	974	*012	*049	*087	*124	*162	*199	46	38
14	8.75 199	236	274	311	348	385	423	45	37
15	423	460	497	534	571	608	645	44	37
16	645	682	719	756	793	830	867	43	37
17	867	904	940	977	*014	*051	*087	42	37
18	8.76 087	124	160	197	233	270	306	41	36
19	306	343	379	416	452	488	525	**40**	36
20	525	561	597	633	669	706	742	39	36
21	742	778	814	850	886	922	958	38	36
22	958	994	*030	*065	*101	*137	*173	**67**	36
23	8.77 173	208	244	280	315	351	387	36	36
24	387	422	458	493	529	564	600	35	36
25	600	635	670	706	741	776	811	34	35
26	811	847	882	917	952	987	*022	33	35
27	8.78 022	057	092	127	162	197	232	32	35
28	232	267	302	337	371	406	441	31	35
29	441	475	510	545	579	614	649	**30**	35
30	649	683	718	752	787	821	855	29	34
31	855	890	924	958	993	*027	*061	28	34
32	8.79 061	096	130	164	198	232	266	27	34
33	266	300	334	368	402	436	470	26	34
34	470	504	538	572	606	639	673	25	34
35	673	707	741	774	808	842	875	24	34
36	875	909	942	976	*009	*043	*076	23	34
37	8.80 076	110	143	177	210	243	277	22	34
38	277	310	343	376	409	443	476	21	33
39	476	509	542	575	608	641	674	**20**	33
40	674	707	740	773	806	839	872	19	33
41	872	905	937	970	*003	*036	*068	18	33
42	8.81 068	101	134	166	199	232	264	17	33
43	264	297	329	362	394	427	459	16	32
44	459	491	524	556	588	621	653	15	32
45	653	685	717	750	782	814	846	14	32
46	846	878	910	942	974	*006	*038	13	32
47	8.82 038	070	102	134	166	198	230	12	32
48	230	262	293	325	357	389	420	11	32
49	420	452	484	515	547	579	610	**10**	32
50	610	642	673	705	736	768	799	9	32
51	799	831	862	893	925	956	987	8	31
52	987	*019	*050	*081	*112	*144	*175	7	31
53	8.83 175	206	237	268	299	330	361	6	31
54	361	392	423	454	485	516	547	5	31
55	547	578	609	640	671	701	732	4	31
56	732	763	794	824	855	886	916	3	31
57	916	947	978	*008	*039	*069	*100	2	31
58	8.84 100	130	161	191	222	252	282	1	30
59	282	313	343	374	404	434	464	**0**	30
	60″	50″	40″	30″	20″	10″	0″	′	d.

P. P.

	41	**40**
1	4.1	4.0
2	8.2	8.0
3	12.3	12.0
4	16.4	16.0
5	20.5	20.0
6	24.6	24.0
7	28.7	28.0
8	32.8	32.0
9	36.9	36.0
	39	**38**
1	3.9	3.8
2	7.8	7.6
3	11.7	11.4
4	15.6	15.2
5	19.5	19.0
6	23.4	22.8
7	27.3	26.6
8	31.2	30.4
9	35.1	34.2
	37	**36**
1	3.7	3.6
2	7.4	7.2
3	11.1	10.8
4	14.8	14.4
5	18.5	18.0
6	22.2	21.6
7	25.9	25.2
8	29.6	28.8
9	33.3	32.4
	35	**34**
1	3.5	3.4
2	7.0	6.8
3	10.5	10.2
4	14.0	13.6
5	17.5	17.0
6	21.0	20.4
7	24.5	23.8
8	28.0	27.2
9	31.5	30.6
	33	**32**
1	3.3	3.2
2	6.6	6.4
3	9.9	9.6
4	13.2	12.8
5	16.5	16.0
6	19.8	19.2
7	23.1	22.4
8	26.4	25.6
9	29.7	28.8
	31	**30**
1	3.1	3.0
2	6.2	6.0
3	9.3	9.0
4	12.4	12.0
5	15.5	15.0
6	18.6	18.0
7	21.7	21.0
8	24.8	24.0
9	27.9	27.0

P. P.

L. Cos. Log Sin. 4°

9.99	′	0″	10″	20″	30″	40″	50″	60″			d.
894	**0**	8.84 358	389	419	449	479	509	539	59	893	30
893	1	539	569	599	629	659	688	718	58	892	30
892	2	718	748	778	808	838	867	897	57	891	30
891	3	897	927	957	986	*016	*045	*075	56	891	30
891	4	8.85 075	105	134	164	193	223	252	55	890	30
890	5	252	282	311	341	370	400	429	54	889	30
889	6	429	458	488	517	546	576	605	53	888	29
888	7	605	634	663	693	722	751	780	52	887	29
887	8	780	809	838	867	896	926	955	51	886	29
886	9	955	984	*013	*042	*070	*099	*128	**50**	885	29
885	**10**	8.86 128	157	186	215	244	273	301	49	884	29
884	11	301	330	359	388	416	445	474	48	883	29
883	12	474	502	531	560	588	617	645	47	882	29
882	13	645	674	703	731	760	788	816	46	881	29
881	14	816	845	873	902	930	958	987	45	880	28
880	15	987	*015	*043	*072	*100	*128	*156	44	879	28
879	16	8.87 156	185	213	241	269	297	325	43	879	28
879	17	325	354	382	410	438	466	494	42	878	28
878	18	494	522	550	578	606	634	661	41	877	28
877	19	661	689	717	745	773	801	829	**40**	876	28
876	**20**	829	856	884	912	940	967	995	39	875	28
875	21	995	*023	*050	*078	*106	*133	*161	38	874	28
874	22	8.88 161	188	216	243	271	298	326	37	873	28
873	23	326	353	381	408	436	463	490	36	872	27
872	24	490	518	545	572	600	627	654	35	871	27
871	25	654	681	709	736	763	790	817	34	870	27
870	26	817	845	872	899	926	953	980	33	869	27
869	27	980	*007	*034	*061	*088	*115	*142	32	868	27
868	28	8.89 142	169	196	223	250	277	304	31	867	27
867	29	304	330	357	384	411	438	464	**30**	866	27
866	**30**	464	491	518	545	571	598	625	29	865	27
865	31	625	651	678	704	731	758	784	28	864	26
864	32	784	811	837	864	890	917	943	27	863	26
863	33	943	970	996	*023	*049	*075	*102	26	862	26
862	34	8.90 102	128	154	181	207	233	260	25	861	26
861	35	260	286	312	338	364	391	417	24	860	26
860	36	417	443	469	495	521	548	574	23	859	26
859	37	574	600	626	652	678	704	730	22	858	26
858	38	730	756	782	808	834	859	885	21	857	26
857	39	885	911	937	963	989	*015	*040	**20**	856	26
856	**40**	8.91 040	066	092	118	143	169	195	19	855	26
855	41	195	221	246	272	298	323	349	18	854	26
854	42	349	374	400	426	451	477	502	17	853	26
853	43	502	528	553	579	604	630	655	16	852	26
852	44	655	680	706	731	757	782	807	15	851	25
851	45	807	833	858	883	909	934	959	14	850	25
850	46	959	984	*010	*035	*060	*085	*110	13	848	25
848	47	8.92 110	135	161	186	211	236	261	12	847	25
847	48	261	286	311	336	361	386	411	11	846	25
846	49	411	436	461	486	511	536	561	**10**	845	25
845	**50**	561	586	611	636	660	685	710	9	844	25
844	51	710	735	760	784	809	834	859	8	843	25
843	52	859	883	908	933	957	982	*007	7	842	25
842	53	8.93 007	031	056	081	105	130	154	6	841	24
841	54	154	179	203	228	253	277	301	5	840	24
840	55	301	326	350	375	399	424	448	4	839	24
839	56	448	472	497	521	546	570	594	3	838	24
838	57	594	619	643	667	691	716	740	2	837	24
837	58	740	764	788	812	837	861	885	1	836	24
836	59	885	909	933	957	981	*006	*030	**0**	834	24
.		60″	50″	40″	30″	20″	10″	0″	′	9.99	d.

P. P.

	31	**30**
1	3.1	3.0
2	6.2	6.0
3	9.3	9.0
4	12.4	12.0
5	15.5	15.0
6	18.6	18.0
7	21.7	21.0
8	24.8	24.0
9	27.9	27.0

	29
1	2.9
2	5.8
3	8.7
4	11.6
5	14.5
6	17.4
7	20.3
8	23.2
9	26.1

	28	**27**
1	2.8	2.7
2	5.6	5.4
3	8.4	8.1
4	11.2	10.8
5	14.0	13.5
6	16.8	16.2
7	19.6	18.9
8	22.4	21.6
9	25.2	24.3

	26
1	2.6
2	5.2
3	7.8
4	10.4
5	13.0
6	15.6
7	18.2
8	20.8
9	23.4

	25	**24**
1	2.5	2.4
2	5.0	4.8
3	7.5	7.2
4	10.0	9.6
5	12.5	12.0
6	15.0	14.4
7	17.5	16.8
8	20.0	19.2
9	22.5	21.6

P. P.

Log Tan. 4°

′	9″	10″	20″	30″	40″	50″	60″		d.
0	8.84 464	495	525	555	585	615	646	59	30
1	646	676	706	736	766	796	826	58	30
2	826	856	886	916	946	976	*006	57	30
3	8.85 006	036	065	095	125	155	185	56	30
4	185	214	244	274	304	333	363	55	30
5	363	392	422	452	481	511	540	54	30
6	540	570	599	629	658	688	717	53	30
7	717	747	776	805	835	864	893	52	29
8	893	922	952	981	*010	*039	*069	51	29
9	8.86 069	098	127	156	185	214	243	**50**	29
10	243	272	301	330	359	388	417	49	29
11	417	447	475	504	533	562	591	48	29
12	591	619	648	677	706	734	763	47	29
13	763	792	821	849	878	907	935	46	29
14	935	964	992	*021	*049	*078	*106	45	28
15	8.87 106	135	163	192	220	249	277	44	28
16	277	305	334	362	390	419	447	43	28
17	447	475	503	532	560	588	616	42	28
18	616	644	673	701	729	757	785	41	28
19	785	813	841	869	897	925	953	**40**	28
20	953	981	*009	*037	*065	*092	*120	39	28
21	8.88 120	148	176	204	231	259	287	38	28
22	287	315	342	370	398	425	453	37	28
23	453	481	508	536	563	591	618	36	28
24	618	646	674	701	728	756	783	35	28
25	783	811	838	866	893	920	948	34	28
26	948	975	*002	*029	*057	*084	*111	33	27
27	8.89 111	138	166	193	220	247	274	32	27
28	274	301	328	355	383	410	437	31	27
29	437	464	491	518	545	571	598	**30**	27
30	598	625	652	679	706	733	760	29	27
31	760	786	813	840	867	894	920	28	27
32	920	947	974	*000	*027	*054	*080	27	27
33	8.90 080	107	134	160	187	213	240	26	27
34	240	266	293	319	346	372	399	25	26
35	399	425	451	478	504	531	557	24	26
36	557	583	610	636	662	688	715	23	26
37	715	741	767	793	820	846	872	22	26
38	872	898	924	950	976	*002	*029	21	26
39	8.91 029	055	081	107	133	159	185	**20**	26
40	185	211	236	262	288	314	340	19	26
41	340	366	392	418	443	469	495	18	26
42	495	521	547	572	598	624	650	17	26
43	650	675	701	727	752	778	803	16	26
44	803	829	855	880	906	931	957	15	26
45	957	982	*008	*033	*059	*084	*110	14	26
46	8.92 110	135	160	186	211	237	262	13	25
47	262	287	313	338	363	388	414	12	25
48	414	439	464	489	515	540	565	11	25
49	565	590	615	640	665	691	716	**10**	25
50	716	741	766	791	816	841	866	9	25
51	866	891	916	941	966	991	*016	8	25
52	8.93 016	040	065	090	115	140	165	7	25
53	165	190	214	239	264	289	313	6	25
54	313	338	363	388	412	437	462	5	25
55	462	486	511	536	560	585	609	4	24
56	609	634	658	683	707	732	756	3	24
57	756	781	805	830	854	879	903	2	24
58	903	928	952	976	*001	*025	*049	1	24
59	8.94 049	074	098	122	147	171	195	**0**	24
	60″	50″	40″	30″	20″	10″	0″	′	d.

P. P.

	31	**30**
1	3.1	3.0
2	6.2	6.0
3	9.3	9.0
4	12.4	12.0
5	15.5	15.0
6	18.6	18.0
7	21.7	21.0
8	24.8	24.0
9	27.9	27.0

	29
1	2.9
2	5.8
3	8.7
4	11.6
5	14.5
6	17.4
7	20.3
8	23.2
9	26.1

	28	**27**
1	2.8	2.7
2	5.6	5.4
3	8.4	8.1
4	11.2	10.8
5	14.0	13.5
6	16.8	16.2
7	19.6	18.9
8	22.4	21.6
9	25.2	24.3

	26
1	2.6
2	5.2
3	7.8
4	10.4
5	13.0
6	15.6
7	18.2
8	20.8
9	23.4

	25	**24**
1	2.5	2.4
2	5.0	4.8
3	7.5	7.2
4	10.0	9.6
5	12.5	12.0
6	15.0	14.4
7	17.5	16.8
8	20.0	19.2
9	22.5	21.6

P. P.

L. Cos. Log Sin. 5°

9.99	′	0″	10″	20″	30″	40″	50″	60″			d.	P. P.
834	**0**	8.94 030	054	078	102	126	150	174	59	833	24	
833	1	174	198	222	246	270	294	317	58	832	24	
832	2	317	341	365	389	413	437	461	57	831	24	
831	3	461	484	508	532	556	580	603	56	830	24	
830	4	603	627	651	675	698	722	746	55	829	24	**24**
												1 2.4
829	5	746	769	793	817	840	864	887	54	828	24	2 4.8
828	6	887	911	935	958	982	*005	*029	53	827	24	3 7.2
827	7	8.95 029	052	076	099	123	146	170	52	825	24	4 9.6
825	8	170	193	216	240	263	287	310	51	824	23	5 12.0
824	9	310	333	357	380	403	427	450	**50**	823	23	6 14.4
												7 16.8
												8 19.2
823	**10**	450	473	496	520	543	566	589	49	822	23	9 21.6
822	11	589	613	636	659	682	705	728	48	821	23	
821	12	728	752	775	798	821	844	867	47	820	23	
820	13	867	890	913	936	959	982	*005	46	819	23	
819	14	8.96 005	028	051	074	097	120	143	45	817	23	
												23
817	15	143	166	189	212	234	257	280	44	816	23	1 2.3
816	16	280	303	326	349	371	394	417	43	815	23	2 4.6
815	17	417	440	462	485	508	531	553	42	814	23	3 6.9
814	18	553	576	599	621	644	667	689	41	813	23	4 9.2
813	19	689	712	735	757	780	802	825	**40**	812	23	5 11.5
												6 13.8
812	**20**	825	847	870	892	915	937	960	39	810	22	7 16.1
810	21	960	982	*005	*027	*050	*072	*095	38	809	22	8 18.4
809	22	8.97 095	117	139	162	184	207	229	37	808	22	9 20.7
808	23	229	251	274	296	318	341	363	36	807	22	
807	24	363	385	407	430	452	474	496	35	806	22	
806	25	496	518	541	563	585	607	629	34	804	22	
804	26	629	651	674	696	718	740	762	33	803	22	**22**
803	27	762	784	806	828	850	872	894	32	802	22	1 2.2
802	28	894	916	938	960	982	*004	*026	31	801	22	2 4.4
801	29	8.98 026	048	070	092	114	135	157	**30**	800	22	3 6.6
												4 8.8
800	**30**	157	179	201	223	245	266	288	29	798	22	5 11.0
798	31	288	310	332	354	375	397	419	28	797	22	6 13.2
797	32	419	441	462	484	506	527	549	27	796	22	7 15.4
796	33	549	571	592	614	636	657	679	26	795	22	8 17.6
795	34	679	701	722	744	765	787	808	25	793	22	9 19.8
793	35	808	830	851	873	894	916	937	24	792	22	
792	36	937	959	980	*002	*023	*045	*066	23	791	22	
791	37	8.99 066	087	109	130	152	173	194	22	790	21	
790	38	194	216	237	258	280	301	322	21	788	21	**21**
788	39	322	343	365	386	407	428	450	**20**	787	21	1 2.1
												2 4.2
787	**40**	450	471	492	513	534	556	577	19	786	21	3 6.3
786	41	577	598	619	640	661	682	704	18	785	21	4 8.4
785	42	704	725	746	767	788	809	830	17	783	21	5 10.5
783	43	830	851	872	893	914	935	956	16	782	21	6 12.6
782	44	956	977	998	*019	*040	*061	*082	15	781	21	7 14.7
												8 16.8
												9 18.9
781	45	9.00 082	103	123	144	165	186	207	14	780	21	
780	46	207	228	249	269	290	311	332	13	778	21	
778	47	332	353	373	394	415	436	456	12	777	21	
777	48	456	477	498	518	539	560	581	11	776	21	
776	49	581	601	622	642	663	684	704	**10**	775	21	**20**
												1 2.0
775	**50**	704	725	746	766	787	807	828	9	773	21	2 4.0
773	51	828	848	869	889	910	930	951	8	772	20	3 6.0
772	52	951	971	992	*012	*033	*053	*074	7	771	20	4 8.0
771	53	9.01 074	094	115	135	155	176	196	6	769	20	5 10.0
769	54	196	217	237	257	278	298	318	5	768	20	6 12.0
												7 14.0
768	55	318	339	359	379	399	420	440	4	767	20	8 16.0
767	56	440	460	480	501	521	541	561	3	765	20	9 18.0
765	57	561	582	602	622	642	662	682	2	764	20	
764	58	682	703	723	743	763	783	803	1	763	20	
763	59	803	823	843	863	883	903	923	**0**	761	20	
		60″	50″	40″	30″	20″	10″	0″	′	9.99	d.	P. P.

Log Tan. 5°

′		0″	10″	20″	30″	40″	50″	60″		d.	P. P.
0	8.94	195	219	244	268	292	316	340	59	24	
1		340	365	389	413	437	461	485	58	24	**25**
2		485	509	533	557	581	606	630	57	24	1 2.5
3		630	654	678	702	725	749	773	56	24	2 5.0
4		773	797	821	845	869	893	917	55	24	3 7.5
5		917	941	964	988	*012	*036	*060	54	24	4 10.0
6	8.95	060	083	107	131	155	178	202	53	24	5 12.5
7		202	226	249	273	297	320	344	52	24	6 15.0
8		344	368	391	415	439	462	486	51	24	7 17.5
9		486	509	533	556	580	603	627	**50**	24	8 20.0
10		627	650	674	697	721	744	767	49	23	9 22.5
11		767	791	814	838	861	884	908	48	23	
12		908	931	954	977	*001	*024	*047	47	23	**24**
13	8.96	047	071	094	117	140	163	187	46	23	1 2.4
14		187	210	233	256	279	302	325	45	23	2 4.8
15		325	349	372	395	418	441	464	44	23	3 7.2
16		464	487	510	533	556	579	602	43	23	4 9.6
17		602	625	648	671	694	717	739	42	23	5 12.0
18		739	762	785	808	831	854	877	41	23	6 14.4
19		877	899	922	945	968	991	*013	**40**	23	7 16.8
20	8.97	013	036	059	081	104	127	150	39	23	8 19.2
21		150	172	195	218	240	263	285	38	23	9 21.6
22		285	308	331	353	376	398	421	37	23	
23		421	443	466	488	511	533	556	36	22	**23**
24		556	578	601	623	646	668	691	35	22	1 2.3
25		691	713	735	758	780	802	825	34	22	2 4.6
26		825	847	869	892	914	936	959	33	22	3 6.9
27		959	981	*003	*025	*048	*070	*092	32	22	4 9.2
28	8.98	092	114	136	159	181	203	225	31	22	5 11.5
29		225	227	269	291	314	336	358	**30**	22	6 13.8
30		358	380	402	424	446	468	490	29	22	7 16.1
31		490	512	534	556	578	600	622	28	22	8 18.4
32		622	644	666	687	709	731	753	27	22	9 20.7
33		753	775	797	819	841	862	884	26	22	
34		884	906	928	950	971	993	*015	25	22	**22**
35	8.99	015	037	058	080	102	123	145	24	22	1 2.2
36		145	167	188	210	232	253	275	23	22	2 4.4
37		275	297	318	340	361	383	405	22	22	3 6.6
38		405	426	448	469	491	512	534	21	22	4 8.8
39		534	555	577	598	620	641	662	**20**	21	5 11.0
40		662	684	705	727	748	769	791	19	21	6 13.2
41		791	812	834	855	876	898	919	18	21	7 15.4
42		919	940	961	983	*004	*025	*046	17	21	8 17.6
43	9.00	046	068	089	110	131	153	174	16	21	9 19.8
44		174	195	216	237	258	280	301	15	21	
45		301	322	343	364	385	406	427	14	21	**21**
46		427	448	469	490	511	532	553	13	21	1 2.1
47		553	574	595	616	637	658	679	12	21	2 4.2
48		679	700	721	742	763	784	805	11	21	3 6.3
49		805	826	846	867	888	909	930	**10**	21	4 8.4
50		930	951	971	992	*013	*034	*055	9	21	5 10.5
51	9.01	055	075	096	117	138	158	179	8	21	6 12.6
52		179	200	220	241	262	282	303	7	21	7 14.7
53		303	324	344	365	386	406	427	6	21	8 16.8
54		427	447	468	489	509	530	550	5	20	9 18.9
55		550	571	591	612	632	653	673	4	20	**20**
56		673	694	714	735	755	776	796	3	20	1 2.0
57		796	816	837	857	878	898	918	2	20	2 4.0
58		918	939	959	979	*000	*020	*040	1	20	3 6.0
59	9.02	040	061	081	101	121	142	162	**0**	20	4 8.0
		60″	50″	40″	30″	20″	10″	0″	′	d.	5 10.0
											P. P.

84° Log Cot.

L. Cos. Log Sin. 6°

9.99	′	0″	10″	20″	30″	40″	50″	60″			d.	P. P.
761	**0**	9.01 923	943	964	984	*004	*024	*043	59	760	20	
760	1	9.02 043	063	083	103	123	143	163	58	759	20	
759	2	163	183	203	223	243	263	283	57	757	20	
757	3	283	302	322	342	362	382	402	56	756	20	**21**
756	4	402	421	441	461	481	501	520	55	755	20	1 2.1
755	5	520	540	560	579	599	619	639	54	753	20	2 4.2
753	6	639	658	678	698	717	737	757	53	752	20	3 6.3
752	7	757	776	796	816	835	855	874	52	751	20	4 8.4
751	8	874	894	914	933	953	972	992	51	749	20	5 10.5
749	9	992	*011	*031	*050	*070	*089	*109	**50**	748	20	6 12.6
												7 14.7
748	**10**	9.03 109	128	148	167	187	206	226	49	747	20	8 16.8
747	11	226	245	265	284	303	323	342	48	745	19	9 18.9
745	12	342	361	381	400	420	439	458	47	744	19	
744	13	458	478	497	516	535	555	574	46	742	19	
742	14	574	593	613	632	651	670	690	45	741	19	
741	15	690	709	728	747	766	786	805	44	740	19	**20**
740	16	805	824	843	862	881	901	920	43	738	19	1 2.0
738	17	920	939	958	977	996	*015	*034	42	737	19	2 4.0
737	18	9.04 034	053	072	091	110	129	149	41	736	19	3 6.0
736	19	149	168	187	206	225	244	262	**40**	734	19	4 8.0
												5 10.0
734	**20**	262	281	300	319	338	357	376	39	733	19	6 12.0
733	21	376	395	414	433	452	471	490	38	731	19	7 14.0
731	22	490	508	527	546	565	584	603	37	730	19	8 16.0
730	23	603	621	640	659	678	697	715	36	728	19	9 18.0
728	24	715	734	753	772	790	809	828	35	727	19	
727	25	828	847	865	884	903	921	940	34	726	19	
726	26	940	959	977	996	*015	*033	*052	33	724	19	**19**
724	27	9.05 052	071	089	108	126	145	164	32	723	19	1 1.9
723	28	164	182	201	219	238	256	275	31	721	18	2 3.8
721	29	275	293	312	330	349	367	386	**30**	720	18	3 5.7
												4 7.6
720	**30**	386	404	423	441	460	478	497	29	718	18	5 9.5
718	31	497	515	533	552	570	589	607	28	717	18	6 11.4
717	32	607	625	644	662	681	699	717	27	716	18	7 13.3
716	33	717	736	754	772	791	809	827	26	714	18	8 15.2
714	34	827	845	864	882	900	918	937	25	713	18	9 17.1
713	35	937	955	973	991	*010	*028	*046	24	711	18	
711	36	9.06 046	064	082	101	119	137	155	23	710	18	
710	37	155	173	191	210	228	246	264	22	708	18	
708	38	264	282	300	318	336	354	372	21	707	18	**18**
707	39	372	390	408	426	445	463	481	**20**	705	18	1 1.8
												2 3.6
705	**40**	481	499	517	535	553	571	589	19	704	18	3 5.4
704	41	589	606	624	642	660	678	696	18	702	18	4 7.2
702	42	696	714	732	750	768	786	804	17	701	18	5 9.0
701	43	804	821	839	857	875	893	911	16	699	18	6 10.8
699	44	911	929	946	964	982	*000	*018	15	698	18	7 12.6
												8 14.4
698	45	9.07 018	035	053	071	089	106	124	14	696	18	9 16.2
696	46	124	142	160	177	195	213	231	13	695	18	
695	47	231	248	266	284	301	319	337	12	693	18	
693	48	337	354	372	390	407	425	442	11	692	18	
692	49	442	460	478	495	513	530	548	**10**	690	18	**17**
												1 1.7
690	**50**	548	566	583	601	618	636	653	9	689	18	2 3.4
689	51	653	671	688	706	723	741	758	8	687	18	3 5.1
687	52	758	776	793	811	828	846	863	7	686	18	4 6.8
686	53	863	881	898	915	933	950	968	6	684	18	5 8.5
684	54	968	985	*002	*020	*037	*055	*072	5	683	17	6 10.2
												7 11.9
683	55	9.08 072	089	107	124	141	159	176	4	681	17	8 13.6
681	56	176	193	211	228	245	262	280	3	680	17	9 15.3
680	57	280	297	314	331	349	366	383	2	678	17	
678	58	383	400	418	435	452	469	486	1	677	17	
677	59	486	504	521	538	555	572	589	**0**	675	17	
		60″	50″	40″	30″	20″	10″	0″	′	9.99	d.	P. P.

83° Log Cos. L. Sin.

Log Tan. 6°

′		0″	10″	20″	30″	40″	50″	60″		d.	P. P.
0	9.02	162	182	203	223	243	263	283	59	20	
1		283	304	324	344	364	384	404	58	20	
2		404	425	445	465	485	505	525	57	20	
3		525	545	565	585	605	625	645	56	20	
4		645	666	686	706	726	746	766	55	20	**21**
											1 2.1
5		766	785	805	825	845	865	885	54	20	2 4.2
6		885	905	925	945	965	985	*005	53	20	3 6.3
7	9.03	005	025	044	064	084	104	124	52	20	4 8.4
8		124	144	163	183	203	223	242	51	20	5 10.5
9		242	262	282	302	321	341	361	**50**	20	6 12.6
											7 14.7
10		361	381	400	420	440	459	479	49	20	8 16.8
11		479	499	518	538	558	577	597	48	20	9 18.9
12		597	616	636	656	675	695	714	47	20	
13		714	734	753	773	793	812	832	46	20	
14		832	851	871	890	910	929	948	45	19	
											20
15		948	968	987	*007	*026	*046	*065	44	19	1 2.0
16	9.04	065	084	104	123	143	162	181	43	19	2 4.0
17		181	201	220	239	259	278	297	42	19	3 6.0
18		297	317	336	355	374	394	413	41	19	4 8.0
19		413	432	451	471	490	509	528	**40**	19	5 10.0
											6 12.0
20		528	548	567	586	605	624	643	39	19	7 14.0
21		643	663	682	701	720	739	758	38	19	8 16.0
22		758	777	796	815	835	854	873	37	19	9 18.0
23		873	892	911	930	949	968	987	36	19	
24		987	*006	*025	*044	*063	*082	*101	35	19	
25	9.05	101	120	139	158	177	195	214	34	19	
26		214	233	252	271	290	309	328	33	19	**19**
27		328	347	365	384	403	422	441	32	19	1 1.9
28		441	460	478	497	516	535	553	31	19	2 3.8
29		553	572	591	610	628	647	666	**30**	19	3 5.7
											4 7.6
30		666	685	703	722	741	759	778	29	19	5 9.5
31		778	797	815	834	853	871	890	28	19	6 11.4
32		890	909	927	946	964	983	*002	27	19	7 13.3
33	9.06	002	020	039	057	076	094	113	26	18	8 15.2
34		113	132	150	169	187	206	224	25	18	9 17.1
35		224	243	261	279	298	316	335	24	18	
36		335	353	372	390	409	427	445	23	18	
37		445	464	482	500	519	537	556	22	18	
38		556	574	592	611	629	647	666	21	18	**18**
39		666	684	702	720	739	757	775	**20**	18	1 1.8
											2 3.6
40		775	793	812	830	848	866	885	19	18	3 5.4
41		885	903	921	939	957	976	994	18	18	4 7.2
42		994	*012	*030	*048	*066	*085	*103	17	18	5 9.0
43	9.07	103	121	139	157	175	193	211	16	18	6 10.8
44		211	229	247	266	284	302	320	15	18	7 12.6
											8 14.4
45		320	338	356	374	392	410	428	14	18	9 16.2
46		428	446	464	482	500	518	536	13	18	
47		536	554	572	589	607	625	643	12	18	
48		643	661	679	697	715	733	751	11	18	
49		751	768	786	804	822	840	858	**10**	18	**17**
											1 1.7
50		858	875	893	911	929	947	964	9	18	2 3.4
51		964	982	*000	*018	*035	*053	*071	8	18	3 5.1
52	9.08	071	089	106	124	142	160	177	7	18	4 6.8
53		177	195	213	230	248	266	283	6	18	5 8.5
54		283	301	319	336	354	371	389	5	18	6 10.2
											7 11.9
55		389	407	424	442	460	477	495	4	18	8 13.6
56		495	512	530	547	565	582	600	3	18	9 15.3
57		600	617	635	653	670	688	705	2	18	
58		705	722	740	757	775	792	810	1	18	
59		810	827	845	862	880	897	914	**0**	17	
		60″	50″	40″	30″	20″	10″	0″	′	d.	P. P.

83° Log Cot.

7°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	
0	9.08 589		9.08 914		0.91 086	9.99 675	**60**
1	9.08 692	103	9.09 019	105	0.90 981	9.99 674	59
2	9.08 795	103	9.09 123	104	0.90 877	9.99 672	58
3	9.08 897	102	9.09 227	104	0.90 773	9.99 670	57
4	9.08 999	102	9.09 330	103	0.90 670	9.99 669	56
5	9.09 101	102	9.09 434	104	0.90 566	9.99 667	55
6	9.09 202	101	9.09 537	103	0.90 463	9.99 666	54
7	9.09 304	102	9.09 640	103	0.90 360	9.99 664	53
8	9.09 405	101	9.09 742	102	0.90 258	9.99 663	52
9	9.09 506	101	9.09 845	103	0.90 155	9.99 661	51
10	9.09 606	100	9.09 947	102	0.90 053	9.99 659	**50**
11	9.09 707	101	9.10 049	102	0.89 951	9.99 658	49
12	9.09 807	100	9.10 150	101	0.89 850	9.99 656	48
13	9.09 907	100	9.10 252	102	0.89 748	9.99 655	47
14	9.10 006	99	9.10 353	101	0.89 647	9.99 653	46
15	9.10 106	100	9.10 454	101	0.89 546	9.99 651	45
16	9.10 205	99	9.10 555	101	0.89 445	9.99 650	44
17	9.10 304	99	9.10 656	101	0.89 344	9.99 648	43
18	9.10 402	98	9.10 756	100	0.89 244	9.99 647	42
19	9.10 501	99	9.10 856	100	0.89 144	9.99 645	41
20	9.10 599	98	9.10 956	100	0.89 044	9.99 643	**40**
21	9.10 697	98	9.11 056	100	0.88 944	9.99 642	39
22	9.10 795	98	9.11 155	99	0.88 845	9.99 640	38
23	9.10 893	98	9.11 254	99	0.88 746	9.99 638	37
24	9.10 990	97	9.11 353	99	0.88 647	9.99 637	36
25	9.11 087	97	9.11 452	99	0.88 548	9.99 635	35
26	9.11 184	97	9.11 551	99	0.88 449	9.99 633	34
27	9.11 281	97	9.11 649	98	0.88 351	9.99 632	33
28	9.11 377	96	9.11 747	98	0.88 253	9.99 630	32
29	9.11 474	97	9.11 845	98	0.88 155	9.99 629	31
30	9.11 570	96	9.11 943	98	0.88 057	9.99 627	**30**
31	9.11 666	96	9.12 040	97	0.87 960	9.99 625	29
32	9.11 761	95	9.12 138	98	0.87 862	9.99 624	28
33	9.11 857	96	9.12 235	97	0.87 765	9.99 622	27
34	9.11 952	95	9.12 332	97	0.87 668	9.99 620	26
35	9.12 047	95	9.12 428	96	0.87 572	9.99 618	25
36	9.12 142	95	9.12 525	97	0.87 475	9.99 617	24
37	9.12 236	94	9.12 621	96	0.87 379	9.99 615	23
38	9.12 331	95	9.12 717	96	0.87 283	9.99 613	22
39	9.12 425	94	9.12 813	96	0.87 187	9.99 612	21
40	9.12 519	94	9.12 909	96	0.87 091	9.99 610	**20**
41	9.12 612	93	9.13 004	95	0.86 996	9.99 608	19
42	9.12 706	94	9.13 099	95	0.86 901	9.99 607	18
43	9.12 799	93	9.13 194	95	0.86 806	9.99 605	17
44	9.12 892	93	9.13 289	95	0.86 711	9.99 603	16
45	9.12 985	93	9.13 384	95	0.86 616	9.99 601	15
46	9.13 078	93	9.13 478	94	0.86 522	9.99 600	14
47	9.13 171	93	9.13 573	95	0.86 427	9.99 598	13
48	9.13 263	92	9.13 667	94	0.86 333	9.99 596	12
49	9.13 355	92	9.13 761	94	0.86 239	9.99 595	11
50	9.13 447	92	9.13 854	93	0.86 146	9.99 593	**10**
51	9.13 539	92	9.13 948	94	0.86 052	9.99 591	9
52	9.13 630	91	9.14 041	93	0.85 959	9.99 589	8
53	9.13 722	92	9.14 134	93	0.85 866	9.99 588	7
54	9.13 813	91	9.14 227	93	0.85 773	9.99 586	6
55	9.13 904	91	9.14 320	93	0.85 680	9.99 584	5
56	9.13 994	90	9.14 412	92	0.85 588	9.99 582	4
57	9.14 085	91	9.14 504	92	0.85 496	9.99 581	3
58	9.14 175	90	9.14 597	93	0.85 403	9.99 579	2
59	9.14 266	91	9.14 688	91	0.85 312	9.99 577	1
60	9.14 356	90	9.14 780	92	0.85 220	9.99 575	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin	′

P. P.

	105	104	103	102
1	1.8	1.7	1.7	1.7
2	3.5	3.5	3.4	3.4
3	5.2	5.2	5.2	5.1
4	7.0	6.9	6.9	6.8
5	8.8	8.7	8.6	8.5
6	10.5	10.4	10.3	10.2
7	12.2	12.1	12.0	11.9
8	14.0	13.9	13.7	13.6
9	15.8	15.6	15.4	15.3
10	17.5	17.3	17.2	17.0
20	35.0	34.7	34.3	34.0
30	52.5	52.0	51.5	51.0
40	70.0	69.3	68.7	68.0
50	87.5	86.7	85.8	85.0

	101	100	99	98
1	1.7	1.7	1.6	1.6
2	3.4	3.3	3.3	3.3
3	5.0	5.0	5.0	4.9
4	6.7	6.7	6.6	6.5
5	8.4	8.3	8.2	8.2
6	10.1	10.0	9.9	9.8
7	11.8	11.7	11.6	11.4
8	13.5	13.3	13.2	13.1
9	15.2	15.0	14.8	14.7
10	16.8	16.7	16.5	16.3
20	33.7	33.3	33.0	32.7
30	50.5	50.0	49.5	49.0
40	67.3	66.7	66.0	65.3
50	84.2	83.3	82.5	81.7

	97	96	95	94
1	1.6	1.6	1.6	1.6
2	3.2	3.2	3.2	3.1
3	4.8	4.8	4.8	4.7
4	6.5	6.4	6.3	6.3
5	8.1	8.0	7.9	7.8
6	9.7	9.6	9.5	9.4
7	11.3	11.2	11.1	11.0
8	12.9	12.8	12.7	12.5
9	14.6	14.4	14.2	14.1
10	16.2	16.0	15.8	15.7
20	32.3	32.0	31.7	31.3
30	48.5	48.0	47.5	47.0
40	64.7	64.0	63.3	62.7
50	80.8	80.0	79.2	78.3

	93	92	91	90
1	1.6	1.5	1.5	1.5
2	3.1	3.1	3.0	3.0
3	4.6	4.6	4.6	4.5
4	6.2	6.1	6.1	6.0
5	7.8	7.7	7.6	7.5
6	9.3	9.2	9.1	9.0
7	10.8	10.7	10.6	10.5
8	12.4	12.3	12.1	12.0
9	14.0	13.8	13.6	13.5
10	15.5	15.3	15.2	15.0
20	31.0	30.7	30.3	30.0
30	46.5	46.0	45.5	45.0
40	62.0	61.3	60.7	60.0
50	77.5	76.7	75.8	75.0

P. P.

8°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	
0	9.14 356		9.14 780		0.85 220	9.99 575	**60**
1	9.14 445	89	9.14 872	92	0.85 128	9.99 574	59
2	9.14 535	90	9.14 963	91	0.85 037	9.99 572	58
3	9.14 624	89	9.15 054	91	0.84 946	9.99 570	57
4	9.14 714	90	9.15 145	91	0.84 855	9.99 568	56
5	9.14 803	89	9.15 236	91	0.84 764	9.99 566	55
6	9.14 891	88	9.15 327	91	0.84 673	9.99 565	54
7	9.14 980	89	9.15 417	90	0.84 583	9.99 563	53
8	9.15 069	89	9.15 508	91	0.84 492	9.99 561	52
9	9.15 157	88	9.15 598	90	0.84 402	9.99 559	51
10	9.15 245	88	9.15 688	90	0.84 312	9.99 557	**50**
11	9.15 333	88	9.15 777	89	0.84 223	9.99 556	49
12	9.15 421	88	9.15 867	90	0.84 133	9.99 554	48
13	9.15 508	87	9.15 956	89	0.84 044	9.99 552	47
14	9.15 596	88	9.16 046	90	0.83 954	9.99 550	46
15	9.15 683	87	9.16 135	89	0.83 865	9.99 548	45
16	9.15 770	87	9.16 224	89	0.83 776	9.99 546	44
17	9.15 857	87	9.16 312	88	0.83 688	9.99 545	43
18	9.15 944	87	9.16 401	89	0.83 599	9.99 543	42
19	9.16 030	86	9.16 489	88	0.83 511	9.99 541	41
20	9.16 116	86	9.16 577	88	0.83 423	9.99 539	**40**
21	9.16 203	87	9.16 665	88	0.83 335	9.99 537	39
22	9.16 289	86	9.16 753	88	0.83 247	9.99 535	38
23	9.16 374	85	9.16 841	88	0.83 159	9.99 533	37
24	9.16 460	86	9.16 928	87	0.83 072	9.99 532	36
25	9.16 545	85	9.17 016	88	0.82 984	9.99 530	35
26	9.16 631	86	9.17 103	87	0.82 897	9.99 528	34
27	9.16 716	85	9.17 190	87	0.82 810	9.99 526	33
28	9.16 801	85	9.17 277	87	0.82 723	9.99 524	32
29	9.16 886	85	9.17 363	86	0.82 637	9.99 522	31
30	9.16 970	84	9.17 450	87	0.82 550	9.99 520	**30**
31	9.17 055	85	9.17 536	86	0.82 464	9.99 518	29
32	9.17 139	84	9.17 622	86	0.82 378	9.99 517	28
33	9.17 223	84	9.17 708	86	0.82 292	9.99 515	27
34	9.17 307	84	9.17 794	86	0.82 206	9.99 513	26
35	9.17 391	84	9.17 880	86	0.82 120	9.99 511	25
36	9.17 474	83	9.17 965	85	0.82 035	9.99 509	24
37	9.17 558	84	9.18 051	86	0.81 949	9.99 507	23
38	9.17 641	83	9.18 136	85	0.81 864	9.99 505	22
39	9.17 724	83	9.18 221	85	0.81 779	9.99 503	21
40	9.17 807	83	9.18 306	85	0.81 694	9.99 501	**20**
41	9.17 890	83	9.18 391	85	0.81 609	9.99 499	19
42	9.17 973	83	9.18 475	84	0.81 525	9.99 497	18
43	9.18 055	82	9.18 560	85	0.81 440	9.99 495	17
44	9.18 137	82	9.18 644	84	0.81 356	9.99 494	16
45	9.18 220	83	9.18 728	84	0.81 272	9.99 492	15
46	9.18 302	82	9.18 812	84	0.81 188	9.99 490	14
47	9.18 383	81	9.18 896	84	0.81 104	9.99 488	13
48	9.18 465	82	9.18 979	83	0.81 021	9.99 486	12
49	9.18 547	82	9.19 063	84	0.80 937	9.99 484	11
50	9.18 628	81	9.19 146	83	0.80 854	9.99 482	**10**
51	9.18 709	81	9.19 229	83	0.80 771	9.99 480	9
52	9.18 790	81	9.19 312	83	0.80 688	9.99 478	8
53	9.18 871	81	9.19 395	83	0.80 605	9.99 476	7
54	9.18 952	81	9.19 478	83	0.80 522	9.99 474	6
55	9.19 033	81	9.19 561	83	0.80 439	9.99 472	5
56	9.19 113	80	9.19 643	82	0.80 357	9.99 470	4
57	9.19 193	80	9.19 725	82	0.80 275	9.99 468	3
58	9.19 273	80	9.19 807	82	0.80 193	9.99 466	2
59	9.19 353	80	9.19 889	82	0.80 111	9.99 464	1
60	9.19 433	80	9.19 971	82	0.80 029	9.99 462	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	′

P. P.

	92	91	90
1	1.5	1.5	1.5
2	3.1	3.0	3.0
3	4.6	4.6	4.5
4	6.1	6.1	6.0
5	7.7	7.6	7.5
6	9.2	9.1	9.0
7	10.7	10.6	10.5
8	12.3	12.1	12.0
9	13.8	13.6	13.5
10	15.3	15.2	15.0
20	30.7	30.3	30.0
30	46.0	45.5	45.0
40	61.3	60.7	60.0
50	76.7	75.8	75.0

	89	88	87
1	1.5	1.5	1.4
2	3.0	2.9	2.9
3	4.4	4.4	4.4
4	5.9	5.9	5.8
5	7.4	7.3	7.2
6	8.9	8.8	8.7
7	10.4	10.3	10.2
8	11.9	11.7	11.6
9	13.4	13.2	13.0
10	14.8	14.7	14.5
20	29.7	29.3	29.0
30	44.5	44.0	43.5
40	59.3	58.7	58.0
50	74.2	73.3	72.5

	86	85	84
1	1.4	1.4	1.4
2	2.9	2.8	2.8
3	4.3	4.2	4.2
4	5.7	5.7	5.6
5	7.2	7.1	7.0
6	8.6	8.5	8.4
7	10.0	9.9	9.8
8	11.5	11.3	11.2
9	12.9	12.8	12.6
10	14.3	14.2	14.0
20	28.7	28.3	28.0
30	43.0	42.5	42.0
40	57.3	56.7	56.0
50	71.7	70.8	70.0

	83	82	81
1	1.4	1.4	1.4
2	2.8	2.7	2.7
3	4.2	4.1	4.0
4	5.5	5.5	5.4
5	6.9	6.8	6.8
6	8.3	8.2	8.1
7	9.7	9.6	9.4
8	11.1	10.9	10.8
9	12.4	12.3	12.2
10	13.8	13.7	13.5
20	27.7	27.3	27.0
30	41.5	41.0	40.5
40	55.3	54.7	54.0
50	69.2	68.3	67.5

P. P.

9°

′	L.Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	
0	9.19 433		9.19 971		0.80 029	9.99 462	**60**
1	9.19 513	80	9.20 053	82	0.79 947	9.99 460	59
2	9.19 592	79	9.20 134	81	0.79 866	9.99 458	58
3	9.19 672	80	9.20 216	82	0.79 784	9.99 456	57
4	9.19 751	79	9.20 297	81	0.79 703	9.99 454	56
5	9.19 830	79	9.20 378	81	0.79 622	9.99 452	55
6	9.19 909	79	9.20 459	81	0.79 541	9.99 450	54
7	9.19 988	79	9.20 540	81	0.79 460	9.99 448	53
8	9.20 067	79	9.20 621	81	0.79 379	9.99 446	52
9	9.20 145	78	9.20 701	80	0.79 299	9.99 444	51
10	9.20 223	78	9.20 782	81	0.79 218	9.99 442	**50**
11	9.20 302	79	9.20 862	80	0.79 138	9.99 440	49
12	9.20 380	78	9.20 942	80	0.79 058	9.99 438	48
13	9.20 458	78	9.21 022	80	0.78 978	9.99 436	47
14	9.20 535	77	9.21 102	80	0.78 898	9.99 434	46
15	9.20 613	78	9.21 182	79	0.78 818	9.99 432	45
16	9.20 691	78	9.21 261	80	0.78 739	9.99 429	44
17	9.20 768	77	9.21 341	79	0.78 659	9.99 427	43
18	9.20 845	77	9.21 420	79	0.78 580	9.99 425	42
19	9.20 922	77	9.21 499	79	0.78 501	9.99 423	41
20	9.20 999	77	9.21 578	79	0.78 422	9.99 421	**40**
21	9.21 076	77	9.21 657	79	0.78 343	9.99 419	39
22	9.21 153	76	9.21 736	78	0.78 264	9.99 417	38
23	9.21 229	77	9.21 814	79	0.78 186	9.99 415	37
24	9.21 306	76	9.21 893	78	0.78 107	9.99 413	36
25	9.21 382	76	9.21 971	78	0.78 029	9.99 411	35
26	9.21 458	76	9.22 049	78	0.77 951	9.99 409	34
27	9.21 534	76	9.22 127	78	0.77 873	9.99 407	33
28	9.21 610	75	9.22 205	78	0.77 795	9.99 404	32
29	9.21 685	76	9.22 283	78	0.77 717	9.99 402	31
30	9.21 761	75	9.22 361	77	0.77 639	9.99 400	**30**
31	9.21 836	76	9.22 438	77	0.77 562	9.99 398	29
32	9.21 912	75	9.22 516	78	0.77 484	9.99 396	28
33	9.21 987	75	9.22 593	77	0.77 407	9.99 394	27
34	9.22 062	75	9.22 670	77	0.77 330	9.99 392	26
35	9.22 137	74	9.22 747	77	0.77 253	9.99 390	25
36	9.22 211	75	9.22 824	77	0.77 176	9.99 388	24
37	9.22 286	75	9.22 901	77	0.77 099	9.99 385	23
38	9.22 361	74	9.22 977	76	0.77 023	9.99 383	22
39	9.22 435	74	9.23 054	77	0.76 946	9.99 381	21
40	9.22 509	74	9.23 130	76	0.76 870	9.99 379	**20**
41	9.22 583	74	9.23 206	76	0.76 794	9.99 377	19
42	9.22 657	74	9.23 283	77	0.76 717	9.99 375	18
43	9.22 731	74	9.23 359	76	0.76 641	9.99 372	17
44	9.22 805	73	9.23 435	76	0.76 565	9.99 370	16
45	9.22 878	74	9.23 510	75	0.76 490	9.99 368	15
46	9.22 952	73	9.23 586	76	0.76 414	9.99 366	14
47	9.23 025	73	9.23 661	75	0.76 339	9.99 364	13
48	9.23 098	73	9.23 737	76	0.76 263	9.99 362	12
49	9.23 171	73	9.23 812	75	0.76 188	9.99 359	11
50	9.23 244	73	9.23 887	75	0.76 113	9.99 357	**10**
51	9.23 317	73	9.23 962	75	0.76 038	9.99 355	9
52	9.23 390	73	9.24 037	75	0.75 963	9.99 353	8
53	9.23 462	72	9.24 112	75	0.75 888	9.99 351	7
54	9.23 535	73	9.24 186	74	0.75 814	9.99 348	6
55	9.23 607	72	9.24 261	75	0.75 739	9.99 346	5
56	9.23 679	72	9.24 335	74	0.75 665	9.99 344	4
57	9.23 752	73	9.24 410	75	0.75 590	9.99 342	3
58	9.23 823	71	9.24 484	74	0.75 516	9.99 340	2
59	9.23 895	72	9.24 558	74	0.75 442	9.99 337	1
60	9.23 967	72	9.24 632	74	0.75 368	9.99 335	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	′

P. P.

	80	79	78	77
1	1.3	1.3	1.3	1.3
2	2.7	2.6	2.6	2.6
3	4.0	4.0	3.9	3.8
4	5.3	5.3	5.2	5.1
5	6.7	6.6	6.5	6.4
6	8.0	7.9	7.8	7.7
7	9.3	9.2	9.1	9.0
8	10.7	10.5	10.4	10.3
9	12.0	11.8	11.7	11.6
10	13.3	13.2	13.0	12.8
20	26.7	26.3	26.0	25.7
30	40.0	39.5	39.0	38.5
40	53.3	52.7	52.0	51.3
50	66.7	65.8	65.0	64.2

	76	75	74	73
1	1.3	1.2	1.2	1.2
2	2.5	2.5	2.5	2.4
3	3.8	3.8	3.7	3.6
4	5.1	5.0	4.9	4.9
5	6.3	6.2	6.2	6.1
6	7.6	7.5	7.4	7.3
7	8.9	8.8	8.6	8.5
8	10.1	10.0	9.9	9.7
9	11.4	11.2	11.1	11.0
10	12.7	12.5	12.3	12.2
20	25.3	25.0	24.7	24.3
30	38.0	37.5	37.0	36.5
40	50.7	50.0	49.3	48.7
50	63.3	62.5	61.7	60.8

	72	71	3	2
1	1.2	1.2	0.0	0.0
2	2.4	2.4	0.1	0.1
3	3.6	3.6	0.2	0.1
4	4.8	4.7	0.2	0.1
5	6.0	5.9	0.2	0.2
6	7.2	7.1	0.3	0.2
7	8.4	8.3	0.4	0.2
8	9.6	9.5	0.4	0.3
9	10.8	10.6	0.4	0.3
10	12.0	11.8	0.5	0.3
20	24.0	23.7	1.0	0.7
30	36.0	35.5	1.5	1.0
40	48.0	47.3	2.0	1.3
50	60.0	59.2	2.5	1.7

P. P.

10°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′
0	9.23 967		9.24 632		0.75 368	9.99 335		**60**
1	9.24 039	72	9.24 706	74	0.75 294	9.99 333	2	59
2	9.24 110	71	9.24 779	73	0.75 221	9.99 331	2	58
3	9.24 181	71	9.24 853	74	0.75 147	9.99 328	3	57
4	9.24 253	72	9.24 926	73	0.75 074	9.99 326	2	56
5	9.24 324	71	9.25 000	74	0.75 000	9.99 324	2	55
6	9.24 395	71	9.25 073	73	0.74 927	9.99 322	2	54
7	9.24 466	71	9.25 146	73	0.74 854	9.99 319	3	53
8	9.24 536	70	9.25 219	73	0.74 781	9.99 317	2	52
9	9.24 607	71	9.25 292	73	0.74 708	9.99 315	2	51
10	9.24 677	70	9.25 365	73	0.74 635	9.99 313	2	**50**
11	9.24 748	71	9.25 437	72	0.74 563	9.99 310	3	49
12	9.24 818	70	9.25 510	73	0.74 490	9.99 308	2	48
13	9.24 888	70	9.25 582	72	0.74 418	9.99 306	2	47
14	9.24 958	70	9.25 655	73	0.74 345	9.99 304	2	46
15	9.25 028	70	9.25 727	72	0.74 273	9.99 301	3	45
16	9.25 098	70	9.25 799	72	0.74 201	9.99 299	2	44
17	9.25 168	70	9.25 871	72	0.74 129	9.99 297	2	43
18	9.25 237	69	9.25 943	72	0.74 057	9.99 294	3	42
19	9.25 307	70	9.26 015	72	0.73 985	9.99 292	2	41
20	9.25 376	69	9.26 086	71	0.73 914	9.99 290	2	**40**
21	9.25 445	69	9.26 158	72	0.73 842	9.99 288	2	39
22	9.25 514	69	9.26 229	71	0.73 771	9.99 285	3	38
23	9.25 583	69	9.26 301	72	0.73 699	9.99 283	2	37
24	9.25 652	69	9.26 372	71	0.73 628	9.99 281	2	36
25	9.25 721	69	9.26 443	71	0.73 557	9.99 278	3	35
26	9.25 790	69	9.26 514	71	0.73 486	9.99 276	2	34
27	9.25 858	68	9.26 585	71	0.73 415	9.99 274	2	33
28	9.25 927	69	9.26 655	70	0.73 345	9.99 271	3	32
29	9.25 995	68	9.26 726	71	0.73 274	9.99 269	2	31
30	9.26 063	68	9.26 797	71	0.73 203	9.99 267	2	**30**
31	9.26 131	68	9.26 867	70	0.73 133	9.99 264	3	29
32	9.26 199	68	9.26 937	70	0.73 063	9.99 262	2	28
33	9.26 267	68	9.27 008	71	0.72 992	9.99 260	2	27
34	9.26 335	68	9.27 078	70	0.72 922	9.99 257	3	26
35	9.26 403	68	9.27 148	70	0.72 852	9.99 255	2	25
36	9.26 470	67	9.27 218	70	0.72 782	9.99 252	3	24
37	9.26 538	68	9.27 288	70	0.72 712	9.99 250	2	23
38	9.26 605	67	9.27 357	69	0.72 643	9.99 248	2	22
39	9.26 672	67	9.27 427	70	0.72 573	9.99 245	3	21
40	9.26 739	67	9.27 496	69	0.72 504	9.99 243	2	**20**
41	9.26 806	67	9.27 566	70	0.72 434	9.99 241	2	19
42	9.26 873	67	9.27 635	69	0.72 365	9.99 238	3	18
43	9.26 940	67	9.27 704	69	0.72 296	9.99 236	2	17
44	9.27 007	67	9.27 773	69	0.72 227	9.99 233	3	16
45	9.27 073	66	9.27 842	69	0.72 158	9.99 231	2	15
46	9.27 140	67	9.27 911	69	0.72 089	9.99 229	2	14
47	9.27 206	66	9.27 980	69	0.72 020	9.99 226	3	13
48	9.27 273	67	9.28 049	69	0.71 951	9.99 224	2	12
49	9.27 339	66	9.28 117	68	0.71 883	9.99 221	2	11
50	9.27 405	66	9.28 186	69	0.71 814	9.99 219	2	**10**
51	9.27 471	66	9.28 254	68	0.71 746	9.99 217	2	9
52	9.27 537	66	9.28 323	69	0.71 677	9.99 214	3	8
53	9.27 602	65	9.28 391	68	0.71 609	9.99 212	2	7
54	9.27 668	66	9.28 459	68	0.71 541	9.99 209	3	6
55	9.27 734	66	9.28 527	68	0.71 473	9.99 207	2	5
56	9.27 799	65	9.28 595	68	0.71 405	9.99 204	3	4
57	9.27 864	65	9.28 662	67	0.71 338	9.99 202	2	3
58	9.27 930	66	9.28 730	68	0.71 270	9.99 200	2	2
59	9.27 995	65	9.28 798	68	0.71 202	9.99 197	3	1
60	9.28 060	65	9.28 865	67	0.71 135	9.99 195	2	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	74	73	72
1	1.2	1.2	1.2
2	2.5	2.4	2.4
3	3.7	3.6	3.6
4	4.9	4.9	4.8
5	6.2	6.1	6.0
6	7.4	7.3	7.2
7	8.6	8.5	8.4
8	9.9	9.7	9.6
9	11.1	11.0	10.8
10	12.3	12.2	12.0
20	24.7	24.3	24.0
30	37.0	36.5	36.0
40	49.3	48.7	48.0
50	61.7	60.8	60.0

	71	70	69
1	1.2	1.2	1.2
2	2.4	2.3	2.3
3	3.6	3.5	3.4
4	4.7	4.7	4.6
5	5.9	5.8	5.8
6	7.1	7.0	6.9
7	8.3	8.2	8.0
8	9.5	9.3	9.2
9	10.6	10.5	10.4
10	11.8	11.7	11.5
20	23.7	23.3	23.0
30	35.5	35.0	34.5
40	47.3	46.7	46.0
50	59.2	58.3	57.5

	68	67	66
1	1.1	1.1	1.1
2	2.3	2.2	2.2
3	3.4	3.4	3.3
4	4.5	4.5	4.4
5	5.7	5.6	5.5
6	6.8	6.7	6.6
7	7.9	7.8	7.7
8	9.1	8.9	8.8
9	10.2	10.0	9.9
10	11.3	11.2	11.0
20	22.7	22.3	22.0
30	34.0	33.5	33.0
40	45.3	44.7	44.0
50	56.7	55.8	55.0

11°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.28 060		9.28 865		0.71 135	9.99 195		**60**
1	9.28 125	65	9.28 933	68	0.71 067	9.99 192	3	59
2	9.28 190	65	9.29 000	67	0.71 000	9.99 190	2	58
3	9.28 254	64	9.29 067	67	0.70 933	9.99 187	3	57
4	9.28 319	65	9.29 134	67	0.70 866	9.99 185	2	56
5	9.28 384	65	9.29 201		0.70 799	9.99 182	3	55
6	9.28 448	64	9.29 268	67	0.70 732	9.99 180	2	54
7	9.28 512	64	9.29 335	67	0.70 665	9.99 177	3	53
8	9.28 577	65	9.29 402	67	0.70 598	9.99 175	2	52
9	9.28 641	64	9.29 468	66	0.70 532	9.99 172	3	51
10	9.28 705	64	9.29 535	67	0.70 465	9.99 170	2	**50**
11	9.28 769	64	9.29 601	66	0.70 399	9.99 167	3	49
12	9.28 833	64	9.29 668	67	0.70 332	9.99 165	2	48
13	9.28 896	63	9.29 734	66	0.70 266	9.99 162	3	47
14	9.28 960	64	9.29 800	66	0.70 200	9.99 160	2	46
15	9.29 024	64	9.29 866	66	0.70 134	9.99 157	3	45
16	9.29 087	63	9.29 932	66	0.70 068	9.99 155	2	44
17	9.29 150	63	9.29 998	66	0.70 002	9.99 152	3	43
18	9.29 214	64	9.30 064	66	0.69 936	9.99 150	2	42
19	9.29 277	63	9.30 130	66	0.69 870	9.99 147	3	41
20	9.29 340	63	9.30 195	65	0.69 805	9.99 145	2	**40**
21	9.29 403	63	9.30 261	66	0.69 739	9.99 142	3	39
22	9.29 466	63	9.30 326	65	0.69 674	9.99 140	2	38
23	9.29 529	63	9.30 391	65	0.69 609	9.99 137	3	37
24	9.29 591	62	9.30 457	66	0.69 543	9.99 135	2	36
25	9.29 654	63	9.30 522	65	0.69 478	9.99 132	3	35
26	9.29 716	62	9.30 587	65	0.69 413	9.99 130	2	34
27	9.29 779	63	9.30 652	65	0.69 348	9.99 127	3	33
28	9.29 841	62	9.30 717	65	0.69 283	9.99 124	3	32
29	9.29 903	62	9.30 782	65	0.69 218	9.99 122	2	31
30	9.29 966	63	9.30 846	64	0.69 154	9.99 119	3	**30**
31	9.30 028	62	9.30 911	65	0.69 089	9.99 117	2	29
32	9.30 090	62	9.30 975	64	0.69 025	9.99 114	3	28
33	9.30 151	61	9.31 040	65	0.68 960	9.99 112	2	27
34	9.30 213	62	9.31 104	64	0.68 896	9.99 109	3	26
35	9.30 275	62	9.31 168	64	0.68 832	9.99 106	3	25
36	9.30 336	61	9.31 233	65	0.68 767	9.99 104	2	24
37	9.30 398	62	9.31 297	64	0.68 703	9.99 101	3	23
38	9.30 459	61	9.31 361	64	0.68 639	9.99 099	2	22
39	9.30 521	62	9.31 425	64	0.68 575	9.99 096	3	21
40	9.30 582	61	9.31 489	64	0.68 511	9.99 093	3	**20**
41	9.30 643	61	9.31 552	63	0.68 448	9.99 091	2	19
42	9.30 704	61	9.31 616	64	0.68 384	9.99 088	3	18
43	9.30 765	61	9.31 679	63	0.68 321	9.99 086	2	17
44	9.30 826	61	9.31 743	64	0.68 257	9.99 083	3	16
45	9.30 887	61	9.31 806	63	0.68 194	9.99 080	3	15
46	9.30 947	60	9.31 870	64	0.68 130	9.99 078	2	14
47	9.31 008	61	9.31 933	63	0.68 067	9.99 075	3	13
48	9.31 068	60	9.31 996	63	0.68 004	9.99 072	3	12
49	9.31 129	61	9.32 059	63	0.67 941	9.99 070	2	11
50	9.31 189	60	9.32 122	63	0.67 878	9.99 067	3	**10**
51	9.31 250	61	9.32 185	63	0.67 815	9.99 064	3	9
52	9.31 310	60	9.32 248	63	0.67 752	9.99 062	2	8
53	9.31 370	60	9.32 311	63	0.67 689	9.99 059	3	7
54	9.31 430	60	9.32 373	62	0.67 627	9.99 056	3	6
55	9.31 490	60	9.32 436	63	0.67 564	9.99 054	2	5
56	9.31 549	59	9.32 498	62	0.67 502	9.99 051	3	4
57	9.31 609	60	9.32 561	63	0.67 439	9.99 048	3	3
58	9.31 669	60	9.32 623	62	0.67 377	9.99 046	2	2
59	9.31 728	59	9.32 685	62	0.67 315	9.99 043	3	1
60	9.31 788	60	9.32 747	62	0.67 253	9.99 040	3	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	65	64	63
1	1.1	1.1	1.0
2	2.2	2.1	2.1
3	3.2	3.2	3.2
4	4.3	4.3	4.2
5	5.4	5.3	5.2
6	6.5	6.4	6.3
7	7.6	7.5	7.4
8	8.7	8.5	8.4
9	9.8	9.6	9.4
10	10.8	10.7	10.5
20	21.7	21.3	21.0
30	32.5	32.0	31.5
40	43.3	42.7	42.0
50	54.2	53.3	52.5

	62	61	60
1	1.0	1.0	1.0
2	2.1	2.0	2.0
3	3.1	3.0	3.0
4	4.1	4.1	4.0
5	5.2	5.1	5.0
6	6.2	6.1	6.0
7	7.2	7.1	7.0
8	8.3	8.1	8.0
9	9.3	9.2	9.0
10	10.3	10.2	10.0
20	20.7	20.3	20.0
30	31.0	30.5	30.0
40	41.3	40.7	40.0
50	51.7	50.8	50.0

	59	3	2
1	1.0	0.0	0.0
2	2.0	0.1	0.1
3	3.0	0.2	0.1
4	3.9	0.2	0.1
5	4.9	0.2	0.2
6	5.9	0.3	0.2
7	6.9	0.4	0.2
8	7.9	0.4	0.3
9	8.8	0.4	0.3
10	9.8	0.5	0.3
20	19.7	1.0	0.7
30	29.5	1.5	1.0
40	39.3	2.0	1.3
50	49.2	2.5	1.7

P. P.

12°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′
0	9.31 788		9.32 747		0.67 253	9.99 040		**60**
1	9.31 847	59	9.32 810	63	0.67 190	9.99 038	2	59
2	9.31 907	60	9.32 872	62	0.67 128	9.99 035	3	58
3	9.31 966	59	9.32 933	61	0.67 067	9.99 032	3	57
4	9.32 025	59	9.32 995	62	0.67 005	9.99 030	2	56
		59		62			3	
5	9.32 084		9.33 057		0.66 943	9.99 027	3	55
6	9.32 143	59	9.33 119	62	0.66 881	9.99 024	3	54
7	9.32 202	59	9.33 180	61	0.66 820	9.99 022	2	53
8	9.32 261	59	9.33 242	62	0.66 758	9.99 019	3	52
9	9.32 319	58	9.33 303	61	0.66 697	9.99 016	3	51
		59		62			3	
10	9.32 378		9.33 365		0.66 635	9.99 013		**50**
11	9.32 437	59	9.33 426	61	0.66 574	9.99 011	2	49
12	9.32 495	58	9.33 487	61	0.66 513	9.99 008	3	48
13	9.32 553	58	9.33 548	61	0.66 452	9.99 005	3	47
14	9.32 612	59	9.33 609	61	0.66 391	9.99 002	3	46
		58		61			2	
15	9.32 670		9.33 670		0.66 330	9.99 000		45
16	9.32 728	58	9.33 731	61	0.66 269	9.98 997	3	44
17	9.32 786	58	9.33 792	61	0.66 208	9.98 994	3	43
18	9.32 844	58	9.33 853	61	0.66 147	9.98 991	3	42
19	9.32 902	58	9.33 913	60	0.66 087	9.98 989	2	41
		58		61			3	
20	9.32 960		9.33 974		0.66 026	9.98 986		**40**
21	9.33 018	58	9.34 034	60	0.65 966	9.98 983	3	39
22	9.33 075	57	9.34 095	61	0.65 905	9.98 980	3	38
23	9.33 133	58	9.34 155	60	0.65 845	9.98 978	2	37
24	9.33 190	57	9.34 215	60	0.65 785	9.98 975	3	36
		58		61			3	
25	9.33 248		9.34 276		0.65 724	9.98 972	3	35
26	9.33 305	57	9.34 336	60	0.65 664	9.98 969	3	34
27	9.33 362	57	9.34 396	60	0.65 604	9.98 967	2	33
28	9.33 420	58	9.34 456	60	0.65 544	9.98 964	3	32
29	9.33 477	57	9.34 516	60	0.65 484	9.98 961	3	31
		57		60			3	
30	9.33 534		9.34 576		0.65 424	9.98 958	3	**30**
31	9.33 591	57	9.34 635	59	0.65 365	9.98 955	3	29
32	9.33 647	56	9.34 695	60	0.65 305	9.98 953	2	28
33	9.33 704	57	9.34 755	60	0.65 245	9.98 950	3	27
34	9.33 761	57	9.34 814	59	0.65 186	9.98 947	3	26
		57		59			3	
35	9.33 818		9.34 874		0.65 126	9.98 944	3	25
36	9.33 874	56	9.34 933	59	0.65 067	9.98 941	3	24
37	9.33 931	57	9.34 992	59	0.65 008	9.98 938	3	23
38	9.33 987	56	9.35 051	59	0.64 949	9.98 936	2	22
39	9.34 043	56	9.35 111	60	0.64 889	9.98 933	3	21
		57		59			3	
40	9.34 100		9.35 170		0.64 830	9.98 930		**20**
41	9.34 156	56	9.35 229	59	0.64 771	9.98 927	3	19
42	9.34 212	56	9.35 288	59	0.64 712	9.98 924	3	18
43	9.34 268	56	9.35 347	59	0.64 653	9.98 921	3	17
44	9.34 324	56	9.35 405	58	0.64 595	9.98 919	2	16
		56		59			3	
45	9.34 380		9.35 464		0.64 536	9.98 916	3	15
46	9.34 436	56	9.35 523	59	0.64 477	9.98 913	3	14
47	9.34 491	55	9.35 581	58	0.64 419	9.98 910	3	13
48	9.34 547	56	9.35 640	59	0.64 360	9.98 907	3	12
49	9.34 602	55	9.35 698	58	0.64 302	9.98 904	3	11
		56		59			3	
50	9.34 658		9.35 757		0.64 243	9.98 901		**10**
51	9.34 713	55	9.35 815	58	0.64 185	9.98 898	3	9
52	9.34 769	56	9.35 873	58	0.64 127	9.98 896	2	8
53	9.34 824	55	9.35 931	58	0.64 069	9.98 893	3	7
54	9.34 879	55	9.35 989	58	0.64 011	9.98 890	3	6
		55		58			3	
55	9.34 934		9.36 047		0.63 953	9.98 887	3	5
56	9.34 989	55	9.36 105	58	0.63 895	9.98 884	3	4
57	9.35 044	55	9.36 163	58	0.63 837	9.98 881	3	3
58	9.35 099	55	9.36 221	58	0.63 779	9.98 878	3	2
59	9.35 154	55	9.36 279	58	0.63 721	9.98 875	3	1
		55		57			3	
60	9.35 209		9.36 336		0.63 664	9.98 872		**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	63	62	61
1	1.0	1.0	1.0
2	2.1	2.1	2.0
3	3.2	3.1	3.0
4	4.2	4.1	4.1
5	5.2	5.2	5.1
6	6.3	6.2	6.1
7	7.4	7.2	7.1
8	8.4	8.3	8.1
9	9.4	9.3	9.2
10	10.5	10.3	10.2
20	21.0	20.7	20.3
30	31.5	31.0	30.5
40	42.0	41.3	40.7
50	52.5	51.7	50.8

	60	59	58
1	1.0	1.0	1.0
2	2.0	2.0	1.9
3	3.0	3.0	2.9
4	4.0	3.9	3.9
5	5.0	4.9	4.8
6	6.0	5.9	5.8
7	7.0	6.9	6.8
8	8.0	7.9	7.7
9	9.0	8.8	8.7
10	10.0	9.8	9.7
20	20.0	19.7	19.3
30	30.0	29.5	29.0
40	40.0	39.3	38.7
50	50.0	49.2	48.3

	57	56	55
1	1.0	0.9	0.9
2	1.9	1.9	1.8
3	2.8	2.8	2.8
4	3.8	3.7	3.7
5	4.8	4.7	4.6
6	5.7	5.6	5.5
7	6.6	6.5	6.4
8	7.6	7.5	7.3
9	8.6	8.4	8.2
10	9.5	9.3	9.2
20	19.0	18.7	18.3
30	28.5	28.0	27.5
40	38.0	37.3	36.7
50	47.5	46.7	45.8

13°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.35 209		9.36 336		0.63 664	9.98 872		**60**
1	9.35 263	54	9.36 394	58	0.63 606	9.98 869	3	59
2	9.35 318	55	9.36 452	58	0.63 548	9.98 867	2	58
3	9.35 373	55	9.36 509	57	0.63 491	9.98 864	3	57
4	9.35 427	54	9.36 566	57	0.63 434	9.98 861	3	56
		54		58			3	
5	9.35 481		9.36 624		0.63 376	9.98 858		55
6	9.35 536	55	9.36 681	57	0.63 319	9.98 855	3	**54**
7	9.35 590	54	9.36 738	57	0.63 262	9.98 852	3	53
8	9.35 644	54	9.36 795	57	0.63 205	9.98 849	3	52
9	9.35 698	54	9.36 852	57	0.63 148	9.98 846	3	51
		54		57			3	
10	9.35 752		9.36 909		0.63 091	9.98 843		**50**
11	9.35 806	54	9.36 966	57	0.63 034	9.98 840	3	49
12	9.35 860	54	9.37 023	57	0.62 977	9.98 837	3	48
13	9.35 914	54	9.37 080	57	0.62 920	9.98 834	3	47
14	9.35 968	54	9.37 137	57	0.62 863	9.98 831	3	46
		54		56			3	
15	9.36 022		9.37 193		0.62 807	9.98 828		45
16	9.36 075	53	9.37 250	57	0.62 750	9.98 825	3	44
17	9.36 129	54	9.37 306	56	0.62 694	9.98 822	3	43
18	9.36 182	53	9.37 363	57	0.62 637	9.98 819	3	42
19	9.36 236	54	9.37 419	56	0.62 581	9.98 816	3	41
		53		57			3	
20	9.36 289		9.37 476		0.62 524	9.98 813		**40**
21	9.36 342	53	9.37 532	56	0.62 468	9.98 810	3	39
22	9.36 395	53	9.37 588	56	0.62 412	9.98 807	3	38
23	9.36 449	54	9.37 644	56	0.62 356	9.98 804	3	37
24	9.36 502	53	9.37 700	56	0.62 300	9.98 801	3	36
		53		56			3	
25	9.36 555		9.37 756		0.62 244	9.98 798		35
26	9.36 608	53	9.37 812	56	0.62 188	9.98 795	3	34
27	9.36 660	52	9.37 868	56	0.62 132	9.98 792	3	33
28	9.36 713	53	9.37 924	56	0.62 076	9.98 789	3	32
29	9.36 766	53	9.37 980	56	0.62 020	9.98 786	3	31
		53		55			3	
30	9.36 819		9.38 035		0.61 965	9.98 783		**30**
31	9.36 871	52	9.38 091	56	0.61 909	9.98 780	3	29
32	9.36 924	53	9.38 147	56	0.61 853	9.98 777	3	28
33	9.36 976	52	9.38 202	55	0.61 798	9.98 774	3	27
34	9.37 028	52	9.38 257	55	0.61 743	9.98 771	3	26
		53		56			3	
35	9.37 081		9.38 313		0.61 687	9.98 768		25
36	9.37 133	52	9.38 368	55	0.61 632	9.98 765	3	24
37	9.37 185	52	9.38 423	55	0.61 577	9.98 762	3	23
38	9.37 237	52	9.38 479	56	0.61 521	9.98 759	3	22
39	9.37 289	52	9.38 534	55	0.61 466	9.98 756	3	21
		52		55			3	
40	9.37 341		9.38 589		0.61 411	9.98 753		**20**
41	9.37 393	52	9.38 644	55	0.61 356	9.98 750	3	19
42	9.37 445	52	9.38 699	55	0.61 301	9.98 746	4	18
43	9.37 497	52	9.38 754	55	0.61 246	9.98 743	3	17
44	9.37 549	52	9.38 808	54	0.61 192	9.98 740	3	16
		51		55			3	
45	9.37 600		9.38 863		0.61 137	9.98 737		15
46	9.37 652	52	9.38 918	55	0.61 082	9.98 734	3	14
47	9.37 703	51	9.38 972	54	0.61 028	9.98 731	3	13
48	9.37 755	52	9.39 027	55	0.60 973	9.98 728	3	12
49	9.37 806	51	9.39 082	55	0.60 918	9.98 725	3	11
		52		54			3	
50	9.37 858		9.39 136		0.60 864	9.98 722		**10**
51	9.37 909	51	9.39 190	54	0.60 810	9.98 719	3	9
52	9.37 960	51	9.39 245	55	0.60 755	9.98 715	4	8
53	9.38 011	51	9.39 299	54	0.60 701	9.98 712	3	7
54	9.38 062	51	9.39 353	54	0.60 647	9.98 709	3	6
		51		54			3	
55	9.38 113		9.39 407		0.60 593	9.98 706		5
56	9.38 164	51	9.39 461	54	0.60 539	9.98 703	3	4
57	9.38 215	51	9.39 515	54	0.60 485	9.98 700	3	3
58	9.38 266	51	9.39 569	54	0.60 431	9.98 697	3	2
59	9.38 317	51	9.39 623	54	0.60 377	9.98 694	3	1
60	9.38 368		9.39 677		0.60 323	9.98 690	4	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	57	56	55
1	1.0	0.9	0.9
2	1.9	1.9	1.8
3	2.8	2.8	2.8
4	3.8	3.7	3.7
5	4.8	4.7	4.6
6	5.7	5.6	5.5
7	6.6	6.5	6.4
8	7.6	7.5	7.3
9	8.6	8.4	8.2
10	9.5	9.3	9.2
20	19.0	18.7	18.3
30	28.5	28.0	27.5
40	38.0	37.3	36.7
50	47.5	46.7	45.8

	54	53	52
1	0.9	0.9	0.9
2	1.8	1.8	1.7
3	2.7	2.6	2.6
4	3.6	3.5	3.5
5	4.5	4.4	4.3
5	5.4	5.3	5.2
7	6.3	6.2	6.1
8	7.2	7.1	6.9
9	8.1	8.0	7.8
10	9.0	8.8	8.7
20	18.0	17.7	17.3
30	27.0	26.5	26.0
40	36.0	35.3	34.7
50	45.0	44.2	43.3

	51	4	3	2
1	0.8	0.1	0.0	0.0
2	1.7	0.1	0.1	0.1
3	2.6	0.2	0.2	0.1
4	3.4	0.3	0.2	0.1
5	4.2	0.3	0.2	0.2
6	5.1	0.4	0.3	0.2
7	6.0	0.5	0.4	0.2
8	6.8	0.5	0.4	0.3
9	7.6	0.6	0.4	0.3
10	8.5	0.7	0.5	0.3
20	17.0	1.3	1.0	0.7
30	25.5	2.0	1.5	1.0
40	34.0	2.7	2.0	1.3
50	42.5	3.3	2.5	1.7

14°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.38 368		9.39 677		0.60 323	9.98 690		**60**
1	9.38 418	50	9.39 731	54	0.60 269	9.98 687	3	59
2	9.38 469	51	9.39 785	54	0.60 215	9.98 684	3	58
3	9.38 519	50	9.39 838	53	0.60 162	9.98 681	3	57
4	9.38 570	51	9.39 892	54	0.60 108	9.98 678	3	56
5	9.38 620	50	9.39 945	53	0.60 055	9.98 675	3	55
6	9.38 670	50	9.39 999	54	0.60 001	9.98 671	4	54
7	9.38 721	51	9.40 052	53	0.59 948	9.98 668	3	53
8	9.38 771	50	9.40 106	54	0.59 894	9.98 665	3	52
9	9.38 821	50	9.40 159	53	0.59 841	9.98 662	3	51
10	9.38 871	50	9.40 212	53	0.59 788	9.98 659	3	**50**
11	9.38 921	50	9.40 266	54	0.59 734	9.98 656	3	49
12	9.38 971	50	9.40 319	53	0.59 681	9.98 652	4	48
13	9.39 021	50	9.40 372	53	0.59 628	9.98 649	3	47
14	9.39 071	50	9.40 425	53	0.59 575	9.98 646	3	46
15	9.39 121	50	9.40 478	53	0.59 522	9.98 643	3	45
16	9.39 170	49	9.40 531	53	0.59 469	9.98 640	4	44
17	9.39 220	50	9.40 584	53	0.59 416	9.98 636	3	43
18	9.39 270	50	9.40 636	52	0.59 364	9.98 633	3	42
19	9.39 319	49	9.40 689	53	0.59 311	9.98 630	3	41
20	9.39 369	50	9.40 742	53	0.59 258	9.98 627	3	**40**
21	9.39 418	49	9.40 795	53	0.59 205	9.98 623	4	39
22	9.39 467	49	9.40 847	52	0.59 153	9.98 620	3	38
23	9.39 517	50	9.40 900	53	0.59 100	9.98 617	3	37
24	9.39 566	49	9.40 952	52	0.59 048	9.98 614	3	36
25	9.39 615	49	9.41 005	53	0.58 995	9.98 610	4	35
26	9.39 664	49	9.41 057	52	0.58 943	9.98 607	3	34
27	9.39 713	49	9.41 109	52	0.58 891	9.98 604	3	33
28	9.39 762	49	9.41 161	52	0.58 839	9.98 601	3	32
29	9.39 811	49	9.41 214	53	0.58 786	9.98 597	4	31
30	9.39 860	49	9.41 266	52	0.58 734	9.98 594	3	**30**
31	9.39 909	49	9.41 318	52	0.58 682	9.98 591	3	29
32	9.39 958	48	9.41 370	52	0.58 630	9.98 588	3	28
33	9.40 006	49	9.41 422	52	0.58 578	9.98 584	4	27
34	9.40 055	48	9.41 474	52	0.58 526	9.98 581	3	26
35	9.40 103	49	9.41 526	52	0.58 474	9.98 578	3	25
36	9.40 152	48	9.41 578	52	0.58 422	9.98 574	4	24
37	9.40 200	49	9.41 629	51	0.58 371	9.98 571	3	23
38	9.40 249	48	9.41 681	52	0.58 319	9.98 568	3	22
39	9.40 297	48	9.41 733	52	0.58 267	9.98 565	3	21
40	9.40 346	49	9.41 784	51	0.58 216	9.98 561	4	**20**
41	9.40 394	48	9.41 836	52	0.58 164	9.98 558	3	19
42	9.40 442	48	9.41 887	51	0.58 113	9.98 555	3	18
43	9.40 490	48	9.41 939	52	0.58 061	9.98 551	4	17
44	9.40 538	48	9.41 990	51	0.58 010	9.98 548	3	16
45	9.40 586	48	9.42 041	51	0.57 959	9.98 545	3	15
46	9.40 634	48	9.42 093	52	0.57 907	9.98 541	4	14
47	9.40 682	48	9.42 144	51	0.57 856	9.98 538	3	13
48	9.40 730	48	9.42 195	51	0.57 805	9.98 535	3	12
49	9.40 778	48	9.42 246	51	0.57 754	9.98 531	4	11
50	9.40 825	47	9.42 297	51	0.57 703	9.98 528	3	**10**
51	9.40 873	48	9.42 348	51	0.57 652	9.98 525	3	9
52	9.40 921	48	9.42 399	51	0.57 601	9.98 521	4	8
53	9.40 968	47	9.42 450	51	0.57 550	9.98 518	3	7
54	9.41 016	48	9.42 501	51	0.57 499	9.98 515	3	6
55	9.41 063	47	9.42 552	51	0.57 448	9.98 511	4	5
56	9.41 111	48	9.42 603	51	0.57 397	9.98 508	3	4
57	9.41 158	47	9.42 653	50	0.57 347	9.98 505	3	3
58	9.41 205	47	9.42 704	51	0.57 296	9.98 501	4	2
59	9.41 252	47	9.42 755	51	0.57 245	9.98 498	3	1
60	9.41 300	48	9.42 805	50	0.57 195	9.98 494	4	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	54	53	52
1	0.9	0.9	0.9
2	1.8	1.8	1.7
3	2.7	2.6	2.6
4	3.6	3.5	3.5
5	4.5	4.4	4.3
6	5.4	5.3	5.2
7	6.3	6.2	6.1
8	7.2	7.1	6.9
9	8.1	8.0	7.8
10	9.0	8.8	8.7
20	18.0	17.7	17.3
30	27.0	26.5	26.0
40	36.0	35.3	34.7
50	45.0	44.2	43.3

	51	50	49
1	0.8	0.8	0.8
2	1.7	1.7	1.6
3	2.6	2.5	2.4
4	3.4	3.3	3.3
5	4.2	4.2	4.1
6	5.1	5.0	4.9
7	6.0	5.8	5.7
8	6.8	6.7	6.5
9	7.6	7.5	7.4
10	8.5	8.3	8.2
20	17.0	16.7	16.3
30	25.5	25.0	24.5
40	34.0	33.3	32.7
50	42.5	41.7	40.8

	48	47	4	3
1	0.8	0.8	0.1	0.0
2	1.6	1.6	0.1	0.1
3	2.4	2.4	0.2	0.2
4	3.2	3.1	0.3	0.2
5	4.0	3.9	0.3	0.2
6	4.8	4.7	0.4	0.3
7	5.6	5.5	0.5	0.4
8	6.4	6.3	0.5	0.4
9	7.2	7.0	0.6	0.4
10	8.0	7.8	0.7	0.5
20	16.0	15.7	1.3	1.0
30	24.0	23.5	2.0	1.5
40	32.0	31.3	2.7	2.0
50	40.0	39.2	3.3	2.5

P. P.

75°

15°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.41 300		9.42 805		0.57 195	9.98 494		**60**
1	9.41 347	47	9.42 856	51	0.57 144	9.98 491	3	59
2	9.41 394	47	9.42 906	50	0.57 094	9.98 488	3	58
3	9.41 441	47	9.42 957	51	0.57 043	9.98 484	4	57
4	9.41 488	47	9.43 007	50	0.56 993	9.98 481	3	56
5	9.41 535	47	9.43 057	50	0.56 943	9.98 477	4	55
6	9.41 582	47	9.43 108	51	0.56 892	9.98 474	3	54
7	9.41 628	46	9.43 158	50	0.56 842	9.98 471	3	53
8	9.41 675	47	9.43 208	50	0.56 792	9.98 467	4	52
9	9.41 722	47	9.43 258	50	0.56 742	9.98 464	3	51
10	9.41 768	46	9.43 308	50	0.56 692	9.98 460	4	**50**
11	9.41 815	47	9.43 358	50	0.56 642	9.98 457	3	49
12	9.41 861	46	9.43 408	50	0.56 592	9.98 453	4	48
13	9.41 908	47	9.43 458	50	0.56 542	9.98 450	3	47
14	9.41 954	46	9.43 508	50	0.56 492	9.98 447	3	46
15	9.42 001	47	9.43 558	50	0.56 442	9.98 443	4	45
16	9.42 047	46	9.43 607	49	0.56 393	9.98 440	3	44
17	9.42 093	46	9.43 657	50	0.56 343	9.98 436	4	43
18	9.42 140	47	9.43 707	50	0.56 293	9.98 433	3	42
19	9.42 186	46	9.43 756	49	0.56 244	9.98 429	4	41
20	9.42 232	46	9.43 806	50	0.56 194	9.98 426	3	**40**
21	9.42 278	46	9.43 855	49	0.56 145	9.98 422	4	39
22	9.42 324	46	9.43 905	50	0.56 095	9.98 419	3	38
23	9.42 370	46	9.43 954	49	0.56 046	9.98 415	4	37
24	9.42 416	46	9.44 004	50	0.55 996	9.98 412	3	36
25	9.42 461	45	9.44 053	49	0.55 947	9.98 409	3	35
26	9.42 507	46	9.44 102	49	0.55 898	9.98 405	4	34
27	9.42 553	46	9.44 151	49	0.55 849	9.98 402	3	33
28	9.42 599	46	9.44 201	50	0.55 799	9.98 398	4	32
29	9.42 644	45	9.44 250	49	0.55 750	9.98 395	3	31
30	9.42 690	46	9.44 299	49	0.55 701	9.98 391	4	**30**
31	9.42 735	45	9.44 348	49	0.55 652	9.98 388	3	29
32	9.42 781	46	9.44 397	49	0.55 603	9.98 384	4	28
33	9.42 826	45	9.44 446	49	0.55 554	9.98 381	3	27
34	9.42 872	46	9.44 495	49	0.55 505	9.98 377	4	26
35	9.42 917	45	9.44 544	49	0.55 456	9.98 373	4	25
36	9.42 962	45	9.44 592	48	0.55 408	9.98 370	3	24
37	9.43 008	46	9.44 641	49	0.55 359	9.98 366	4	23
38	9.43 053	45	9.44 690	49	0.55 310	9.98 363	3	22
39	9.43 098	45	9.44 738	48	0.55 262	9.98 359	4	21
40	9.43 143	45	9.44 787	49	0.55 213	9.98 356	3	**20**
41	9.43 188	45	9.44 836	49	0.55 164	9.98 352	4	19
42	9.43 233	45	9.44 884	48	0.55 116	9.98 349	3	18
43	9.43 278	45	9.44 933	49	0.55 067	9.98 345	4	17
44	9.43 323	45	9.44 981	48	0.55 019	9.98 342	3	16
45	9.43 367	44	9.45 029	48	0.54 971	9.98 338	4	15
46	9.43 412	45	9.45 078	49	0.54 922	9.98 334	4	14
47	9.43 457	45	9.45 126	48	0.54 874	9.98 331	3	13
48	9.43 502	45	9.45 174	48	0.54 826	9.98 327	4	12
49	9.43 546	44	9.45 222	48	0.54 778	9.98 324	3	11
50	9.43 591	45	9.45 271	49	0.54 729	9.98 320	4	**10**
51	9.43 635	44	9.45 319	48	0.54 681	9.98 317	3	9
52	9.43 680	45	9.45 367	48	0.54 633	9.98 313	4	8
53	9.43 724	44	9.45 415	48	0.54 585	9.98 309	4	7
54	9.43 769	45	9.45 463	48	0.54 537	9.98 306	3	6
55	9.43 813	44	9.45 511	48	0.54 489	9.98 302	4	5
56	9.43 857	44	9.45 559	48	0.54 441	9.98 299	3	4
57	9.43 901	44	9.45 606	47	0.54 394	9.98 295	4	3
58	9.43 946	45	9.45 654	48	0.54 346	9.98 291	4	2
59	9.43 990	44	9.45 702	48	0.54 298	9.98 288	3	1
60	9.44 034	44	9.45 750	48	0.54 250	9.98 284	4	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	51	50	49
1	0.8	0.8	0.8
2	1.7	1.7	1.6
3	2.6	2.5	2.4
4	3.4	3.3	3.3
5	4.2	4.2	4.1
6	5.1	5.0	4.9
7	6.0	5.8	5.7
8	6.8	6.7	6.5
9	7.6	7.5	7.4
10	8.5	8.3	8.2
20	17.0	16.7	16.3
30	25.5	25.0	24.5
40	34.0	33.3	32.7
50	42.5	41.7	40.8

	48	47	46
1	0.8	0.8	0.8
2	1.6	1.6	1.5
3	2.4	2.4	2.3
4	3.2	3.1	3.1
5	4.0	3.9	3.8
6	4.8	4.7	4.6
7	5.6	5.5	5.4
8	6.4	6.3	6.1
9	7.2	7.0	6.9
10	8.0	7.8	7.7
20	16.0	15.7	15.3
30	24.0	23.5	23.0
40	32.0	31.3	30.7
50	40.0	39.2	38.3

	45	44	4	3
1	0.8	0.7	0.1	0.0
2	1.5	1.5	0.1	0.1
3	2.2	2.2	0.2	0.2
4	3.0	2.9	0.3	0.2
5	3.8	3.7	0.3	0.2
6	4.5	4.4	0.4	0.3
7	5.2	5.1	0.5	0.4
8	6.0	5.9	0.5	0.4
9	6.8	6.6	0.6	0.4
10	7.5	7.3	0.7	0.5
20	15.0	14.7	1.3	1.0
30	22.5	22.0	2.0	1.5
40	30.0	29.3	2.7	2.0
50	37.5	36.7	3.3	2.5

P. P.

74°

16°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.44 034		9.45 750		0.54 250	9.98 284		**60**
1	9.44 078	44	9.45 797	47	0.54 203	9.98 281	3	59
2	9.44 122	44	9.45 845	48	0.54 155	9.98 277	4	58
3	9.44 166	44	9.45 892	47	0.54 108	9.98 273	4	57
4	9.44 210	44	9.45 940	48	0.54 060	9.98 270	3	56
5	9.44 253	43	9.45 987	47	0.54 013	9.98 266	4	55
6	9.44 297	44	9.46 035	48	0.53 965	9.98 262	4	54
7	9.44 341	44	9.46 082	47	0.53 918	9.98 259	3	53
8	9.44 385	44	9.46 130	48	0.53 870	9.98 255	4	52
9	9.44 428	43	9.46 177	47	0.53 823	9.98 251	4	51
10	9.44 472	44	9.46 224	47	0.53 776	9.98 248	3	**50**
11	9.44 516	44	9.46 271	47	0.53 729	9.98 244	4	49
12	9.44 559	43	9.46 319	48	0.53 681	9.98 240	4	48
13	9.44 602	43	9.46 366	47	0.53 634	9.98 237	3	47
14	9.44 646	44	9.46 413	47	0.53 587	9.98 233	4	46
15	9.44 689	43	9.46 460	47	0.53 540	9.98 229	4	45
16	9 44 733	44	9.46 507	47	0.53 493	9.98 226	3	44
17	9.44 776	43	9.46 554	47	0.53 446	9.98 222	4	43
18	9.44 819	43	9.46 601	47	0.53 399	9.98 218	4	42
19	9.44 862	43	9.46 648	47	0.53 352	9.98 215	3	41
20	9.44 905	43	9.46 694	46	0.53 306	9.98 211	4	**40**
21	9.44 948	44	9.46 741	47	0.53 259	9.98 207	4	39
22	9.44 992	43	9.46 788	47	0.53 212	9.98 204	3	38
23	9.45 035	42	9.46 835	47	0.53 165	9.98 200	4	37
24	9.45 077	43	9.46 881	46	0.53 119	9.98 196	4	36
25	9.45 120	43	9.46 928	47	0.53 072	9.98 192	4	35
26	9.45 163	43	9.46 975	47	0.53 025	9.98 189	3	34
27	9.45 206	43	9.47 021	46	0.52 979	9.98 185	4	33
28	9.45 249	43	9.47 068	47	0.52 932	9.98 181	4	32
29	9.45 292	43	9.47 114	46	0.52 886	9.98 177	4	31
30	9.45 334	42	9.47 160	46	0.52 840	9.98 174	3	**30**
31	9.45 377	43	9.47 207	47	0.52 793	9.98 170	4	29
32	9.45 419	42	9.47 253	46	0.52 747	9.98 166	4	28
33	9.45 462	43	9.47 299	46	0.52 701	9.98 162	4	27
34	9.45 504	42	9.47 346	47	0.52 654	9.98 159	3	26
35	9.45 547	43	9.47 392	46	0.52 608	9.98 155	4	25
36	9.45 589	42	9.47 438	46	0.52 562	9.98 151	4	24
37	9.45 632	43	9.47 484	46	0.52 516	9.98 147	4	23
38	9.45 674	42	9.47 530	46	0.52 470	9.98 144	3	22
39	9.45 716	42	9.47 576	46	0.52 424	9.98 140	4	21
40	9.45 758	42	9.47 622	46	0.52 378	9.98 136	4	**20**
41	9.45 801	43	9.47 668	46	0.52 332	9.98 132	4	19
42	9.45 843	42	9.47 714	46	0.52 286	9.98 129	3	18
43	9.45 885	42	9.47 760	46	0.52 240	9.98 125	4	17
44	9.45 927	42	9.47 806	46	0.52 194	9.98 121	4	16
45	9.45 969	42	9.47 852	46	0.52 148	9.98 117	4	15
46	9.46 011	42	9.47 897	45	0.52 103	9.98 113	4	14
47	9.46 053	42	9.47 943	46	0.52 057	9.98 110	3	13
48	9.46 095	42	9.47 989	46	0.52 011	9.98 106	4	12
49	9.46 136	41	9.48 035	46	0.51 965	9.98 102	4	11
50	9.46 178	42	9.48 080	45	0.51 920	9.98 098	4	**10**
51	9.46 220	42	9.48 126	46	0.51 874	9.98 094	4	9
52	9.46 262	42	9.48 171	45	0.51 829	9.98 090	4	8
53	9.46 303	41	9.48 217	46	0.51 783	9.98 087	3	7
54	9.46 345	42	9.48 262	45	0.51 738	9.98 083	4	6
55	9.46 386	41	9.48 307	45	0.51 693	9.98 079	4	5
56	9.46 428	42	9.48 353	46	0.51 647	9.98 075	4	4
57	9.46 469	41	9.48 398	45	0.51 602	9.98 071	4	3
58	9.46 511	42	9.48 443	45	0.51 557	9.98 067	4	2
59	9.46 552	41	9.48 489	46	0.51 511	9.98 063	4	1
60	9.46 594	42	9.48 534	45	0.51 466	9.98 060	3	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	48	47	46
1	0.8	0.8	0.8
2	1.6	1.6	1.5
3	2.4	2.4	2.3
4	3.2	3.1	3.1
5	4.0	3.9	3.8
6	4.8	4.7	4.6
7	5.6	5.5	5.4
8	6.4	6.3	6.1
9	7.2	7.0	6.9
10	8.0	7.8	7.7
20	16.0	15.7	15.3
30	24.0	23.5	23.0
40	32.0	31.3	30.7
50	40.0	39.2	38.3

	45	44	43
1	0.8	0.7	0.7
2	1.5	1.5	1.4
3	2.2	2.2	2.2
4	3.0	2.9	2.9
5	3.8	3.7	3.6
6	4.5	4.4	4.3
7	5.2	5.1	5.0
8	6.0	5.9	5.7
9	6.8	6.6	6.4
10	7.5	7.3	7.2
20	15.0	14.7	14.3
30	22.5	22.0	21.5
40	30.0	29.3	28.7
50	37.5	36.7	35.8

	42	41	4	3
1	0.7	0.7	0.1	0.0
2	1.4	1.4	0.1	0.1
3	2.1	2.0	0.2	0.2
4	2.8	2.7	0.3	0.2
5	3.5	3.4	0.3	0.2
6	4.2	4.1	0.4	0.3
7	4.9	4.8	0.5	0.4
8	5.6	5.5	0.5	0.4
9	6.3	6.2	0.6	0.4
10	7.0	6.8	0.7	0.5
20	14.0	13.7	1.3	1.0
30	21.0	20.5	2.0	1.5
40	28.0	27.3	2.7	2.0
50	35.0	34.2	3.3	2.5

P. P.

17°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.46 594		9.48 534		0.51 466	9.98 060		**60**
1	9.46 635	41	9.48 579	45	0.51 421	9.98 056	4	59
2	9.46 676	41	9.48 624	45	0.51 376	9.98 052	4	58
3	9.46 717	41	9.48 669	45	0.51 331	9.98 048	4	57
4	9.46 758	41	9.48 714	45	0.51 286	9.98 044	4	56
5	9.46 800	42	9.48 759	45	0.51 241	9.98 040	4	55
6	9.46 841	41	9.48 804	45	0.51 196	9.98 036	4	54
7	9.46 882	41	9.48 849	45	0.51 151	9.98 032	4	53
8	9.46 923	41	9.48 894	45	0.51 106	9.98 029	3	52
9	9.46 964	41	9.48 939	45	0.51 061	9.98 025	4	51
10	9.47 005	41	9.48 984	45	0.51 016	9.98 021	4	**50**
11	9.47 045	40	9.49 029	45	0.50 971	9.98 017	4	49
12	9.47 086	41	9.49 073	44	0.50 927	9.98 013	4	48
13	9.47 127	41	9.49 118	45	0.50 882	9.98 009	4	47
14	9.47 168	41	9.49 163	45	0.50 837	9.98 005	4	46
15	9.47 209	41	9.49 207	44	0.50 793	9.98 001	4	45
16	9.47 249	40	9.49 252	45	0.50 748	9.97 997	4	44
17	9.47 290	41	9.49 296	44	0.50 704	9.97 993	4	43
18	9.47 330	40	9.49 341	45	0.50 659	9.97 989	4	42
19	9.47 371	41	9.49 385	44	0.50 615	9.97 986	3	41
20	9.47 411	40	9.49 430	45	0.50 570	9.97 982	4	**40**
21	9.47 452	41	9.49 474	44	0.50 526	9.97 978	4	39
22	9.47 492	40	9.49 519	45	0.50 481	9.97 974	4	38
23	9.47 533	41	9.49 563	44	0.50 437	9.97 970	4	37
24	9.47 573	40	9.49 607	44	0.50 393	9.97 966	4	36
25	9.47 613	40	9.49 652	45	0.50 348	9.97 962	4	35
26	9.47 654	41	9.49 696	44	0.50 304	9.97 958	4	34
27	9.47 694	40	9.49 740	44	0.50 260	9.97 954	4	33
28	9.47 734	40	9.49 784	44	0.50 216	9.97 950	4	32
29	9.47 774	40	9.49 828	44	0.50 172	9.97 946	4	31
30	9.47 814	40	9.49 872	44	0.50 128	9.97 942	4	**30**
31	9.47 854	40	9.49 916	44	0.50 084	9.97 938	4	29
32	9.47 894	40	9.49 960	44	0.50 040	9.97 934	4	28
33	9.47 934	40	9.50 004	44	0.49 996	9.97 930	4	27
34	9.47 974	40	9.50 048	44	0.49 952	9.97 926	4	26
35	9.48 014	40	9.50 092	44	0.49 908	9.97 922	4	25
36	9.48 054	40	9.50 136	44	0.49 864	9.97 918	4	24
37	9.48 094	40	9.50 180	44	0.49 820	9.97 914	4	23
38	9.48 133	39	9.50 223	43	0.49 777	9.97 910	4	22
39	9.48 173	40	9.50 267	44	0.49 733	9.97 906	4	21
40	9.48 213	40	9.50 311	44	0.49 689	9.97 902	4	**20**
41	9.48 252	39	9.50 355	44	0.49 645	9.97 898	4	19
42	9.48 292	40	9.50 398	43	0.49 602	9.97 894	4	18
43	9.48 332	40	9.50 442	44	0.49 558	9.97 890	4	17
44	9.48 371	39	9.50 485	43	0.49 515	9.97 886	4	16
45	9.48 411	40	9.50 529	44	0.49 471	9.97 882	4	15
46	9.48 450	39	9.50 572	43	0.49 428	9.97 878	4	14
47	9.48 490	40	9.50 616	44	0.49 384	9.97 874	4	13
48	9.48 529	39	9.50 659	43	0.49 341	9.97 870	4	12
49	9.48 568	39	9.50 703	44	0.49 297	9.97 866	4	11
50	9.48 607	39	9.50 746	43	0.49 254	9.97 861	5	**10**
51	9.48 647	40	9.50 789	43	0.49 211	9.97 857	4	9
52	9.48 686	39	9.50 833	44	0.49 167	9.97 853	4	8
53	9.48 725	39	9.50 876	43	0.49 124	9.97 849	4	7
54	9.48 764	39	9.50 919	43	0.49 081	9.97 845	4	6
55	9.48 803	39	9.50 962	43	0.49 038	9.97 841	4	5
56	9.48 842	39	9.51 005	43	0.48 995	9.97 837	4	4
57	9.48 881	39	9.51 048	43	0.48 952	9.97 833	4	3
58	9.48 920	39	9.51 092	44	0.48 908	9.97 829	4	2
59	9.48 959	39	9.51 135	43	0.48 865	9.97 825	4	1
60	9.48 998		9.51 178	43	0.48 822	9.97 821	4	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	45	44	43
1	0.8	0.7	0.7
2	1.5	1.5	1.4
3	2.2	2.2	2.2
4	3.0	2.9	2.9
5	3.8	3.7	3.6
6	4.5	4.4	4.3
7	5.2	5.1	5.0
8	6.0	5.9	5.7
9	6.8	6.6	6.4
10	7.5	7.3	7.2
20	15.0	14.7	14.3
30	22.5	22.0	21.5
40	30.0	29.3	28.7
50	37.5	36.7	35.8

	42	41	40
1	0.7	0.7	0.7
2	1.4	1.4	1.3
3	2.1	2.0	2.0
4	2.8	2.7	2.7
5	3.5	3.4	3.3
6	4.2	4.1	4.0
7	4.9	4.8	4.7
8	5.6	5.5	5.3
9	6.3	6.2	6.0
10	7.0	6.8	6.7
20	14.0	13.7	13.3
30	21.0	20.5	20.0
40	28.0	27.3	26.7
50	35.0	34.2	33.3

	39	5	4	3
1	0.6	0.1	0.1	0.0
2	1.3	0.2	0.1	0.1
3	2.0	0.2	0.2	0.2
4	2.6	0.3	0.3	0.2
5	3.2	0.4	0.3	0.2
6	3.9	0.5	0.4	0.3
7	4.6	0.6	0.5	0.4
8	5.2	0.7	0.5	0.4
9	5.8	0.8	0.6	0.4
10	6.5	0.8	0.7	0.5
20	13.0	1.7	1.3	1.0
30	19.5	2.5	2.0	1.5
40	26.0	3.3	2.7	2.0
50	32.5	4.2	3.3	2.5

18°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.48 998		9.51 178		0.48 822	9.97 821		**60**
1	9.49 037	39	9.51 221	43	0.48 779	9.97 817	4	59
2	9.49 076	39	9.51 264	43	0.48 736	9.97 812	5	58
3	9.49 115	39	9.51 306	42	0.48 694	9.97 808	4	57
4	9.49 153	38	9.51 349	43	0.48 651	9.97 804	4	56
		39		43			4	
5	9.49 192		9.51 392		0.48 608	9.97 800		55
6	9.49 231	39	9.51 435	43	0.48 565	9.97 796	4	54
7	9.49 269	38	9.51 478	43	0.48 522	9.97 792	4	53
8	9.49 308	39	9.51 520	42	0.48 480	9.97 788	4	52
9	9.49 347	39	9.51 563	43	0.48 437	9.97 784	4	51
		38		43			5	
10	9.49 385		9.51 606		0.48 394	9.97 779		**50**
11	9.49 424	39	9.51 648	42	0.48 352	9.97 775	4	49
12	9.49 462	38	9.51 691	43	0.48 309	9.97 771	4	48
13	9.49 500	38	9.51 734	43	0.48 266	9.97 767	4	47
14	9.49 539	39	9.51 776	42	0.48 224	9.97 763	4	46
		38		43			4	
15	9.49 577		9.51 819		0.48 181	9.97 759		45
16	9.49 615	38	9.51 861	42	0.48 139	9.97 754	5	44
17	9.49 654	39	9.51 903	42	0.48 097	9.97 750	4	43
18	9.49 692	38	9.51 946	43	0.48 054	9.97 746	4	42
19	9.49 730	38	9.51 988	42	0.48 012	9.97 742	4	41
		38		43			4	
20	9.49 768		9.52 031		0.47 969	9.97 738		**40**
21	9.49 806	38	9.52 073	42	0.47 927	9.97 734	4	39
22	9.49 844	38	9.52 115	42	0.47 885	9.97 729	5	38
23	9.49 882	38	9.52 157	42	0.47 843	9.97 725	4	37
24	9.49 920	38	9.52 200	43	0.47 800	9.97 721	4	36
		38		42			4	
25	9.49 958		9.52 242		0.47 758	9.97 717		35
26	9.49 996	38	9.52 284	42	0.47 716	9.97 713	4	34
27	9.50 034	38	9.52 326	42	0.47 674	9.97 708	5	33
28	9.50 072	38	9.52 368	42	0.47 632	9.97 704	4	32
29	9.50 110	38	9.52 410	42	0.47 590	9.97 700	4	31
		38		42			4	
30	9.50 148		9.52 452		0.47 548	9.97 696		**30**
31	9.50 185	37	9.52 494	42	0.47 506	9.97 691	5	29
32	9.50 223	38	9.52 536	42	0.47 464	9.97 687	4	28
33	9.50 261	38	9.52 578	42	0.47 422	9.97 683	4	27
34	9.50 298	37	9.52 620	42	0.47 380	9.97 679	4	26
		38		41			5	
35	9.50 336		9.52 661		0.47 339	9.97 674		25
36	9.50 374	38	9.52 703	42	0.47 297	9.97 670	4	24
37	9.50 411	37	9.52 745	42	0.47 255	9.97 666	4	23
38	9.50 449	38	9.52 787	42	0.47 213	9.97 662	4	22
39	9.50 486	37	9.52 829	42	0.47 171	9.97 657	5	21
		37		41			4	
40	9.50 523		9.52 870		0.47 130	9.97 653		**20**
41	9.50 561	38	9.52 912	42	0.47 088	9.97 649	4	19
42	9.50 598	37	9.52 953	41	0.47 047	9.97 645	4	18
43	9.50 635	37	9.52 995	42	0.47 005	9.97 640	5	17
44	9.50 673	38	9.53 037	42	0.46 963	9.97 636	4	16
		37		41			4	
45	9.50 710		9.53 078		0.46 922	9.97 632		15
46	9.50 747	37	9.53 120	42	0.46 880	9.97 628	4	14
47	9.50 784	37	9.53 161	41	0.46 839	9.97 623	5	13
48	9.50 821	37	9.53 202	41	0.46 798	9.97 619	4	12
49	9.50 858	37	9.53 244	42	0.46 756	9.97 615	4	11
		38		41			5	
50	9.50 896		9.53 285		0.46 715	9.97 610		**10**
51	9.50 933	37	9.53 327	42	0.46 673	9.97 606	4	9
52	9.50 907	37	9.53 368	41	0.46 632	9.97 602	4	8
53	9.51 007	37	9.53 409	41	0.46 591	9.97 597	5	7
54	9.51 043	36	9.53 450	41	0.46 550	9.97 593	4	6
		37		42			4	
55	9.51 080		9.53 492		0.46 508	9.97 589		5
56	9.51 117	37	9.53 533	41	0.46 467	9.97 584	5	4
57	9.51 154	37	9.53 574	41	0.46 426	9.97 580	4	3
58	9.51 191	37	9.53 615	41	0.46 385	9.97 576	4	2
59	9.51 227	36	9.53 656	41	0.46 344	9.97 571	5	1
		37		41			4	
60	9.51 264		9.53 697		0.46 303	9.97 567		**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	43	42	41
1	0.7	0.7	0.7
2	1.4	1.4	1.4
3	2.2	2.1	2.0
4	2.9	2.8	2.7
5	3.6	3.5	3.4
6	4.3	4.2	4.1
7	5.0	4.9	4.8
8	5.7	5.6	5.5
9	6.4	6.3	6.2
10	7.2	7.0	6.8
20	14.3	14.0	13.7
30	21.5	21.0	20.5
40	28.7	28.0	27.3
50	35.8	35.0	34.2

	39	38	37
1	0.6	0.6	0.6
2	1.3	1.3	1.2
3	2.0	1.9	1.8
4	2.6	2.5	2.5
5	3.2	3.2	3.1
6	3.9	3.8	3.7
7	4.6	4.4	4.3
8	5.2	5.1	4.9
9	5.8	5.7	5.6
10	6.5	6.3	6.2
20	13.0	12.7	12.3
30	19.5	19.0	18.5
40	26.0	25.3	24.7
50	32.5	31.7	30.8

	36	5	4
1	0.6	0.1	0.1
2	1.2	0.2	0.1
3	1.8	0.2	0.2
4	2.4	0.3	0.3
5	3.0	0.4	0.3
6	3.6	0.5	0.4
7	4.2	0.6	0.5
8	4.8	0.7	0.5
9	5.4	0.8	0.6
10	6.0	0.8	0.7
20	12.0	1.7	1.3
30	18.0	2.5	2.0
40	24.0	3.3	2.7
50	30.0	4.2	3.3

P. P.

19°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.51 264		9.53 697		0.46 303	9.97 567		**60**
1	9.51 301	37	9.53 738	41	0.46 262	9.97 563	4	59
2	9.51 338	37	9.53 779	41	0.46 221	9.97 558	5	58
3	9.51 374	36	9.53 820	41	0.46 180	9.97 554	4	57
4	9.51 411	37	9.53 861	41	0.46 139	9.97 550	4	56
5	9.51 447	36	9.53 902	41	0.46 098	9.97 545	5	55
6	9.51 484	37	9.53 943	41	0.46 057	9.97 541	4	54
7	9.51 520	36	9.53 984	41	0.46 016	9.97 536	5	53
8	9.51 557	37	9.54 025	41	0.45 975	9.97 532	4	52
9	9.51 593	36	9.54 065	40	0.45 935	9.97 528	4	51
10	9.51 629	36	9.54 106	41	0.45 894	9.97 523	5	**50**
11	9.51 666	37	9.54 147	41	0.45 853	9.97 519	4	49
12	9.51 702	36	9.54 187	40	0.45 813	9.97 515	4	48
13	9.51 738	36	9.54 228	41	0.45 772	9.97 510	5	47
14	9.51 774	36	9.54 269	41	0.45 731	9.97 506	4	46
15	9.51 811	37	9.54 309	40	0.45 691	9.97 501	5	45
16	9.51 847	36	9.54 350	41	0.45 650	9.97 497	4	44
17	9.51 883	36	9.54 390	40	0.45 610	9.97 492	5	43
18	9.51 919	36	9.54 431	41	0.45 569	9.97 488	4	42
19	9.51 955	36	9.54 471	40	0.45 529	9.97 484	4	41
20	9.51 991	36	9.54 512	41	0.45 488	9.97 479	5	**40**
21	9.52 027	36	9.54 552	40	0.45 448	9.97 475	4	39
22	9.52 063	36	9.54 593	41	0.45 407	9.97 470	5	38
23	9.52 099	36	9.54 633	40	0.45 367	9.97 466	4	37
24	9.52 135	36	9.54 673	40	0.45 327	9.97 461	5	36
25	9.52 171	36	9.54 714	41	0.45 286	9.97 457	4	35
26	9.52 207	36	9.54 754	40	0.45 246	9.97 453	4	34
27	9.52 242	35	9.54 794	40	0.45 206	9.97 448	5	33
28	9.52 278	36	9.54 835	41	0.45 165	9.97 444	4	32
29	9.52 314	36	9.54 875	40	0.45 125	9.97 439	5	31
30	9.52 350	36	9.54 915	40	0.45 085	9.97 435	4	**30**
31	9.52 385	35	9.54 955	40	0.45 045	9.97 430	5	29
32	9.52 421	36	9.54 995	40	0.45 005	9.97 426	4	28
33	9.52 456	35	9.55 035	40	0.44 965	9.97 421	5	27
34	9.52 492	36	9.55 075	40	0.44 925	9.97 417	4	26
35	9.52 527	35	9.55 115	40	0.44 885	9.97 412	5	25
36	9.52 563	36	9.55 155	40	0.44 845	9.97 408	4	24
37	9.52 598	35	9.55 195	40	0.44 805	9.97 403	5	23
38	9.52 634	36	9.55 235	40	0.44 765	9.97 399	4	22
39	9.52 669	35	9.55 275	40	0.44 725	9.97 394	5	21
40	9.52 705	36	9.55 315	40	0.44 685	9.97 390	4	**20**
41	9.52 740	35	9.55 355	40	0.44 645	9.97 385	5	19
42	9.52 775	35	9.55 395	40	0.44 605	9.97 381	4	18
43	9.52 811	36	9.55 434	39	0.44 566	9.97 376	5	17
44	9.52 846	35	9.55 474	40	0.44 526	9.97 372	4	16
45	9.52 881	35	9.55 514	40	0.44 486	9.97 367	5	15
46	9.52 916	35	9.55 554	40	0.44 446	9.97 363	4	14
47	9.52 951	35	9.55 593	39	0.44 407	9.97 358	5	13
48	9.52 986	35	9.55 633	40	0.44 367	9.97 353	5	12
49	9.53 021	35	9.55 673	40	0.44 327	9.97 349	4	11
50	9.53 056	35	9.55 712	39	0.44 288	9.97 344	5	**10**
51	9.53 092	36	9.55 752	40	0.44 248	9.97 340	4	9
52	9.53 126	34	9.55 791	39	0.44 209	9.97 335	5	8
53	9.53 161	35	9.55 831	40	0.44 169	9.97 331	4	7
54	9.53 196	35	9.55 870	39	0.44 130	9.97 326	5	6
55	9.53 231	35	9.55 910	40	0.44 090	9.97 322	4	5
56	9.53 266	35	9.55 949	39	0.44 051	9.97 317	5	4
57	9.53 301	35	9.55 989	40	0.44 011	9.97 312	5	3
58	9.53 336	35	9.56 028	39	0.43 972	9.97 308	4	2
59	9.53 370	34	9.56 067	39	0.43 933	9.97 303	5	1
60	9.53 405	35	9.56 107	40	0.43 893	9.97 299	4	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	41	40	39
1	0.7	0.7	0.6
2	1.4	1.3	1.3
3	2.0	2.0	2.0
4	2.7	2.7	2.6
5	3.4	3.3	3.2
6	4.1	4.0	3.9
7	4.8	4.7	4.6
8	5.5	5.3	5.2
9	6.2	6.0	5.8
10	6.8	6.7	6.5
20	13.7	13.3	13.0
30	20.5	20.0	19.5
40	27.3	26.7	26.0
50	34.2	33.3	32.5

P. P.

	37	36	35
1	0.6	0.6	0.6
2	1.2	1.2	1.2
3	1.8	1.8	1.8
4	2.5	2.4	2.3
5	3.1	3.0	2.9
6	3.7	3.6	3.5
7	4.3	4.2	4.1
8	4.9	4.8	4.7
9	5.6	5.4	5.2
10	6.2	6.0	5.8
20	12.3	12.0	11.7
30	18.5	18.0	17.5
40	24.7	24.0	23.3
50	30.8	30.0	29.2

P. P.

	34	5	4
1	0.6	0.1	0.1
2	1.1	0.2	0.1
3	1.7	0.2	0.2
4	2.3	0.3	0.3
5	2.8	0.4	0.3
6	3.4	0.5	0.4
7	4.0	0.6	0.5
8	4.5	0.7	0.5
9	5.1	0.8	0.6
10	5.7	0.8	0.7
20	11.3	1.7	1.3
30	17.0	2.5	2.0
40	22.7	3.3	2.7
50	28.3	4.2	3.3

20°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.53 405		9.56 107		0.43 893	9.97 299		**60**
1	9.53 440	35	9.56 146	39	0.43 854	9.97 294	5	59
2	9.53 475	35	9.56 185	39	0.43 815	9.97 289	5	58
3	9.53 509	34	9.56 224	39	0.43 776	9.97 285	4	57
4	9.53 544	35	9.56 264	40	0.43 736	9.97 280	5	56
5	9.53 578	34	9.56 303	39	0.43 697	9.97 276	4	55
6	9.53 613	35	9.56 342	39	0.43 658	9.97 271	5	54
7	9.53 647	34	9.56 381	39	0.43 619	9.97 266	5	53
8	9.53 682	35	9.56 420	39	0.43 580	9.97 262	4	52
9	9.53 716	34	9.56 459	39	0.43 541	9.97 257	5	51
10	9.53 751	35	9.56 498	39	0.43 502	9.97 252	5	**50**
11	9.53 785	34	9.56 537	39	0.43 463	9.97 248	4	49
12	9.53 819	34	9.56 576	39	0.43 424	9.97 243	5	48
13	9.53 854	35	9.56 615	39	0.43 385	9.97 238	5	47
14	9.53 888	34	9.56 654	39	0.43 346	9.97 234	4	46
15	9.53 922	34	9.56 693	39	0.43 307	9.97 229	5	45
16	9.53 957	35	9.56 732	39	0.43 268	9.97 224	5	44
17	9.53 991	34	9.56 771	39	0.43 229	9.97 220	4	43
18	9.54 025	34	9.56 810	39	0.43 190	9.97 215	5	42
19	9.54 059	34	9.56 849	39	0.43 151	9.97 210	5	41
20	9.54 093	34	9.56 887	38	0.43 113	9.97 206	4	**40**
21	9.54 127	34	9.56 926	39	0.43 074	9.97 201	5	39
22	9.54 161	34	9.56 965	39	0.43 035	9.97 196	5	38
23	9.54 195	34	9.57 004	39	0.42 996	9.97 192	4	37
24	9.54 229	34	9.57 042	38	0.42 958	9.97 187	5	36
25	9.54 263	34	9.57 081	39	0.42 919	9.97 182	5	35
26	9.54 297	34	9.57 120	39	0.42 880	9.97 178	4	34
27	9.54 331	34	9.57 158	38	0.42 842	9.97 173	5	33
28	9.54 365	34	9.57 197	39	0.42 803	9.97 168	5	32
29	9.54 399	34	9.57 235	38	0.42 765	9.97 163	5	31
30	9.54 433	33	9.57 274	39	0.42 726	9.97 159	4	**30**
31	9.54 466	34	9.57 312	38	0.42 688	9.97 154	5	29
32	9.54 500	34	9.57 351	39	0.42 649	9.97 149	5	28
33	9.54 534	34	9.57 389	38	0.42 611	9.97 145	4	27
34	9.54 567	33	9.57 428	39	0.42 572	9.97 140	5	26
35	9.54 601	34	9.57 466	38	0.42 534	9.97 135	5	25
36	9.54 635	34	9.57 504	38	0.42 496	9.97 130	5	24
37	9.54 668	33	9.57 543	39	0.42 457	9.97 126	4	23
38	9.54 702	34	9.57 581	38	0.42 419	9.97 121	5	22
39	9.54 735	33	9.57 619	38	0.42 381	9.97 116	5	21
40	9.54 769	34	9.57 658	39	0.42 342	9.97 111	5	**20**
41	9.54 802	33	9.57 696	38	0.42 304	9.97 107	4	19
42	9.54 836	34	9.57 734	38	0.42 266	9.97 102	5	18
43	9.54 869	33	9.57 772	38	0.42 228	9.97 097	5	17
44	9.54 903	34	9.57 810	38	0.42 190	9.97 092	5	16
45	9.54 936	33	9.57 849	39	0.42 151	9.97 087	5	15
46	9.54 969	33	9.57 887	38	0.42 113	9.97 083	4	14
47	9.55 003	34	9.57 925	38	0.42 075	9.97 078	5	13
48	9.55 036	33	9.57 963	38	0.42 037	9.97 073	5	12
49	9.55 069	33	9.58 001	38	0.41 999	9.97 068	5	11
50	9.55 102	33	9.58 039	38	0.41 961	9.97 063	5	**10**
51	9.55 136	34	9.58 077	38	0.41 923	9.97 059	4	9
52	9.55 169	33	9.58 115	38	0.41 885	9.97 054	5	8
53	9.55 202	33	9.58 153	38	0.41 847	9.97 049	5	7
54	9.55 235	33	9.58 191	38	0.41 809	9.97 044	5	6
55	9.55 268	33	9.58 229	38	0.41 771	9.97 039	5	**5**
56	9.55 301	33	9.58 267	38	0.41 733	9.97 035	4	4
57	9.55 334	33	9.58 304	37	0.41 696	9.97 030	5	3
58	9.55 367	33	9.58 342	38	0.41 658	9.97 025	5	2
59	9.55 400	33	9.58 380	38	0.41 620	9.97 020	5	1
60	9.55 433	33	9.58 418	38	0.41 582	9.97 015	5	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	40	39	38
1	0.7	0.6	0.6
2	1.3	1.3	1.3
3	2.0	2.0	1.9
4	2.7	2.6	2.5
5	3.3	3.2	3.2
6	4.0	3.9	3.8
7	4.7	4.6	4.4
8	5.3	5.2	5.1
9	6.0	5.8	5.7
10	6.7	6.5	6.3
20	13.3	13.0	12.7
30	20.0	19.5	19.0
40	26.7	26.0	25.3
50	33.3	32.5	31.7

	37	35	34
1	0.6	0.6	0.6
2	1.2	1.2	1.1
3	1.8	1.8	1.7
4	2.5	2.3	2.3
5	3.1	2.9	2.8
6	3.7	3.5	3.4
7	4.3	4.1	4.0
8	4.9	4.7	4.5
9	5.6	5.2	5.1
10	6.2	5.8	5.7
20	12.3	11.7	11.3
30	18.5	17.5	17.0
40	24.7	23.3	22.7
50	30.8	29.2	28.3

	33	5	4
1	0.6	0.1	0.1
2	1.1	0.2	0.1
3	1.6	0.2	0.2
4	2.2	0.3	0.3
5	2.8	0.4	0.3
6	3.3	0.5	0.4
7	3.8	0.6	0.5
8	4.4	0.7	0.5
9	5.0	0.8	0.6
10	5.5	0.8	0.7
20	11.0	1.7	1.3
30	16.5	2.5	2.0
40	22.0	3.3	2.7
50	27.5	4.2	3.3

21°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′
0	9.55 433		9.58 418		0.41 582	9.97 015		**60**
1	9.55 466	33	9.58 455	37	0.41 545	9.97 010	5	59
2	9.55 499	33	9.58 493	38	0.41 507	9.97 005	5	58
3	9.55 532	33	9.58 531	38	0.41 469	9.97 001	4	57
4	9.55 564	32	9.58 569	38	0.41 431	9.96 996	5	56
5	9.55 597	33	9.58 606	37	0.41 394	9.96 991	5	55
6	9.55 630	33	9.58 644	38	0.41 356	9.96 986	5	54
7	9.55 663	33	9.58 681	37	0.41 319	9.96 981	5	53
8	9.55 695	32	9.58 719	38	0.41 281	9.96 976	5	52
9	9.55 728	33	9.58 757	38	0.41 243	9.96 971	5	51
10	9.55 761	33	9.58 794	37	0.41 206	9.96 966	5	**50**
11	9.55 793	32	9.58 832	38	0.41 168	9.96 962	4	49
12	9.55 826	33	9.58 869	37	0.41 131	9.96 957	5	48
13	9.55 858	32	9.58 907	38	0.41 093	9.96 952	5	47
14	9.55 891	33	9.58 944	37	0.41 056	9.96 947	5	46
15	9.55 923	32	9.58 981	37	0.41 019	9.96 942	5	45
16	9.55 956	33	9.59 019	38	0.40 981	9.96 937	5	44
17	9.55 988	32	9.59 056	37	0.40 944	9.96 932	5	43
18	9.56 021	33	9.59 094	38	0.40 906	9.96 927	5	42
19	9.56 053	32	9.59 131	37	0.40 869	9.96 922	5	41
20	9.56 085	32	9.59 168	37	0.40 832	9.96 917	5	**40**
21	9.56 118	33	9.59 205	37	0.40 795	9.96 912	5	39
22	9.56 150	32	9.59 243	38	0.40 757	9.96 907	5	38
23	9.56 182	32	9.59 280	37	0.40 720	9.96 903	4	37
24	9.56 215	33	9.59 317	37	0.40 683	9.96 898	5	36
25	9.56 247	32	9.59 354	37	0.40 646	9.96 893	5	35
26	9.56 279	32	9.59 391	37	0.40 609	9.96 888	5	34
27	9.56 311	32	9.59 429	38	0.40 571	9.96 883	5	33
28	9.56 343	32	9.59 466	37	0.40 534	9.96 878	5	32
29	9.56 375	32	9.59 503	37	0.40 497	9.96 873	5	31
30	9.56 408	33	9.59 540	37	0.40 460	9.96 868	5	**30**
31	9.56 440	32	9.59 577	37	0.40 423	9.96 863	5	29
32	9.56 472	32	9.59 614	37	0.40 386	9.96 858	5	28
33	9.56 504	32	9.59 651	37	0.40 349	9.96 853	5	27
34	9.56 536	32	9.59 688	37	0.40 312	9.96 848	5	26
35	9.56 568	32	9.59 725	37	0.40 275	9.96 843	5	25
36	9.56 599	31	9.59 762	37	0.40 238	9.96 838	5	24
37	9.56 631	32	9.59 799	37	0.40 201	9.96 833	5	23
38	9.56 663	32	9.59 835	36	0.40 165	9.96 828	5	22
39	9.56 695	32	9.59 872	37	0.40 128	9.96 823	5	21
40	9.56 727	32	9.59 909	37	0.40 091	9.96 818	5	**20**
41	9.56 759	32	9.59 946	37	0.40 054	9.96 813	5	19
42	9.56 790	31	9.59 983	37	0.40 017	9.96 808	5	18
43	9.56 822	32	9.60 019	36	0.39 981	9.96 803	5	17
44	9.56 854	32	9.60 056	37	0.39 944	9.96 798	5	16
45	9.56 886	32	9.60 093	37	0.39 907	9.96 793	5	15
46	9.56 917	31	9.60 130	37	0.39 870	9.96 788	5	14
47	9.56 949	32	9.60 166	36	0.39 834	9.96 783	5	13
48	9.56 980	31	9.60 203	37	0.39 797	9.96 778	5	12
49	9.57 012	32	9.60 240	37	0.39 760	9.96 772	6	11
50	9.57 044	32	9.60 276	36	0.39 724	9.96 767	5	**10**
51	9.57 075	31	9.60 313	37	0.39 687	9.96 762	5	9
52	9.57 107	32	9.60 349	36	0.39 651	9.96 757	5	8
53	9.57 138	31	9.60 386	37	0.39 614	9.96 752	5	7
54	9.57 169	31	9.60 422	36	0.39 578	9.96 747	5	6
55	9.57 201	32	9.60 459	37	0.39 541	9.96 742	5	5
56	9.57 232	31	9.60 495	36	0.39 505	9.96 737	5	4
57	9.57 264	32	9.60 532	37	0.39 468	9.96 732	5	3
58	9.57 295	31	9.60 568	36	0.39 432	9.96 727	5	2
59	9.57 326	31	9.60 605	37	0.39 395	9.96 722	5	1
60	9.57 358	32	9.60 641	36	0.39 359	9.96 717	5	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	38	37	36
1	0.6	0.6	0.6
2	1.3	1.2	1.2
3	1.9	1.8	1.8
4	2.5	2.5	2.4
5	3.2	3.1	3.0
6	3.8	3.7	3.6
7	4.4	4.3	4.2
8	5.1	4.9	4.8
9	5.7	5.6	5.4
10	6.3	6.2	6.0
20	12.7	12.3	12.0
30	19.0	18.5	18.0
40	25.3	24.7	24.0
50	31.7	30.8	30.0

	33	32	31
1	0.6	0.5	0.5
2	1.1	1.1	1.0
3	1.6	1.6	1.6
4	2.2	2.1	2.1
5	2.8	2.7	2.6
6	3.3	3.2	3.1
7	3.8	3.7	3.6
8	4.4	4.3	4.1
9	5.0	4.8	4.6
10	5.5	5.3	5.2
20	11.0	10.7	10.3
30	16.5	16.0	15.5
40	22.0	21.3	20.7
50	27.5	26.7	25.8

	6	5	4
1	0.1	0.1	0.1
2	0.2	0.2	0.1
3	0.3	0.2	0.2
4	0.4	0.3	0.3
5	0.5	0.4	0.3
6	0.6	0.5	0.4
7	0.7	0.6	0.5
8	0.8	0.7	0.5
9	0.9	0.8	0.6
10	1.0	0.8	0.7
20	2.0	1.7	1.3
30	3.0	2.5	2.0
40	4.0	3.3	2.7
50	5.0	4.2	3.3

22°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.57 358		9.60 641		0.39 359	9.96 717		**60**
1	9.57 389	31	9.60 677	36	0.39 323	9.96 711	6	59
2	9.57 420	31	9.60 714	37	0.39 286	9.96 706	5	58
3	9.57 451	31	9.60 750	36	0.39 250	9.96 701	5	57
4	9.57 482	31	9.60 786	36	0.39 214	9.96 696	5	56
5	9.57 514	32	9.60 823	37	0.39 177	9.96 691	5	55
6	9.57 545	31	9.60 859	36	0.39 141	9.96 686	5	54
7	9.57 576	31	9.60 895	36	0.39 105	9.96 681	5	53
8	9.57 607	31	9.60 931	36	0.39 069	9.96 676	5	52
9	9.57 638	31	9.60 967	36	0.39 033	9.96 670	6	51
10	9.57 669	31	9.61 004	37	0.38 996	9.96 665	5	**50**
11	9.57 700	31	9.61 040	36	0.38 960	9.96 660	5	49
12	9.57 731	31	9.61 076	36	0.38 924	9.96 655	5	48
13	9.57 762	31	9.61 112	36	0.38 888	9.96 650	5	47
14	9.57 793	31	9.61 148	36	0.38 852	9.96 645	5	46
15	9.57 824	31	9.61 184	36	0.38 816	9.96 640	5	45
16	9.57 855	31	9.61 220	36	0.38 780	9.96 634	6	44
17	9.57 885	30	9.61 256	36	0.38 744	9.96 629	5	43
18	9.57 916	31	9.61 292	36	0.38 708	9.96 624	5	42
19	9.57 947	31	9.61 328	36	0.38 672	9.96 619	5	41
20	9.57 978	31	9.61 364	36	0.38 636	9.96 614	5	**40**
21	9.58 008	30	9.61 400	36	0.38 600	9.96 608	6	39
22	9.58 039	31	9.61 436	36	0.38 564	9.96 603	5	38
23	9.58 070	31	9.61 472	36	0.38 528	9.96 598	5	37
24	9.58 101	31	9.61 508	36	0.38 492	9.96 593	5	36
25	9.58 131	30	9.61 544	36	0.38 456	9.96 588	5	35
26	9.58 162	31	9.61 579	35	0.38 421	9.96 582	6	34
27	9.58 192	30	9.61 615	36	0.38 385	9.96 577	5	33
28	9.58 223	31	9.61 651	36	0.38 349	9.96 572	5	32
29	9.58 253	30	9.61 687	36	0.38 313	9.96 567	5	31
30	9.58 284	31	9.61 722	35	0.38 278	9.96 562	5	**30**
31	9.58 314	30	9.61 758	36	0.38 242	9.96 556	6	29
32	9.58 345	31	9.61 794	36	0.38 206	9.96 551	5	28
33	9.58 375	30	9.61 830	36	0.38 170	9.96 546	5	27
34	9.58 406	31	9.61 865	35	0.38 135	9.96 541	5	26
35	9.58 436	30	9.61 901	36	0.38 099	9.96 535	6	25
36	9.58 467	31	9.61 936	35	0.38 064	9.96 530	5	24
37	9.58 497	30	9.61 972	36	0.38 028	9.96 525	5	23
38	9.58 527	30	9.62 008	36	0.37 992	9.96 520	5	22
39	9.58 557	30	9.62 043	35	0.37 957	9.96 514	6	21
40	9.58 588	31	9.62 079	36	0.37 921	9.96 509	5	**20**
41	9.58 618	30	9.62 114	35	0.37 886	9.96 504	5	19
42	9.58 648	30	9.62 150	36	0.37 850	9.96 498	6	18
43	9.58 678	30	9.62 185	35	0.37 815	9.96 493	5	17
44	9.58 709	31	9.62 221	36	0.37 779	9.96 488	5	16
45	9.58 739	30	9.62 256	35	0.37 744	9.96 483	5	15
46	9.58 769	30	9.62 292	36	0.37 708	9.96 477	6	14
47	9.58 799	30	9.62 327	35	0.37 673	9.96 472	5	13
48	9.58 829	30	9.62 362	35	0.37 638	9.96 467	5	12
49	9.58 859	30	9.62 398	36	0.37 602	9.96 461	6	11
50	9.58 889	30	9.62 433	35	0.37 567	9.96 456	5	**10**
51	9.58 919	30	9.62 468	35	0.37 532	9.96 451	5	9
52	9.58 949	30	9.62 504	36	0.37 496	9.96 445	6	8
53	9.58 979	30	9.62 539	35	0.37 461	9.96 440	5	7
54	9.59 009	30	9.62 574	35	0.37 426	9.96 435	5	6
55	9.59 039	30	9.62 609	35	0.37 391	9.96 429	6	5
56	9.59 069	30	9.62 645	36	0.37 355	9.96 424	5	4
57	9.59 098	29	9.62 680	35	0.37 320	9.96 419	5	3
58	9.59 128	30	9.62 715	35	0.37 285	9.96 413	6	2
59	9.59 158	30	9.62 750	35	0.37 250	9.96 408	5	1
60	9.59 188	30	9.62 785	35	0.37 215	9.96 403	5	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	37	36	35
1	0.6	0.6	0.6
2	1.2	1.2	1.2
3	1.8	1.8	1.8
4	2.5	2.4	2.3
5	3.1	3.0	2.9
6	3.7	3.6	3.5
7	4.3	4.2	4.1
8	4.9	4.8	4.7
9	5.6	5.4	5.2
10	6.2	6.0	5.8
20	12.3	12.0	11.7
30	18.5	18.0	17.5
40	24.7	24.0	23.3
50	30.8	30.0	29.2

P. P.

	32	31	30
1	0.5	0.5	0.5
2	1.1	1.0	1.0
3	1.6	1.6	1.5
4	2.1	2.1	2.0
5	2.7	2.6	2.5
6	3.2	3.1	3.0
7	3.7	3.6	3.5
8	4.3	4.1	4.0
9	4.8	4.6	4.5
10	5.3	5.2	5.0
20	10.7	10.3	10.0
30	16.0	15.5	15.0
40	21.3	20.7	20.0
50	26.7	25.8	25.0

P. P.

	29	6	5
1	0.5	0.1	0.1
2	1.0	0.2	0.2
3	1.4	0.3	0.2
4	1.9	0.4	0.3
5	2.4	0.5	0.4
6	2.9	0.6	0.5
7	3.4	0.7	0.6
8	3.9	0.8	0.7
9	4.4	0.9	0.8
10	4.8	1.0	0.8
20	9.7	2.0	1.7
30	14.5	3.0	2.5
40	19.3	4.0	3.3
50	24.2	5.0	4.2

23°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′
0	9.59 188		9.62 785		0.37 215	9.96 403		**60**
1	9.59 218	30	9.62 820	35	0.37 180	9.96 397	6	59
2	9.59 247	29	9.62 855	35	0.37 145	9.96 392	5	58
3	9.59 277	30	9.62 890	35	0.37 110	9.96 387	5	57
4	9.59 307	30	9.62 926	36	0.37 074	9.96 381	6	56
		29		35			5	
5	9.59 336		9.62 961		0.37 039	9.96 376	5	55
6	9.59 366	30	9.62 996	35	0.37 004	9.96 370	6	54
7	9.59 396	30	9.63 031	35	0.36 969	9.96 365	5	53
8	9.59 425	29	9.63 066	35	0.36 934	9.96 360	5	52
9	9.59 455	30	9.63 101	35	0.36 899	9.96 354	6	51
		29		34			5	
10	9.59 484		9.63 135		0.36 865	9.96 349		**50**
11	9.59 514	30	9.63 170	35	0.36 830	9.96 343	6	49
12	9.59 543	29	9.63 205	35	0.36 795	9.96 338	5	48
13	9.59 573	30	9.63 240	35	0.36 760	9.96 333	5	47
14	9.59 602	29	9.63 275	35	0.36 725	9.96 327	6	46
		30		35			5	
15	9.59 632		9.63 310		0.36 690	9.96 322		45
16	9.59 661	29	9.63 345	35	0.36 655	9.96 316	6	44
17	9.59 690	29	9.63 379	34	0.36 621	9.96 311	5	43
18	9.59 720	30	9.63 414	35	0.36 586	9.96 305	6	42
19	9.59 749	29	9.63 449	35	0.36 551	9.96 300	5	41
		29		35			6	
20	9.59 778		9.63 484		0.36 516	9.96 294		**40**
21	9.59 808	30	9.63 519	35	0.36 481	9.96 289	5	39
22	9.59 837	29	9.63 553	34	0.36 447	9.96 284	5	38
23	9.59 866	29	9.63 588	35	0.36 412	9.96 278	6	37
24	9.59 895	29	9.63 623	35	0.36 377	9.96 273	5	36
		29		34			6	
25	9.59 924		9.63 657		0.36 343	9.96 267		35
26	9.59 954	30	9.63 692	35	0.36 308	9.96 262	5	34
27	9.59 983	29	9.63 726	34	0.36 274	9.96 256	6	33
28	9.60 012	29	9.63 761	35	0.36 239	9.96 251	5	32
29	9.60 041	29	9.63 796	35	0.36 204	9.96 245	6	31
		29		34			5	
30	9.60 070		9.63 830		0.36 170	9.96 240		**30**
31	9.60 099	29	9.63 865	35	0.36 135	9.96 234	6	29
32	9.60 128	29	9.63 899	34	0.36 101	9.96 229	5	28
33	9.60 157	29	9.63 934	35	0.36 066	9.96 223	6	27
34	9.60 186	29	9.63 968	34	0.36 032	9.96 218	5	26
		29		35			6	
35	9.60 215		9.64 003		0.35 997	9.96 212		25
36	9.60 244	29	9.64 037	34	0.35 963	9.96 207	5	24
37	9.60 273	29	9.64 072	35	0.35 928	9.96 201	6	23
38	9.60 302	29	9.64 106	34	0.35 894	9.96 196	5	22
39	9.60 331	29	9.64 140	34	0.35 860	9.96 190	6	21
		28		35			5	
40	9.60 359		9.64 175		0.35 825	9.96 185	6	**20**
41	9.60 388	29	9.64 209	34	0.35 791	9.96 179	5	19
42	9.60 417	29	9.64 243	34	0.35 757	9.96 174	6	18
43	9.60 446	29	9.64 278	35	0.35 722	9.96 168	6	17
44	9.60 474	28	9.64 312	34	0.35 688	9.96 162	6	16
		29		34			5	
45	9.60 503		9.64 346		0.35 654	9.96 157		15
46	9.60 532	29	9.64 381	35	0.35 619	9.96 151	6	14
47	9.60 561	29	9.64 415	34	0.35 585	9.96 146	5	13
48	9.60 589	28	9.64 449	34	0.35 551	9.96 140	6	12
49	9.60 618	29	9.64 483	34	0.35 517	9.96 135	5	11
		28		34			6	
50	9.60 646		9.64 517		0.35 483	9.96 129		**10**
51	9.60 675	29	9.64 552	35	0.35 448	9.96 123	6	9
52	9.60 704	29	9.64 586	34	0.35 414	9.96 118	5	8
53	9.60 732	28	9.64 620	34	0.35 380	9.96 112	6	7
54	9.60 761	29	9.64 654	34	0.35 346	9.96 107	5	6
		28		34			6	
55	9.60 789		9.64 688		0.35 312	9.96 101		5
56	9.60 818	29	9.64 722	34	0.35 278	9.96 095	6	4
57	9.60 846	28	9.64 756	34	0.35 244	9.96 090	5	**3**
58	9.60 875	29	9.64 790	34	0.35 210	9.96 084	6	**2**
59	9.60 903	28	9.64 824	34	0.35 176	9.96 079	5	**1**
60	9.60 931	28	9.64 858	34	0.35 142	9.96 073	6	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	36	35	34
1	0.6	0.6	0.6
2	1.2	1.2	1.1
3	1.8	1.8	1.7
4	2.4	2.3	2.3
5	3.0	2.9	2.8
6	3.6	3.5	3.4
7	4.2	4.1	4.0
8	4.8	4.7	4.5
9	5.4	5.2	5.1
10	6.0	5.8	5.7
20	12.0	11.7	11.3
30	18.0	17.5	17.0
40	24.0	23.3	22.7
50	30.0	29.2	28.3

	30	29	28
1	0.5	0.5	0.5
2	1.0	1.0	0.9
3	1.5	1.4	1.4
4	2.0	1.9	1.9
5	2.5	2.4	2.3
6	3.0	2.9	2.8
7	3.5	3.4	3.3
8	4.0	3.9	3.7
9	4.5	4.4	4.2
10	5.0	4.8	4.7
20	10.0	9.7	9.3
30	15.0	14.5	14.0
40	20.0	19.3	18.7
50	25.0	24.2	23.3

	6	5
1	0.1	0.1
2	0.2	0.2
3	0.3	0.2
4	0.4	0.3
5	0.5	0.4
6	0.6	0.5
7	0.7	0.6
8	0.8	0.7
9	0.9	0.8
10	1.0	0.8
20	2.0	1.7
30	3.0	2.4
40	4.0	3.3
50	5.0	4.2

24°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.60 931		9.64 858		0.35 142	9.96 073		**60**
1	9.60 960	29	9.64 892	34	0.35 108	9.96 067	6	59
2	9.60 988	28	9.64 926	34	0.35 074	9.96 062	5	58
3	9.61 016	28	9.64 960	34	0.35 040	9.96 056	6	57
4	9.61 045	29	9.64 994	34	0.35 006	9.96 050	6	56
		28		34			5	
5	9.61 073		9.65 028		0.34 972	9.96 045		55
6	9.61 101	28	9.65 062	34	0.34 938	9.96 039	6	54
7	9.61 129	28	9.65 096	34	0.34 904	9.96 034	5	53
8	9.61 158	29	9.65 130	34	0.34 870	9.96 028	6	52
9	9.61 186	28	9.65 164	34	0.34 836	9.96 022	6	51
		28		33			5	
10	9.61 214		9.65 197		0.34 803	9.96 017		**50**
11	9.61 242	28	9.65 231	34	0.34 769	9.96 011	6	49
12	9.61 270	28	9.65 265	34	0.34 735	9.96 005	6	48
13	9.61 298	28	9.65 299	34	0.34 701	9.96 000	5	47
14	9.61 326	28	9.65 333	34	0.34 667	9.95 994	6	46
		28		33			6	
15	9.61 354		9.65 366		0.34 634	9.95 988		45
16	9.61 382	28	9.65 400	34	0.34 600	9.95 982	6	44
17	9.61 411	29	9.65 434	34	0.34 566	9.95 977	5	43
18	9.61 438	27	9.65 467	33	0.34 533	9.95 971	6	42
19	9.61 466	28	9.65 501	34	0.34 499	9.95 965	6	41
		28		34			5	
20	9.61 494		9.65 535		0.34 465	9.95 960		**40**
21	9.61 522	28	9.65 568	33	0.34 432	9.95 954	6	39
22	9.61 550	28	9.65 602	34	0.34 398	9.95 948	6	38
23	9.61 578	28	9.65 636	34	0.34 364	9.95 942	6	37
24	9.61 606	28	9.65 669	33	0.34 331	9.95 937	5	36
		28		34			6	
25	9.61 634		9.65 703		0.34 297	9.95 931		35
26	9.61 662	28	9.65 736	33	0.34 264	9.95 925	6	34
27	9.61 689	27	9.65 770	34	0.34 230	9.95 920	5	33
28	9.61 717	28	9.65 803	33	0.34 197	9.95 914	6	32
29	9.61 745	28	9.65 837	34	0.34 163	9.95 908	6	31
		28		33			6	
30	9.61 773		9.65 870		0.34 130	9.95 902		**30**
31	9.61 800	27	9.65 904	34	0.34 096	9.95 897	5	29
32	9.61 828	28	9.65 937	33	0.34 063	9.95 891	6	28
33	9.61 856	28	9.65 971	34	0.34 029	9.95 885	6	27
34	9.61 883	27	9.66 004	33	0.33 996	9.95 879	6	26
		28		34			6	
35	9.61 911		9.66 038		0.33 962	9.95 873		25
36	9.61 939	28	9.66 071	33	0.33 929	9.95 868	5	24
37	9.61 966	27	9.66 104	33	0.33 896	9.95 862	6	23
38	9.61 994	28	9.66 138	34	0.33 862	9.95 856	6	22
39	9.62 021	27	9.66 171	33	0.33 829	9.95 850	6	21
		28		33			6	
40	9.62 049		9.66 204		0.33 796	9.95 844		**20**
41	9.62 076	27	9.66 238	34	0.33 762	9.95 839	5	19
42	9.62 104	28	9.66 271	33	0.33 729	9.95 833	6	18
43	9.62 131	27	9.66 304	33	0.33 696	9.95 827	6	17
44	9.62 159	28	9.66 337	33	0.33 663	9.95 821	6	16
		27		34			6	
45	9.62 186		9.66 371		0.33 629	9.95 815		15
46	9.62 214	28	9.66 404	33	0.33 596	9.95 810	5	14
47	9.62 241	27	9.66 437	33	0.33 563	9.95 804	6	13
48	9.62 268	27	9.66 470	33	0.33 530	9.95 798	6	12
49	9.62 296	28	9.66 503	33	0.33 497	9.95 792	6	11
		27		34			6	
50	9.62 323		9.66 537		0.33 463	9.95 786		**10**
51	9.62 350	27	9.66 570	33	0.33 430	9.95 780	6	9
52	9.62 377	27	9.66 603	33	0.33 397	9.95 775	5	8
53	9.62 405	28	9.66 636	33	0.33 364	9.95 769	6	7
54	9.62 432	27	9.66 669	33	0.33 331	9.95 763	6	6
		27		33			6	
55	9.62 459		9.66 702		0.33 298	9.95 757		5
56	9.62 486	27	9.66 735	33	0.33 265	9.95 751	6	4
57	9.62 513	27	9.66 768	33	0.33 232	9.95 745	6	3
58	9.62 541	28	9.66 801	33	0.33 199	9.95 739	6	2
59	9.62 568	27	9.66 834	33	0.33 166	9.95 733	6	1
		27		33			5	
60	9.62 595		9.66 867		0.33 133	9.95 728		**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	34	33
1	0.6	0.6
2	1.1	1.1
3	1.7	1.6
4	2.3	2.2
5	2.8	2.8
6	3.4	3.3
7	4.0	3.8
8	4.5	4.4
9	5.1	5.0
10	5.7	5.5
20	11.3	11.0
30	17.0	16.5
40	22.7	22.0
50	28.3	27.5

	29	28	27
1	0.5	0.5	0.4
2	1.0	0.9	0.9
3	1.4	1.4	1.4
4	1.9	1.9	1.8
5	2.4	2.3	2.2
6	2.9	2.8	2.7
7	3.4	3.3	3.2
8	3.9	3.7	3.6
9	4.4	4.2	4.0
10	4.8	4.7	4.5
20	9.7	9.3	9.0
30	14.5	14.0	13.5
40	19.3	18.7	18.0
50	24.2	23.3	22.5

	6	5
1	0.1	0.1
2	0.2	0.2
3	0.3	0.2
4	0.4	0.3
5	0.5	0.4
6	0.6	0.5
7	0.7	0.6
8	0.8	0.7
9	0.9	0.8
10	1.0	0.8
20	2.0	1.7
30	3.0	2.5
40	4.0	3.3
50	5.0	4.2

P. P.

25°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′
0	9.62 595		9.66 867		0.33 133	9.95 728		**60**
1	9.62 622	27	9.66 900	33	0.33 100	9.95 722	6	59
2	9.62 649	27	9.66 933	33	0.33 067	9.95 716	6	58
3	9.62 676	27	9.66 966	33	0.33 034	9.95 710	6	57
4	9.62 703	27	9.66 999	33	0.33 001	9.95 704	6	56
5	9.62 730	27	9.67 032	33	0.32 968	9.95 698	6	55
6	9.62 757	27	9.67 065	33	0.32 935	9.95 692	6	54
7	9.62 784	27	9.67 098	33	0.32 902	9.95 686	6	53
8	9.62 811	27	9.67 131	33	0.32 869	9.95 680	6	52
9	9.62 838	27	9.67 163	32	0.32 837	9.95 674	6	51
10	9.62 865	27	9 67 196	33	0.32 804	9.95 668	5	**50**
11	9.62 892	27	9.67 229	33	0.32 771	9.95 663	6	49
12	9.62 918	26	9.67 262	33	0.32 738	9.95 657	6	48
13	9.62 945	27	9.67 295	33	0.32 705	9.95 651	6	47
14	9.62 972	27	9.67 327	32	0.32 673	9.95 645	6	46
15	9.62 999	27	9.67 360	33	0.32 640	9.95 639	6	45
16	9.63 026	27	9.67 393	33	0.32 607	9.95 633	6	44
17	9.63 052	26	9.67 426	33	0.32 574	9.95 627	6	43
18	9.63 079	27	9.67 458	32	0.32 542	9.95 621	6	42
19	9.63 106	27	9.67 491	33	0.32 509	9.95 615	6	41
20	9.63 133	27	9.67 524	33	0.32 476	9.95 609	6	**40**
21	9.63 159	26	9.67 556	32	0.32 444	9.95 603	6	39
22	9.63 186	27	9.67 589	33	0.32 411	9.95 597	6	38
23	9.63 213	27	9.67 622	33	0.32 378	9.95 591	6	37
24	9.63 239	26	9.67 654	32	0.32 346	9.95 585	6	36
25	9.63 266	27	9.67 687	33	0.32 313	9.95 579	6	35
26	9.63 292	26	9.67 719	32	0.32 281	9.95 573	6	34
27	9.63 319	27	9.67 752	33	0.32 248	9.95 567	6	33
28	9.63 345	26	9.67 785	33	0.32 215	9.95 561	6	32
29	9.63 372	27	9.67 817	32	0.32 183	9.95 555	6	31
30	9.63 398	26	9.67 850	33	0.32 150	9.95 549	6	**30**
31	9.63 425	27	9.67 882	32	0.32 118	9.95 543	6	29
32	9.63 451	26	9.67 915	33	0.32 085	9.95 537	6	28
33	9.63 478	27	9.67 947	32	0.32 053	9.95 531	6	27
34	9.63 504	26	9.67 980	33	0.32 020	9.95 525	6	26
35	9.63 531	27	9.68 012	32	0.31 988	9.95 519	6	25
36	9.63 557	26	9.68 044	32	0.31 956	9.95 513	6	24
37	9.63 583	26	9.68 077	33	0.31 923	9.95 507	6	23
38	9.63 610	27	9.68 109	32	0.31 891	9.95 500	7	22
39	9.63 636	26	9.68 142	33	0.31 858	9.95 494	6	21
40	9.63 662	26	9.68 174	32	0.31 826	9.95 488	6	**20**
41	9.63 689	27	9.68 206	32	0.31 794	9.95 482	6	19
42	9.63 715	26	9.68 239	33	0.31 761	9.95 476	6	18
43	9.63 741	26	9.68 271	32	0.31 729	9.95 470	6	17
44	9.63 767	26	9.68 303	32	0.31 697	9.95 464	6	16
45	9.63 794	27	9.68 336	33	0.31 664	9.95 458	6	15
46	9.63 820	26	9.68 368	32	0.31 632	9.95 452	6	14
47	9.63 846	26	9.68 400	32	0.31 600	9.95 446	6	13
48	9.63 872	26	9.68 432	32	0.31 568	9.95 440	6	12
49	9.63 898	26	9.68 465	33	0.31 535	9.95 434	6	11
50	9.63 924	26	9.68 497	32	0.31 503	9.95 427	7	**10**
51	9.63 950	26	9.68 529	32	0.31 471	9.95 421	6	9
52	9.63 976	26	9.68 561	32	0.31 439	9.95 415	6	8
53	9.64 002	26	9.68 593	32	0.31 407	9.95 409	6	7
54	9.64 028	26	9.68 626	33	0.31 374	9.95 403	6	6
55	9.64 054	26	9.68 658	32	0.31 342	9.95 397	6	5
56	9.64 080	26	9.68 690	32	0.31 310	9.95 391	6	4
57	9.64 106	26	9.68 722	32	0.31 278	9.95 384	7	3
58	9.64 132	26	9.68 754	32	0.31 246	9.95 378	6	2
59	9.64 158	26	9.68 786	32	0.31 214	9.95 372	6	1
60	9.64 184		9.68 818		0.31 182	9.95 366		**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	33	32
1	0.6	0.5
2	1.1	1.1
3	1.6	1.6
4	2.2	2.1
5	2.8	2.7
6	3.3	3.2
7	3.8	3.7
8	4.4	4.3
9	5.0	4.8
10	5.5	5.3
20	11.0	10.7
30	16.5	16.0
40	22.0	21.3
50	27.5	26.7

	27	26
1	0.4	0.4
2	0.9	0.9
3	1.4	1.3
4	1.8	1.7
5	2.2	2.2
6	2.7	2.6
7	3.2	3.0
8	3.6	3.5
9	4.0	3.9
10	4.5	4.3
20	9.0	8.7
30	13.5	13.0
40	18.0	17.3
50	22.5	21.7

	7	6	5
1	0.1	0.1	0.1
2	0.2	0.2	0.2
3	0.4	0.3	0.2
4	0.5	0.4	0.3
5	0.6	0.5	0.4
6	0.7	0.6	0.5
7	0.8	0.7	0.6
8	0.9	0.8	0.7
9	1.0	0.9	0.8
10	1.2	1.0	0.8
20	2.3	2.0	1.7
30	3.5	3.0	2.5
40	4.7	4.0	3.3
50	5.8	5.0	4.2

26°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′
0	9.64 184		9.68 818		0.31 182	9.95 366		**60**
1	9.64 210	26	9.68 850	32	0.31 150	9.95 360	6	59
2	9.64 236	26	9.68 882	32	0.31 118	9.95 354	6	58
3	9.64 262	26	9.68 914	32	0.31 086	9.95 348	6	57
4	9.64 288	26	9.68 946	32	0.31 054	9.95 341	7	56
		25		32			6	
5	9.64 313		9.68 978		0.31 022	9.95 335		55
6	9.64 339	26	9.69 010	32	0.30 990	9.95 329	6	54
7	9.64 365	26	9.69 042	32	0.30 958	9.95 323	6	53
8	9.64 391	26	9.69 074	32	0.30 926	9.95 317	6	52
9	9.64 417	26	9.69 106	32	0.30 894	9.95 310	7	51
		25		32			6	
10	9.64 442		9.69 138		0.30 862	9.95 304		**50**
11	9.64 468	26	9.69 170	32	0.30 830	9.95 298	6	49
12	9.64 494	26	9.69 202	32	0.30 798	9.95 292	6	48
13	9.64 519	25	9.69 234	32	0.30 766	9.95 286	6	47
14	9.64 545	26	9.69 266	32	0.30 734	9.95 279	7	46
		26		32			6	
15	9.64 571		9.69 298		0.30 702	9.95 273		45
16	9.64 596	25	9.69 329	31	0.30 671	9.95 267	6	44
17	9.64 622	26	9.69 361	32	0.30 639	9.95 261	6	43
18	9.64 647	25	9.69 393	32	0.30 607	9.95 254	7	42
19	9.64 673	26	9.69 425	32	0.30 575	9.95 248	6	41
		25		32			6	
20	9.64 698		9.69 457		0.30 543	9.95 242		**40**
21	9.64 724	26	9.69 488	31	0.30 512	9.95 236	6	39
22	9.64 749	25	9.69 520	32	0.30 480	9.95 229	7	38
23	9.64 775	26	9.69 552	32	0.30 448	9.95 223	6	37
24	9.64 800	25	9.69 584	32	0.30 416	9.95 217	6	36
		26		31			6	
25	9.64 826		9.69 615		0.30 385	9.95 211		35
26	9.64 851	25	9.69 647	32	0.30 353	9.95 204	7	34
27	9.64 877	26	9.69 679	32	0.30 321	9.95 198	6	33
28	9.64 902	25	9.69 710	31	0.30 290	9.95 192	6	32
29	9.64 927	25	9.69 742	32	0.30 258	9.95 185	7	31
		26		32			6	
30	9.64 953		9.69 774		0.30 226	9.95 179		**30**
31	9.64 978	25	9.69 805	31	0.30 195	9.95 173	6	29
32	9.65 003	25	9.69 837	32	0.30 163	9.95 167	6	28
33	9.65 029	26	9.69 868	31	0.30 132	9.95 160	7	27
34	9.65 054	25	9.69 900	32	0.30 100	9.95 154	6	26
		25		32			6	
35	9.65 079		9.69 932		0.30 068	9.95 148		25
36	9.65 104	25	9.69 963	31	0.30 037	9.95 141	7	24
37	9.65 130	26	9.69 995	32	0.30 005	9.95 135	6	23
38	9.65 155	25	9.70 026	31	0.29 974	9.95 129	6	22
39	9.65 180	25	9.70 058	32	0.29 942	9.95 122	7	21
		25		31			6	
40	9.65 205		9.70 089		0.29 911	9.95 116		**20**
41	9.65 230	25	9.70 121	32	0.29 879	9.95 110	6	19
42	9.65 255	25	9.70 152	31	0.29 848	9.95 103	7	18
43	9.65 281	26	9.70 184	32	0.29 816	9.95 097	6	17
44	9.65 306	25	9.70 215	31	0.29 785	9.95 090	7	16
		25		32			6	
45	9.65 331		9.70 247		0.29 753	9.95 084		15
46	9.65 356	25	9.70 278	31	0.29 722	9.95 078	6	14
47	9.65 381	25	9.70 309	31	0.29 691	9.95 071	7	13
48	9.65 406	25	9.70 341	32	0.29 659	9.95 065	6	12
49	9.65 431	25	9.70 372	31	0.29 628	9.95 059	6	11
		25		32			7	
50	9.65 456		9.70 404		0.29 596	9.95 052		**10**
51	9.65 481	25	9.70 435	31	0.29 565	9.95 046	6	9
52	9.65 506	25	9.70 466	31	0.29 534	9.95 039	7	8
53	9.65 531	25	9.70 498	32	0.29 502	9.95 033	6	7
54	9.65 556	25	9.70 529	31	0.29 471	9.95 027	6	6
		24		31			7	
55	9.65 580		9.70 560		0.29 440	9.95 020		5
56	9.65 605	25	9.70 592	32	0.29 408	9.95 014	6	4
57	9.65 630	25	9.70 623	31	0.29 377	9.95 007	7	3
58	9.65 655	25	9.70 654	31	0.29 346	9.95 001	6	2
59	9.65 680	25	9.70 685	31	0.29 315	9.94 995	6	1
60	9.65 705		9.70 717	32	0.29 283	9.94 988	7	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	32	31
1	0.5	0.5
2	1.1	1.0
3	1.6	1.6
4	2.1	2.1
5	2.7	2.6
6	3.2	3.1
7	3.7	3.6
8	4.3	4.1
9	4.8	4.6
10	5.3	5.2
20	10.7	10.3
30	16 0	15.5
40	21.3	20.7
50	26.7	25.8

	26	25	24
1	0.4	0.4	0.4
2	0.9	0.8	0.8
3	1.3	1.2	1.2
4	1.7	1.7	1.6
5	2.2	2.1	2.0
6	2.6	2.5	2.4
7	3.0	2.9	2.8
8	3.5	3.3	3.2
9	3.9	3.8	3.6
10	4.3	4.2	4.0
20	8.7	8.3	8.0
30	13.0	12.5	12.0
40	17.3	16.7	16.0
50	21.7	20.8	20.0

	7	6
1	0.1	0.1
2	0.2	0.2
3	0.4	0.3
4	0.5	0.4
5	0.6	0.5
6	0.7	0.6
7	0.8	0.7
8	0.9	0.8
9	1.0	0.9
10	1.2	1.0
20	2.3	2.0
30	3.5	3.0
40	4.7	4.0
50	5.8	5.0

27°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′
0	9.65 705		9.70 717		0.29 283	9.94 988		**60**
1	9.65 729	24	9.70 748	31	0.29 252	9.94 982	6	59
2	9.65 754	25	9.70 779	31	0.29 221	9.94 975	7	58
3	9.65 779	25	9.70 810	31	0.29 190	9.94 969	6	57
4	9.65 804	25	9.70 841	31	0.29 159	9.94 962	7	56
5	9.65 828	24	9.70 873	32	0.29 127	9.94 956	6	55
6	9.65 853	25	9.70 904	31	0.29 096	9.94 949	7	54
7	9.65 878	25	9.70 935	31	0.29 065	9.94 943	6	53
8	9.65 902	24	9.70 966	31	0.29 034	9.94 936	7	52
9	9.65 927	25	9.70 997	31	0.29 003	9.94 930	6	51
10	9.65 952	25	9.71 028	31	0.28 972	9.94 923	7	**50**
11	9.65 976	24	9.71 059	31	0.28 941	9.94 917	6	49
12	9.66 001	25	9.71 090	31	0.28 910	9.94 911	6	48
13	9.66 025	24	9.71 121	31	0.28 879	9.94 904	7	47
14	9.66 050	25	9.71 153	32	0.28 847	9.94 898	6	46
15	9.66 075	25	9.71 184	31	0.28 816	9.94 891	7	45
16	9.66 099	24	9.71 215	31	0.28 785	9.94 885	6	44
17	9.66 124	25	9.71 246	31	0.28 754	9.94 878	7	43
18	9.66 148	24	9.71 277	31	0.28 723	9.94 871	7	42
19	9.66 173	25	9.71 308	31	0.28 692	9.94 865	6	41
20	9.66 197	24	9.71 339	31	0.28 661	9.94 858	7	**40**
21	9.66 221	24	9.71 370	31	0.28 630	9.94 852	6	39
22	9.66 246	25	9.71 401	31	0.28 599	9.94 845	7	38
23	9.66 270	24	9.71 431	30	0.28 569	9.94 839	6	37
24	9.66 295	25	9.71 462	31	0.28 538	9.94 832	7	36
25	9.66 319	24	9.71 493	31	0.28 507	9.94 826	6	35
26	9.66 343	24	9.71 524	31	0.28 476	9.94 819	7	34
27	9.66 368	25	9.71 555	31	0.28 445	9.94 813	6	33
28	9.66 392	24	9.71 586	31	0.28 414	9.94 806	7	32
29	9.66 416	24	9.71 617	31	0.28 383	9.94 799	7	31
30	9.66 441	25	9.71 648	31	0.28 352	9.94 793	6	**30**
31	9.66 465	24	9.71 679	31	0.28 321	9.94 786	7	29
32	9.66 489	24	9.71 709	30	0.28 291	9.94 780	6	28
33	9.66 513	24	9.71 740	31	0.28 260	9.94 773	7	27
34	9.66 537	24	9.71 771	31	0.28 229	9.94 767	6	26
35	9.66 562	25	9.71 802	31	0.28 198	9.94 760	7	25
36	9.66 586	24	9.71 833	31	0.28 167	9.94 753	7	24
37	9.66 610	24	9.71 863	30	0.28 137	9.94 747	6	23
38	9.66 634	24	9.71 894	31	0.28 106	9.94 740	7	22
39	9.66 658	24	9.71 925	31	0.28 075	9.94 734	6	21
40	9.66 682	24	9.71 955	30	0.28 045	9.94 727	7	**20**
41	9.66 706	24	9.71 986	31	0.28 014	9.94 720	7	19
42	9.66 731	25	9.72 017	31	0.27 983	9.94 714	6	18
43	9.66 755	24	9.72 048	31	0.27 952	9.94 707	7	17
44	9.66 779	24	9.72 078	30	0.27 922	9.94 700	7	16
45	9.66 803	24	9.72 109	31	0.27 891	9.94 694	6	15
46	9.66 827	24	9.72 140	31	0.27 860	9.94 687	7	14
47	9.66 851	24	9.72 170	30	0.27 830	9.94 680	7	13
48	9.66 875	24	9.72 201	31	0.27 799	9.94 674	6	12
49	9.66 899	24	9.72 231	30	0.27 769	9.94 667	7	11
50	9.66 922	23	9.72 262	31	0.27 738	9.94 660	7	**10**
51	9.66 946	24	9.72 293	31	0.27 707	9.94 654	6	9
52	9.66 970	24	9.72 323	30	0.27 677	9.94 647	7	8
53	9.66 994	24	9.72 354	31	0.27 646	9.94 640	7	7
54	9.67 018	24	9.72 384	30	0.27 616	9.94 634	6	6
55	9.67 042	24	9.72 415	31	0.27 585	9.94 627	7	5
56	9.67 066	24	9.72 445	30	0.27 555	9.94 620	7	4
57	9.67 090	24	9.72 476	31	0.27 524	9.94 614	6	3
58	9.67 113	23	9.72 506	30	0.27 494	9.94 607	7	2
59	9.67 137	24	9.72 537	31	0.27 463	9.94 600	7	1
60	9.67 161	24	9.72 567	30	0.27 433	9.94 593	7	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	32	31	30
1	0.5	0.5	0.5
2	1.1	1.0	1.0
3	1.6	1.6	1.5
4	2.1	2.1	2.0
5	2.7	2.6	2.5
6	3.2	3.1	3.0
7	3.7	3.6	3.5
8	4.3	4.1	4.0
9	4.8	4.6	4.5
10	5.3	5.2	5.0
20	10.7	10.3	10.0
30	16.0	15.5	15.0
40	21.3	20.7	20.0
50	26.7	25.8	25.0

	25	24	23
1	0.4	0.4	0.4
2	0.8	0.8	0.8
3	1.2	1.2	1.2
4	1.7	1.6	1.5
5	2.1	2.0	1.9
6	2.5	2.4	2.3
7	2.9	2.8	2.7
8	3.3	3.2	3.1
9	3.8	3.6	3.4
10	4.2	4.0	3.8
20	8.3	8.0	7.7
30	12.5	12.0	11.5
40	16.7	16.0	15.3
50	20.8	20.0	19.2

	7	6
1	0.1	0.1
2	0.2	0.2
3	0.4	0.3
4	0.5	0.4
5	0.6	0.5
6	0.7	0.6
7	0.8	0.7
8	0.9	0.8
9	1.0	0.9
10	1.2	1.0
20	2.3	2.0
30	3.5	3.0
40	4.7	4.0
50	5.8	5.0

P. P.

28°

′	L. Sin	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.67 161		9.72 567		0.27 433	9.94 593		**60**
1	9.67 185	24	9.72 598	31	0.27 402	9.94 587	6	59
2	9.67 208	23	9.72 628	30	0.27 372	9.94 580	7	58
3	9.67 232	24	9.72 659	31	0.27 341	9.94 573	7	57
4	9.67 256	24	9.72 689	30	0.27 311	9.94 567	6	56
5	9.67 280	24	9.72 720	31	0.27 280	9.94 560	7	55
6	9.67 303	23	9.72 750	30	0.27 250	9.94 553	7	54
7	9.67 327	24	9.72 780	30	0.27 220	9.94 546	7	53
8	9.67 350	23	9.72 811	31	0.27 189	9.94 540	6	52
9	9.67 374	24	9.72 841	30	0.27 159	9.94 533	7	51
10	9.67 398	24	9.72 872	31	0.27 128	9.94 526	7	**50**
11	9.67 421	23	9.72 902	30	0.27 098	9.94 519	7	49
12	9.67 445	24	9.72 932	30	0.27 068	9.94 513	6	48
13	9.67 468	23	9.72 963	31	0.27 037	9.94 506	7	47
14	9.67 492	24	9.72 993	30	0.27 007	9.94 499	7	46
15	9.67 515	23	9.73 023	30	0.26 977	9.94 492	7	45
16	9.67 539	24	9.73 054	31	0.26 946	9.94 485	7	44
17	9.67 562	23	9.73 084	30	0.26 916	9.94 479	6	43
18	9.67 586	24	9.73 114	30	0.26 886	9.94 472	7	42
19	9.67 609	23	9.73 144	30	0.26 856	9.94 465	7	41
20	9.67 633	24	9.73 175	31	0.26 825	9.94 458	7	**40**
21	9.67 656	23	9.73 205	30	0.26 795	9.94 451	7	39
22	9.67 680	24	9.73 235	30	0.26 765	9.94 445	6	38
23	9.67 703	23	9.73 265	30	0.26 735	9.94 438	7	37
24	9.67 726	23	9.73 295	30	0.26 705	9.94 431	7	36
25	9.67 750	24	9.73 326	31	0.26 674	9.94 424	7	35
26	9.67 773	23	9.73 356	30	0.26 644	9.94 417	7	34
27	9.67 796	23	9.73 386	30	0.26 614	9.94 410	7	33
28	9.67 820	24	9.73 416	30	0.26 584	9.94 404	6	32
29	9.67 843	23	9.73 446	30	0.26 554	9.94 397	7	31
30	9.67 866	23	9.73 476	30	0.26 524	9.94 390	7	**30**
31	9.67 890	24	9.73 507	31	0.26 493	9.94 383	7	29
32	9.67 913	23	9.73 537	30	0.26 463	9.94 376	7	28
33	9.67 936	23	9.73 567	30	0.26 433	9.94 369	7	27
34	9.67 959	23	9.73 597	30	0.26 403	9.94 362	7	26
35	9.67 982	23	9.73 627	30	0.26 373	9.94 355	7	25
36	9.68 006	24	9.73 657	30	0.26 343	9.94 349	6	24
37	9.68 029	23	9.73 687	30	0.26 313	9.94 342	7	23
38	9.68 052	23	9.73 717	30	0.26 283	9.94 335	7	22
39	9.68 075	23	9.73 747	30	0.26 253	9.94 328	7	21
40	9.68 098	23	9.73 777	30	0.26 223	9.94 321	7	**20**
41	9.68 121	23	9.73 807	30	0.26 193	9.94 314	7	19
42	9.68 144	23	9.73 837	30	0.26 163	9.94 307	7	18
43	9.68 167	23	9.73 867	30	0.26 133	9.94 300	7	17
44	9.68 190	23	9.73 897	30	0.26 103	9.94 293	7	16
45	9.68 213	24	9.73 927	30	0.26 073	9.94 286	7	15
46	9.68 237	23	9.73 957	30	0.26 043	9.94 279	7	14
47	9.68 260	23	9.73 987	30	0.26 013	9.94 273	6	13
48	9.68 283	23	9.74 017	30	0.25 983	9.94 266	7	12
49	9.68 305	22	9.74 047	30	0.25 953	9.94 259	7	11
50	9.68 328	23	9.74 077	30	0.25 923	9.94 252	7	**10**
51	9.68 351	23	9.74 107	30	0.25 893	9.94 245	7	9
52	9.68 374	23	9.74 137	30	0.25 863	9.94 238	7	8
53	9.68 397	23	9.74 166	29	0.25 834	9.94 231	7	7
54	9.68 420	23	9.74 196	30	0.25 804	9.94 224	7	6
55	9.68 443	23	9.74 226	30	0.25 774	9.94 217	7	5
56	9.68 466	23	9.74 256	30	0.25 744	9.94 210	7	4
57	9.68 489	23	9.74 286	30	0.25 714	9.94 203	7	3
58	9.68 512	23	9.74 316	30	0.25 684	9.94 196	7	2
59	9.68 534	22	9.74 345	29	0.25 655	9.94 189	7	**1**
60	9.68 557	23	9.74 375	30	0.25 625	9.94 182	7	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	31	30	29
1	0.5	0.5	0.5
2	1.0	1.0	1.0
3	1.6	1.5	1.4
4	2.1	2.0	1.9
5	2.6	2.5	2.4
6	3.1	3.0	2.9
7	3.6	3.5	3.4
8	4.1	4.0	3.9
9	4.6	4.5	4.4
10	5.2	5.0	4.8
20	10.3	10.0	9.7
30	15.5	15.0	14.5
40	20.7	20.0	19.3
50	25.8	25.0	24.2

	24	23	22
1	0.4	0.4	0.4
2	0.8	0.8	0.7
3	1.2	1.2	1.1
4	1.6	1.5	1.5
5	2.0	1.9	1.8
6	2.4	2.3	2.2
7	2.8	2.7	2.6
8	3.2	3.1	2.9
9	3.6	3.4	3.3
10	4.0	3.8	3.7
20	8.0	7.7	7.3
30	12.0	11.5	11.0
40	16.0	15.3	14.7
50	20.0	19.2	18.3

	7	6
1	0.1	0.1
2	0.2	0.2
3	0.4	0.3
4	0.5	0.4
5	0.6	0.5
6	0.7	0.6
7	0.8	0.7
8	0.9	0.8
9	1.0	0.9
10	1.2	1.0
20	2.3	2.0
30	3.5	3.0
40	4.7	4.0
50	5.8	5.0

29°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.68 557		9.74 375		0.25 625	9.94 182	7	**60**
1	9.68 580	23	9.74 405	30	0.25 595	9.94 175	7	59
2	9.68 603	23	9.74 435	30	0.25 565	9.94 168	7	58
3	9.68 625	22	9.74 465	30	0.25 535	9.94 161	7	57
4	9.68 648	23	9.74 494	29	0.25 506	9.94 154	7	56
5	9.68 671	23	9.74 524	30	0.25 476	9.94 147	7	55
6	9.68 694	23	9.74 554	30	0.25 446	9.94 140	7	54
7	9.68 716	22	9.74 583	29	0.25 417	9.94 133	7	53
8	9.68 739	23	9.74 613	30	0.25 387	9.94 126	7	52
9	9.68 762	23	9.74 643	30	0.25 357	9.94 119	7	51
10	9.68 784	22	9.74 673	30	0.25 327	9.94 112	7	**50**
11	9.68 807	23	9.74 702	29	0.25 298	9.94 105	7	49
12	9.68 829	22	9.74 732	30	0.25 268	9.94 098	7	48
13	9.68 852	23	9.74 762	30	0.25 238	9.94 090	8	47
14	9.68 875	23	9.74 791	29	0.25 209	9.94 083	7	46
15	9.68 897	22	9.74 821	30	0.25 179	9.94 076	7	45
16	9.68 920	23	9.74 851	30	0.25 149	9.94 069	7	44
17	9.68 942	22	9.74 880	29	0.25 120	9.94 062	7	43
18	9.68 965	23	9.74 910	30	0.25 090	9.94 055	7	42
19	9.68 987	22	9.74 939	29	0.25 061	9.94 048	7	41
20	9.69 010	23	9.74 969	30	0.25 031	9.94 041	7	**40**
21	9.69 032	22	9.74 998	29	0.25 002	9.94 034	7	39
22	9.69 055	23	9.75 028	30	0.24 972	9.94 027	7	38
23	9.69 077	22	9.75 058	30	0.24 942	9.94 020	7	37
24	9.69 100	23	9.75 087	29	0.24 913	9.94 012	8	36
25	9.69 122	22	9.75 117	30	0.24 883	9.94 005	7	35
26	9.69 144	22	9.75 146	29	0.24 854	9.93 998	7	34
27	9.69 167	23	9.75 176	30	0.24 824	9.93 991	7	33
28	9.69 189	22	9.75 205	29	0.24 795	9.93 984	7	32
29	9.69 212	23	9.75 235	30	0.24 765	9.93 977	7	31
30	9.69 234	22	9.75 264	29	0.24 736	9.93 970	7	**30**
31	9.69 256	22	9.75 294	30	0.24 706	9.93 963	7	29
32	9.69 279	23	9.75 323	29	0.24 677	9.93 955	8	28
33	9.69 301	22	9.75 353	30	0.24 647	9.93 948	7	27
34	9.69 323	22	9.75 382	29	0.24 618	9.93 941	7	26
35	9.69 345	22	9.75 411	29	0.24 589	9.93 934	7	25
36	9.69 368	23	9.75 441	30	0.24 559	9.93 927	7	24
37	9.69 390	22	9.75 470	29	0.24 530	9.93 920	7	23
38	9.69 412	22	9.75 500	30	0.24 500	9.93 912	8	22
39	9.69 434	22	9.75 529	29	0.24 471	9.93 905	7	21
40	9.69 456	22	9.75 558	29	0.24 442	9.93 898	7	**20**
41	9.69 479	23	9.75 588	30	0.24 412	9.93 891	7	19
42	9.69 501	22	9.75 617	29	0.24 383	9.93 884	7	18
43	9.69 523	22	9.75 647	30	0.24 353	9.93 876	8	17
44	9.69 545	22	9.75 676	29	0.24 324	9.93 869	7	16
45	9.69 567	22	9.75 705	29	0.24 295	9.93 862	7	15
46	9.69 589	22	9.75 735	30	0.24 265	9.93 855	7	14
47	9.69 611	22	9.75 764	29	0.24 236	9.93 847	8	13
48	9.69 633	22	9.75 793	29	0.24 207	9.93 840	7	12
49	9.69 655	22	9.75 822	29	0.24 178	9.93 833	7	11
50	9.69 677	22	9.75 852	30	0.24 148	9.93 826	7	**10**
51	9.69 699	22	9.75 881	29	0.24 119	9.93 819	7	9
52	9.69 721	22	9.75 910	29	0.24 090	9.93 811	8	8
53	9.69 743	22	9.75 939	29	0.24 061	9.93 804	7	7
54	9.69 765	22	9.75 969	30	0.24 031	9.93 797	7	6
55	9.69 787	22	9.75 998	29	0.24 002	9.93 789	8	5
56	9.69 809	22	9.76 027	29	0.23 973	9.93 782	7	4
57	9.69 831	22	9.76 056	29	0.23 944	9.93 775	7	3
58	9.69 853	22	9.76 086	30	0.23 914	9.93 768	7	2
59	9.69 875	22	9.76 115	29	0.23 885	9.93 760	8	1
60	9.69 897	22	9.76 144	29	0.23 856	9.93 753	7	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	30	**29**	**23**
1	0.5	0.5	0.4
2	1.0	1.0	0.8
3	1.5	1.4	1.2
4	2.0	1.9	1.5
5	2.5	2.4	1.9
6	3.0	2.9	2.3
7	3.5	3.4	2.7
8	4.0	3.9	3.1
9	4.5	4.4	3.4
10	5.0	4.8	3.8
20	10.0	9.7	7.7
30	15.0	14.5	11.5
40	20.0	19.3	15.3
50	25.0	24.2	19.2

P. P.

	22	**8**	**7**
1	0.4	0.1	0.1
2	0.7	0.3	0.2
3	1.1	0.4	0.4
4	1.5	0.5	0.5
5	1.8	0.7	0.6
6	2.2	0.8	0.7
7	2.6	0.9	0.8
8	2.9	1.1	0.9
9	3.3	1.2	1.0
10	3.7	1.3	1.2
20	7.3	2.7	2.3
30	11.0	4.0	3.5
40	14.7	5.3	4.7
50	18.3	6.7	5.8

30°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′
0	9.69 897		9.76 144		0.23 856	9.93 753		**60**
1	9.69 919	22	9.76 173	29	0.23 827	9.93 746	7	59
2	9.69 941	22	9.76 202	29	0.23 798	9.93 738	8	58
3	9.69 963	22	9.76 231	29	0.23 769	9.93 731	7	57
4	9.69 984	21	9.76 261	30	0.23 739	9.93 724	7	56
5	9.70 006	22	9.76 290	29	0.23 710	9.93 717	7	55
6	9.70 028	22	9.76 319	29	0.23 681	9.93 709	8	54
7	9.70 050	22	9.76 348	29	0.23 652	9.93 702	7	53
8	9.70 072	22	9.76 377	29	0.23 623	9.93 695	7	52
9	9.70 093	21	9.76 406	29	0.23 594	9.93 687	8	51
10	9.70 115	22	9.76 435	29	0.23 565	9.93 680	7	**50**
11	9.70 137	22	9.76 464	29	0.23 536	9.93 673	7	49
12	9.70 159	22	9.76 493	29	0.23 507	9.93 665	8	48
13	9.70 180	21	9.76 522	29	0.23 478	9.93 658	7	47
14	9.70 202	22	9.76 551	29	0.23 449	9.93 650	8	46
15	9.70 224	22	9.76 580	29	0.23 420	9.93 643	7	45
16	9.70 245	21	9.76 609	29	0.23 391	9.93 636	7	44
17	9.70 267	22	9.76 639	30	0.23 361	9.93 628	8	43
18	9.70 288	21	9.76 668	29	0.23 332	9.93 621	7	42
19	9.70 310	22	9.76 697	29	0.23 303	9.93 614	7	41
20	9.70 332	22	9.76 725	28	0.23 275	9.93 606	8	**40**
21	9.70 353	21	9.76 754	29	0.23 246	9.93 599	7	39
22	9.70 375	22	9.76 783	29	0.23 217	9.93 591	8	38
23	9.70 396	21	9.76 812	29	0.23 188	9.93 584	7	37
24	9.70 418	22	9.76 841	29	0.23 159	9.93 577	7	36
25	9.70 439	21	9.76 870	29	0.23 130	9.93 569	8	35
26	9.70 461	22	9.76 899	29	0.23 101	9.93 562	7	34
27	9.70 482	21	9.76 928	29	0.23 072	9.93 554	8	33
28	9.70 504	22	9.76 957	29	0.23 043	9.93 547	7	32
29	9.70 525	21	9.76 986	29	0.23 014	9.93 539	8	31
30	9.70 547	22	9.77 015	29	0.22 985	9.93 532	7	**30**
31	9.70 568	21	9.77 044	29	0.22 956	9.93 525	7	29
32	9.70 590	22	9.77 073	28	0.22 927	9.93 517	8	28
33	9.70 611	21	9.77 101	29	0.22 899	9.93 510	7	27
34	9.70 633	22	9.77 130	29	0.22 870	9.93 502	8	26
35	9.70 654	21	9.77 159	29	0.22 841	9.93 495	7	25
36	9.70 675	21	9.77 188	29	0.22 812	9.93 487	8	24
37	9.70 697	22	9.77 217	29	0.22 783	9.93 480	7	23
38	9.70 718	21	9.77 246	29	0.22 754	9.93 472	8	22
39	9.70 739	21	9.77 274	28	0.22 726	9.93 465	7	21
40	9.70 761	22	9.77 303	29	0.22 697	9.93 457	8	**20**
41	9.70 782	21	9.77 332	29	0.22 668	9.93 450	7	19
42	9.70 803	21	9.77 361	29	0.22 639	9.93 442	8	18
43	9.70 824	21	9.77 390	29	0.22 610	9.93 435	7	17
44	9.70 846	22	9.77 418	28	0.22 582	9.93 427	8	16
45	9.70 867	21	9.77 447	29	0.22 553	9.93 420	7	15
46	9.70 888	21	9.77 476	29	0.22 524	9.93 412	8	14
47	9.70 909	21	9.77 505	29	0.22 495	9.93 405	7	13
48	9.70 931	22	9.77 533	28	0.22 467	9.93 397	8	12
49	9.70 952	21	9.77 562	29	0.22 438	9.93 390	7	11
50	9.70 973	21	9.77 591	29	0.22 409	9.93 382	8	**10**
51	9.70 994	21	9.77 619	28	0.22 381	9.93 375	7	9
52	9.71 015	21	9.77 648	29	0.22 352	9.93 367	8	8
53	9.71 036	21	9.77 677	29	0.22 323	9.93 360	7	7
54	9.71 058	22	9.77 706	29	0.22 294	9.93 352	8	6
55	9.71 079	21	9.77 734	28	0.22 266	9.93 344	8	5
56	9.71 100	21	9.77 763	29	0.22 237	9.93 337	7	4
57	9.71 121	21	9.77 791	28	0.22 209	9.93 329	8	3
58	9.71 142	21	9.77 820	29	0.22 180	9.93 322	7	2
59	9.71 163	21	9.77 849	29	0.22 151	9.93 314	8	1
60	9.71 184	21	9.77 877	28	0.22 123	9.93 307	7	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	30	29	28
1	0.5	0.5	0.5
2	1.0	1.0	0.9
3	1.5	1.4	1.4
4	2.0	1.9	1.9
5	2.5	2.4	2.3
6	3.0	2.9	2.8
7	3.5	3.4	3.3
8	4.0	3.9	3.7
9	4.5	4.4	4.2
10	5.0	4.8	4.7
20	10.0	9.7	9.3
30	15.0	14.5	14.0
40	20.0	19.3	18.7
50	25.0	24.2	23.3

	22	21
1	0.4	0.4
2	0.7	0.7
3	1.1	1.0
4	1.5	1.4
5	1.8	1.8
6	2.2	2.1
7	2.6	2.4
8	2.9	2.8
9	3.3	3.2
10	3.7	3.5
20	7.3	7.0
30	11.0	10.5
40	14.7	14.0
50	18.3	17.5

	8	7
1	0.1	0.1
2	0.3	0.2
3	0.4	0.4
4	0.5	0.5
5	0.7	0.6
6	0.8	0.7
7	0.9	0.8
8	1.1	0.9
9	1.2	1.0
10	1.3	1.2
20	2.7	2.3
30	4.0	3.5
40	5.3	4.7
50	6.7	5.8

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′	P. P.
0	9.71 184		9.77 877		0.22 123	9.93 307		60	
1	9.71 205	21	9.77 906	29	0.22 094	9.93 299	8	59	
2	9.71 226	21	9.77 935	29	0.22 065	9.93 291	8	58	
3	9.71 247	21	9.77 963	28	0.22 037	9.93 284	7	57	
4	9.71 268	21	9.77 992	29	0.22 008	9.93 276	8	56	
5	9.71 289	21	9.78 020	28	0.21 980	9.93 269	7	55	
6	9.71 310	21	9.78 049	29	0.21 951	9.93 261	8	54	
7	9.71 331	21	9.78 077	28	0.21 923	9.93 253	8	53	29 28
8	9.71 352	21	9.78 106	29	0.21 894	9.93 246	7	52	1 2.9 2.8
9	9.71 373	20	9.78 135	28	0.21 865	9.93 238	8	51	2 5.8 5.6
10	9.71 393	21	9.78 163	28	0.21 837	9.93 230	7	50	3 8.7 8.4
11	9.71 414	21	9.78 192	29	0.21 808	9.93 223	8	49	4 11.6 11.2
12	9.71 435	21	9.78 220	28	0.21 780	9.93 215	8	48	5 14.5 14.0
13	9.71 456	21	9.78 249	29	0.21 751	9.93 207	7	47	6 17.4 16.8
14	9.71 477	21	9.78 277	28	0.21 723	9.93 200	8	46	7 20.3 19.6
15	9.71 498	21	9.78 306	29	0.21 694	9.93 192	8	45	8 23.2 22.4
16	9.71 519	21	9.78 334	28	0.21 666	9.93 184	7	44	9 26.1 25.2
17	9.71 539	20	9.78 363	29	0.21 637	9.93 177	8	43	
18	9.71 560	21	9.78 391	28	0.21 609	9.93 169	8	42	
19	9.71 581	21	9.78 419	28	0.21 581	9.93 161	8	41	
20	9.71 602	21	9.78 448	29	0.21 552	9.93 154	7	40	
21	9.71 622	20	9.78 476	28	0.21 524	9.93 146	8	39	
22	9.71 643	21	9.78 505	29	0.21 495	9.93 138	8	38	
23	9.71 664	21	9.78 533	28	0.21 467	9.93 131	7	37	
24	9.71 685	21	9.78 562	29	0.21 438	9.93 123	8	36	
25	9.71 705	20	9.78 590	28	0.21 410	9.93 115	8	35	29 28
26	9.71 726	21	9.78 618	28	0.21 382	9.93 108	7	34	1 2.9 2.8
27	9.71 747	21	9.78 647	29	0.21 353	9.93 100	8	33	2 5.8 5.6
28	9.71 767	20	9.78 675	28	0.21 325	9.93 092	8	32	3 8.7 8.4
29	9.71 788	21	9.78 704	29	0.21 296	9.93 084	8	31	4 11.6 11.2
30	9.71 809	21	9.78 732	28	0.21 268	9.93 077	7	30	5 14.5 14.0
31	9.71 829	20	9.78 760	28	0.21 240	9.93 069	8	29	6 17.4 16.8
32	9.71 850	21	9.78 789	29	0.21 211	9.93 061	8	28	7 20.3 19.6
33	9.71 870	20	9.78 817	28	0.21 183	9.93 053	8	27	8 23.2 22.4
34	9.71 891	21	9.78 845	28	0.21 155	9.93 046	7	26	9 26.1 25.2
35	9.71 911	20	9.78 874	29	0.21 126	9.93 038	8	25	
36	9.71 932	21	9.78 902	28	0.21 098	9.93 030	8	24	
37	9.71 952	20	9.78 930	28	0.21 070	9.93 022	8	23	
38	9.71 973	21	9.78 959	29	0.21 041	9.93 014	8	22	
39	9.71 994	21	9.78 987	28	0.21 013	9.93 007	7	21	
40	9.72 014	20	9.79 015	28	0.20 985	9.92 999	8	20	
41	9.72 034	20	9.79 043	28	0.20 957	9.92 991	8	19	
42	9.72 055	21	9.79 072	29	0.20 928	9.92 983	8	18	
43	9.72 075	20	9.79 100	28	0.20 900	9.92 976	7	17	
44	9.72 096	21	9.79 128	28	0.20 872	9.92 968	8	16	28 27
45	9.72 116	20	9.79 156	28	0.20 844	9.92 960	8	15	1 2.8 2.7
46	9.72 137	21	9.79 185	29	0.20 815	9.92 952	8	14	2 5.6 5.4
47	9.72 157	20	9.79 213	28	0.20 787	9.92 944	8	13	3 8.4 8.1
48	9.72 177	20	9.79 241	28	0.20 759	9.92 936	8	12	4 11.2 10.8
49	9.72 198	21	9.79 269	28	0.20 731	9.92 929	7	11	5 14.0 13.5
50	9.72 218	20	9.79 297	28	0.20 703	9.92 921	8	10	6 16.8 16.2
51	9.72 238	20	9.79 326	29	0.20 674	9.92 913	8	9	7 19.6 18.9
52	9.72 259	21	9.79 354	28	0.20 646	9.92 905	8	8	8 22.4 21.6
53	9.72 279	20	9.79 382	28	0.20 618	9.92 897	8	7	9 25.2 24.3
54	9.72 299	20	9.79 410	28	0.20 590	9.92 889	8	6	
55	9.72 320	21	9.79 438	28	0.20 562	9.92 881	8	5	
56	9.72 340	20	9.79 466	28	0.20 534	9.92 874	7	4	
57	9.72 360	20	9.79 495	29	0.20 505	9.92 866	8	3	
58	9.72 381	21	9.79 523	28	0.20 477	9.92 858	8	2	
59	9.72 401	20	9.79 551	28	0.20 449	9.92 850	8	1	
60	9.72 421		9.79 579		0.20 421	9.92 842		0	
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′	P. P.

32°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′
0	9.72 421		9.79 579		0.20 421	9.92 842		**60**
1	9.72 441	20	9.79 607	28	0.20 393	9.92 834	8	59
2	9.72 461	20	9.79 635	28	0.20 365	9.92 826	8	58
3	9.72 482	21	9.79 663	28	0.20 337	9.92 818	8	57
4	9.72 502	20	9.79 691	28	0.20 309	9.92 810	8	56
5	9.72 522	20	9.79 719	28	0.20 281	9.92 803	7	55
6	9.72 542	20	9.79 747	28	0.20 253	9.92 795	8	54
7	9.72 562	20	9.79 776	29	0.20 224	9.92 787	8	53
8	9.72 582	20	9.79 804	28	0.20 196	9.92 779	8	52
9	9.72 602	20	9.79 832	28	0.20 168	9.92 771	8	51
10	9.72 622		9.79 860	28	0.20 140	9.92 763	8	**50**
11	9.72 643	21	9.79 888	28	0.20 112	9.92 755	8	49
12	9.72 663	20	9.79 916	28	0.20 084	9.92 747	8	48
13	9.72 683	20	9.79 944	28	0.20 056	9.92 739	8	47
14	9.72 703	20	9.79 972	28	0.20 028	9.92 731	8	46
15	9.72 723	20	9.80 000	28	0.20 000	9.92 723	8	45
16	9.72 743	20	9.80 028	28	0.19 972	9.92 715	8	44
17	9.72 763	20	9.80 056	28	0.19 944	9.92 707	8	43
18	9.72 783	20	9.80 084	28	0.19 916	9.92 699	8	42
19	9.72 803	20	9.80 112	28	0.19 888	9.92 691	8	41
20	9.72 823		9.80 140	28	0.19 860	9.92 683	8	**40**
21	9.72 843	20	9.80 168	27	0.19 832	9.92 675	8	39
22	9.72 863	20	9.80 195	28	0.19 805	9.92 667	8	38
23	9.72 883	20	9.80 223	28	0.19 777	9.92 659	8	37
24	9.72 902	19	9.80 251	28	0.19 749	9.92 651	8	36
25	9.72 922	20	9.80 279	28	0.19 721	9.92 643	8	35
26	9.72 942	20	9.80 307	28	0.19 693	9.92 635	8	34
27	9.72 962	20	9.80 335	28	0.19 665	9.92 627	8	33
28	9.72 982	20	9.80 363	28	0.19 637	9.92 619	8	32
29	9.73 002	20	9.80 391	28	0.19 609	9.92 611	8	31
30	9.73 022		9.80 419	28	0.19 581	9.92 603	8	**30**
31	9.73 041	19	9.80 447	27	0.19 553	9.92 595	8	29
32	9.73 061	20	9.80 474	28	0.19 526	9.92 587	8	28
33	9.73 081	20	9.80 502	28	0.19 498	9.92 579	8	27
34	9.73 101	20	9.80 530	28	0.19 470	9.92 571	8	26
35	9.73 121	20	9.80 558	28	0.19 442	9.92 563	8	25
36	9.73 140	19	9.80 586	28	0.19 414	9.92 555	8	24
37	9.73 160	20	9.80 614	28	0.19 386	9.92 546	9	23
38	9.73 180	20	9.80 642	28	0.19 358	9.92 538	8	22
39	9.73 200	20	9.80 669	27	0.19 331	9.92 530	8	21
40	9.73 219	19	9.80 697	28	0.19 303	9.92 522	8	**20**
41	9.73 239	20	9.80 725	28	0.19 275	9.92 514	8	19
42	9.73 259	20	9.80 753	28	0.19 247	9.92 506	8	18
43	9.73 278	19	9.80 781	28	0.19 219	9.92 498	8	17
44	9.73 298	20	9.80 808	27	0.19 192	9.92 490	8	16
45	9.73 318	20	9.80 836	28	0.19 164	9.92 482	8	15
46	9.73 337	19	9.80 864	28	0.19 136	9.92 473	9	14
47	9.73 357	20	9.80 892	28	0.19 108	9.92 465	8	13
48	9.73 377	20	9.80 919	27	0.19 081	9.92 457	8	12
49	9.73 396	19	9.80 947	28	0.19 053	9.92 449	8	11
50	9.73 416	20	9.80 975	28	0.19 025	9.92 441	8	**10**
51	9.73 435	19	9.81 003	27	0.18 997	9.92 433	8	9
52	9.73 455	20	9.81 030	28	0.18 970	9.92 425	9	8
53	9.73 474	19	9.81 058	28	0.18 942	9.92 416	8	7
54	9.73 494	20	9.81 086	28	0.18 914	9.92 408	8	6
55	9.73 513	19	9.81 113	27	0.18 887	9.92 400	8	5
56	9.73 533	20	9.81 141	28	0.18 859	9.92 392	8	4
57	9.73 552	19	9.81 169	28	0.18 831	9.92 384	8	3
58	9.73 572	20	9.81 196	27	0.18 804	9.92 376	8	2
59	9.73 591	19	9.81 224	28	0.18 776	9.92 367	9	1
60	9.73 611	20	9.81 252	28	0.18 748	9.92 359	8	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	29	**28**	**27**
1	0.5	0.5	0.4
2	1.0	0.9	0.9
3	1.4	1.4	1.4
4	1.9	1.9	1.8
5	2.4	2.3	2.2
6	2.9	2.8	2.7
7	3.4	3.3	3.2
8	3.9	3.7	3.6
9	4.4	4.2	4.0
10	4.8	4.7	4.5
20	9.7	9.3	9.0
30	14.5	14.0	13.5
40	19.3	18.7	18.0
50	24.2	23.3	22.5

	21	**20**	**19**
1	0.4	0.3	0.3
2	0.7	0.7	0.6
3	1.0	1.0	1.0
4	1.4	1.3	1.3
5	1.8	1.7	1.6
6	2.1	2.0	1.9
7	2.4	2.3	2.2
8	2.8	2.7	2.5
9	3.2	3.0	2.8
10	3.5	3.3	3.2
20	7.0	6.7	6.3
30	10.5	10.0	9.5
40	14.0	13.3	12.7
50	17.5	16.7	15.8

	9	**8**	**7**
1	0.2	0.1	0.1
2	0.3	0.3	0.2
3	0.4	0.4	0.4
4	0.6	0.5	0.5
5	0.8	0.7	0.6
6	0.9	0.8	0.7
7	1.0	0.9	0.8
8	1.2	1.1	0.9
9	1.4	1.2	1.0
10	1.5	1.3	1.2
20	3.0	2.7	2.3
30	4.5	4.0	3.5
40	6.0	5.3	4.7
50	7.5	6.7	5.8

57°

33°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.73 611		9.81 252		0.18 748	9.92 359		**60**
1	9.73 630	19	9.81 279	27	0.18 721	9.92 351	8	59
2	9.73 650	20	9.81 307	28	0.18 693	9.92 343	8	58
3	9.73 669	19	9.81 335	28	0.18 665	9.92 335	8	57
4	9.73 689	20	9.81 362	27	0.18 638	9.92 326	9	56
		19		28			8	
5	9.73 708		9.81 390		0.18 610	9.92 318		55
6	9.73 727	19	9.81 418	28	0.18 582	9.92 310	8	54
7	9.73 747	20	9.81 445	27	0.18 555	9.92 302	8	53
8	9.73 766	19	9.81 473	28	0.18 527	9.92 293	9	52
9	9.73 785	19	9.81 500	27	0.18 500	9.92 285	8	51
		20		28			8	
10	9.73 805		9.81 528		0.18 472	9.92 277		**50**
11	9.73 824	19	9.81 556	28	0.18 444	9.92 269	8	49
12	9.73 843	19	9.81 583	27	0.18 417	9.92 260	9	48
13	9.73 863	20	9.81 611	28	0.18 389	9.92 252	8	47
14	9.73 882	19	9.81 638	27	0.18 362	9.92 244	8	46
		19		28			9	
15	9.73 901		9.81 666		0.18 334	9.92 235	8	45
16	9.73 921	20	9.81 693	27	0.18 307	9.92 227	8	44
17	9.73 940	19	9.81 721	28	0.18 279	9.92 219	8	43
18	9.73 959	19	9.81 748	27	0.18 252	9.92 211	8	42
19	9.73 978	19	9.81 776	28	0.18 224	9.92 202	9	41
		19		27			8	
20	9.73 997		9.81 803		0.18 197	9.92 194	8	**40**
21	9.74 017	20	9.81 831	28	0.18 169	9.92 186	8	39
22	9.74 036	19	9.81 858	27	0.18 142	9.92 177	9	38
23	9.74 055	19	9.81 886	28	0.18 114	9.92 169	8	37
24	9.74 074	19	9.81 913	27	0.18 087	9.92 161	8	36
		19		28			9	
25	9.74 093		9.81 941		0.18 059	9.92 152	8	35
26	9.74 113	20	9.81 968	27	0.18 032	9.92 144	8	34
27	9.74 132	19	9.81 996	28	0.18 004	9.92 136	9	33
28	9.74 151	19	9.82 023	27	0.17 977	9.92 127	8	32
29	9.74 170	19	9.82 051	28	0.17 949	9.92 119	8	31
		19		27			9	
30	9.74 189		9.82 078		0.17 922	9.92 111	8	**30**
31	9.74 208	19	9.82 106	28	0.17 894	9.92 102	9	29
32	9.74 227	19	9.82 133	27	0.17 867	9.92 094	8	28
33	9.74 246	19	9.82 161	28	0.17 839	9.92 086	8	27
34	9.74 265	19	9.82 188	27	0.17 812	9.92 077	9	26
		19		27			8	
35	9.74 284		9.82 215		0.17 785	9.92 069	8	25
36	9.74 303	19	9.82 243	28	0.17 757	9.92 060	9	24
37	9.74 322	19	9.82 270	27	0.17 730	9.92 052	8	23
38	9.74 341	19	9.82 298	28	0.17 702	9.92 044	8	22
39	9.74 360	19	9.82 325	27	0.17 675	9.92 035	9	21
		19		27			8	
40	9.74 379		9.82 352		0.17 648	9.92 027	8	**20**
41	9.74 398	19	9.82 380	28	0.17 620	9.92 018	9	19
42	9.74 417	19	9.82 407	27	0.17 593	9.92 010	8	18
43	9.74 436	19	9.82 435	28	0.17 565	9.92 002	8	17
44	9.74 455	19	9.82 462	27	0.17 538	9.91 993	9	16
		19		27			8	
45	9.74 474		9.82 489		0.17 511	9.91 985	8	15
46	9.74 493	19	9.82 517	28	0.17 483	9.91 976	9	14
47	9.74 512	19	9.82 544	27	0.17 456	9.91 968	8	13
48	9.74 531	19	9.82 571	27	0.17 429	9.91 959	9	12
49	9.74 549	18	9.82 599	28	0.17 401	9.91 951	8	11
		19		27			9	
50	9.74 568		9.82 626		0.17 374	9.91 942	9	**10**
51	9.74 587	19	9.82 653	27	0.17 347	9.91 934	8	9
52	9.74 606	19	9.82 681	28	0.17 319	9.91 925	9	8
53	9.74 625	19	9.82 708	27	0.17 292	9.91 917	8	7
54	9.74 644	19	9.82 735	27	0.17 265	9.91 908	9	6
		18		27			8	
55	9.74 662		9.82 762		0.17 238	9.91 900	9	5
56	9.74 681	19	9.82 790	28	0.17 210	9.91 891	9	4
57	9.74 700	19	9.82 817	27	0.17 183	9.91 883	8	3
58	9.74 719	19	9.82 844	27	0.17 156	9.91 874	9	2
59	9.74 737	18	9.82 871	27	0.17 129	9.91 866	8	1
60	9.74 756	19	9.82 899	28	0.17 101	9.91 857	9	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	28	27
1	0.5	0.4
2	0.9	0.9
3	1.4	1.4
4	1.9	1.8
5	2.3	2.2
6	2.8	2.7
7	3.3	3.2
8	3.7	3.6
9	4.2	4.0
10	4.7	4.5
20	9.3	9.0
30	14.0	13.5
40	18.7	18.0
50	23.3	22.5

P. P.

	20	19	18
1	0.3	0.3	0.3
2	0.7	0.6	0.6
3	1.0	1.0	0.9
4	1.3	1.3	1.2
5	1.7	1.6	1.5
6	2.0	1.9	1.8
7	2.3	2.2	2.1
8	2.7	2.5	2.4
9	3.0	2.8	2.7
10	3.3	3.2	3.0
20	6.7	6.3	6.0
30	10.0	9.5	9.0
40	13.3	12.7	12.0
50	16.7	15.8	15.0

P. P.

	9	8
1	0.2	0.1
2	0.3	0.3
3	0.4	0.4
4	0.6	0.5
5	0.8	0.7
6	0.9	0.8
7	1.0	0.9
8	1.2	1.1
9	1.4	1.2
10	1.5	1.3
20	3.0	2.7
30	4.5	4.0
40	6.0	5.3
50	7.5	6.7

34°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.74 756		9.82 899		0.17 101	9.91 857		**60**
1	9.74 775	19	9.82 926	27	0.17 074	9.91 849	8	59
2	9.74 794	19	9.82 953	27	0.17 047	9.91 840	9	58
3	9.74 812	18	9.82 980	27	0.17 020	9.91 832	8	57
4	9.74 831	19	9.83 008	28	0.16 992	9.91 823	9	56
5	9.74 850	19	9.83 035	27	0.16 965	9.91 815	8	55
6	9.74 868	18	9.83 062	27	0.16 938	9.91 806	9	54
7	9.74 887	19	9.83 089	27	0.16 911	9.91 798	8	53
8	9.74 906	19	9.83 117	28	0.16 883	9.91 789	9	52
9	9.74 924	18	9.83 144	27	0.16 856	9.91 781	8	51
10	9.74 943	19	9.83 171	27	0.16 829	9.91 772	9	**50**
11	9.74 961	18	9.83 198	27	0.16 802	9.91 763	9	49
12	9.74 980	19	9.83 225	27	0.16 775	9.91 755	8	48
13	9.74 999	19	9.83 252	27	0.16 748	9.91 746	9	47
14	9.75 017	18	9.83 280	28	0.16 720	9.91 738	8	46
15	9.75 036	19	9.83 307	27	0.16 693	9.91 729	9	45
16	9.75 054	18	9.83 334	27	0.16 666	9.91 720	9	44
17	9.75 073	19	9.83 361	27	0.16 639	9.91 712	8	43
18	9.75 091	18	9.83 388	27	0.16 612	9.91 703	9	42
19	9.75 110	19	9.83 415	27	0.16 585	9.91 695	8	41
20	9.75 128	18	9.83 442	27	0.16 558	9.91 686	9	**40**
21	9.75 147	19	9.83 470	28	0.16 530	9.91 677	9	39
22	9.75 165	18	9.83 497	27	0.16 503	9.91 669	8	38
23	9.75 184	19	9.83 524	27	0.16 476	9.91 660	9	37
24	9.75 202	18	9.83 551	27	0.16 449	9.91 651	9	36
25	9.75 221	19	9.83 578	27	0.16 422	9.91 643	8	35
26	9.75 239	18	9.83 605	27	0.16 395	9.91 634	9	34
27	9.75 258	19	9.83 632	27	0.16 368	9.91 625	9	33
28	9.75 276	18	9.83 659	27	0.16 341	9.91 617	8	32
29	9.75 294	18	9.83 686	27	0.16 314	9.91 608	9	31
30	9.75 313	19	9.83 713	27	0.16 287	9.91 599	9	**30**
31	9.75 331	18	9.83 740	27	0.16 260	9.91 591	8	29
32	9.75 350	19	9.83 768	28	0.16 232	9.91 582	9	28
33	9.75 368	18	9.83 795	27	0.16 205	9.91 573	9	27
34	9.75 386	18	9.83 822	27	0.16 178	9.91 565	8	26
35	9.75 405	19	9.83 849	27	0.16 151	9.91 556	9	25
36	9.75 423	18	9.83 876	27	0.16 124	9.91 547	9	24
37	9.75 441	18	9.83 903	27	0.16 097	9.91 538	9	23
38	9.75 459	18	9.83 930	27	0.16 070	9.91 530	8	22
39	9.75 478	19	9.83 957	27	0.16 043	9.91 521	9	21
40	9.75 496	18	9.83 984	27	0.16 016	9.91 512	9	**20**
41	9.75 514	18	9.84 011	27	0.15 989	9.91 504	8	19
42	9.75 533	19	9.84 038	27	0.15 962	9.91 495	9	18
43	9.75 551	18	9.84 065	27	0.15 935	9.91 486	9	17
44	9.75 569	18	9.84 092	27	0.15 908	9.91 477	9	16
45	9.75 587	18	9.84 119	27	0.15 881	9.91 469	8	15
46	9.75 605	18	9.84 146	27	0.15 854	9.91 460	9	14
47	9.75 624	19	9.84 173	27	0.15 827	9.91 451	9	13
48	9.75 642	18	9.84 200	27	0.15 800	9.91 442	9	12
49	9.75 660	18	9.84 227	27	0.15 773	9.91 433	9	11
50	9.75 678	18	9.84 254	27	0.15 746	9.91 425	8	**10**
51	9.75 696	18	9.84 280	26	0.15 720	9.91 416	9	9
52	9.75 714	18	9.84 307	27	0.15 693	9.91 407	9	8
53	9.75 733	18	9.84 334	27	0.15 666	9.91 398	9	7
54	9.75 751	18	9.84 361	27	0.15 639	9.91 389	9	6
55	9.75 769	18	9.84 388	27	0.15 612	9.91 381	8	5
56	9.75 787	18	9.84 415	27	0.15 585	9.91 372	9	4
57	9.75 805	18	9.84 442	27	0.15 558	9.91 363	9	3
58	9.75 823	18	9.84 469	27	0.15 531	9.91 354	9	2
59	9.75 841	18	9.84 496	27	0.15 504	9.91 345	9	1
60	9.75 859		9.84 523		0.15 477	9.91 336	9	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	28	27	26
1	0.5	0.4	0.4
2	0.9	0.9	0.9
3	1.4	1.4	1.3
4	1.9	1.8	1.7
5	2.3	2.2	2.2
6	2.8	2.7	2.6
7	3.3	3.2	3.0
8	3.7	3.6	3.5
9	4.2	4.0	3.9
10	4.7	4.5	4.3
20	9.3	9.0	8.7
30	14.0	13.5	13.0
40	18.7	18.0	17.3
50	23.3	22.5	21.7

	19	18
1	0.3	0.3
2	0.6	0.6
3	1.0	0.9
4	1.3	1.2
5	1.6	1.5
6	1.9	1.8
7	2.2	2.1
8	2.5	2.4
9	2.8	2.7
10	3.2	3.0
20	6.3	6.0
30	9.5	9.0
40	12.7	12.0
50	15.8	15.0

	9	8
1	0.2	0.1
2	0.3	0.3
3	0.4	0.4
4	0.6	0.5
5	0.8	0.7
6	0.9	0.8
7	1.0	0.9
8	1.2	1.1
9	1.4	1.2
10	1.5	1.3
20	3.0	2.7
30	4.5	4.0
40	6.0	5.3
50	7.5	6.7

55°

35°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′
0	9.75 859		9.84 523		0.15 477	9.91 336		**60**
1	9.75 877	18	9.84 550	27	0.15 450	9.91 328	8	59
2	9.75 895	18	9.84 576	26	0.15 424	9.91 319	9	58
3	9.75 913	18	9.84 603	27	0.15 397	9.91 310	9	57
4	9.75 931	18	9.84 630	27	0.15 370	9.91 301	9	56
5	9.75 949	18	9.84 657	27	0.15 343	9.91 292	9	55
6	9.75 967	18	9.84 684	27	0.15 316	9.91 283	9	54
7	9.75 985	18	9.84 711	27	0.15 289	9.91 274	9	53
8	9.76 003	18	9.84 738	27	0.15 262	9.91 266	8	52
9	9.76 021	18	9.84 764	26	0.15 236	9.91 257	9	51
10	9.76 039	18	9.84 791	27	0.15 209	9.91 248	9	**50**
11	9.76 057	18	9.84 818	27	0.15 182	9.91 239	9	49
12	9.76 075	18	9.84 845	27	0.15 155	9.91 230	9	48
13	9.76 093	18	9.84 872	27	0.15 128	9.91 221	9	47
14	9.76 111	18	9.84 899	27	0.15 101	9.91 212	9	46
15	9.76 129	18	9.84 925	26	0.15 075	9.91 203	9	45
16	9.76 146	17	9.84 952	27	0.15 048	9.91 194	9	44
17	9.76 164	18	9.84 979	27	0.15 021	9.91 185	9	43
18	9.76 182	18	9.85 006	27	0.14 994	9.91 176	9	42
19	9.76 200	18	9.85 033	27	0.14 967	9.91 167	9	41
20	9.76 218	18	9.85 059	26	0.14 941	9.91 158	9	**40**
21	9.76 236	18	9.85 086	27	0.14 914	9.91 149	9	39
22	9.76 253	17	9.85 113	27	0.14 887	9.91 141	8	38
23	9.76 271	18	9.85 140	27	0.14 860	9.91 132	9	37
24	9.76 289	18	9.85 166	26	0.14 834	9.91 123	9	36
25	9.76 307	18	9.85 193	27	0.14 807	9.91 114	9	35
26	9.76 324	17	9.85 220	27	0.14 780	9.91 105	9	34
27	9.76 342	18	9.85 247	27	0.14 753	9.91 096	9	33
28	9.76 360	18	9.85 273	26	0.14 727	9.91 087	9	32
29	9.76 378	18	9.85 300	27	0.14 700	9.91 078	9	31
30	9.76 395	17	9.85 327	27	0.14 673	9.91 069	9	**30**
31	9.76 413	18	9.85 354	27	0.14 646	9.91 060	9	29
32	9.76 431	18	9.85 380	26	0.14 620	9.91 051	9	28
33	9.76 448	17	9.85 407	27	0.14 593	9.91 042	9	27
34	9.76 466	18	9.85 434	27	0.14 566	9.91 033	9	26
35	9.76 484	18	9.85 460	26	0.14 540	9.91 023	10	25
36	9.76 501	17	9.85 487	27	0.14 513	9.91 014	9	24
37	9.76 519	18	9.85 514	27	0.14 486	9.91 005	9	23
38	9.76 537	18	9.85 540	26	0.14 460	9.90 996	9	22
39	9.76 554	17	9.85 567	27	0.14 433	9.90 987	9	21
40	9.76 572	18	9.85 594	27	0.14 406	9.90 978	9	**20**
41	9.76 590	18	9.85 620	26	0.14 380	9.90 969	9	19
42	9.76 607	17	9.85 647	27	0.14 353	9.90 960	9	18
43	9.76 625	18	9.85 674	27	0.14 326	9.90 951	9	17
44	9.76 642	17	9.85 700	26	0.14 300	9.90 942	9	16
45	9.76 660	18	9.85 727	27	0.14 273	9.90 933	9	15
46	9.76 677	17	9.85 754	27	0.14 246	9.90 924	9	14
47	9.76 695	18	9.85 780	26	0.14 220	9.90 915	9	13
48	9.76 712	17	9.85 807	27	0.14 193	9.90 906	9	12
49	9.76 730	18	9.85 834	27	0.14 166	9.90 896	10	11
50	9.76 747	17	9.85 860	26	0.14 140	9.90 887	9	**10**
51	9.76 765	18	9.85 887	27	0.14 113	9.90 878	9	9
52	9.76 782	17	9.85 913	26	0.14 087	9.90 869	9	8
53	9.76 800	18	9.85 940	27	0.14 060	9.90 860	9	7
54	9.76 817	17	9.85 967	27	0.14 033	9.90 851	9	6
55	9.76 835	18	9.85 993	26	0.14 007	9.90 842	9	5
56	9.76 852	17	9.86 020	27	0.13 980	9.90 832	10	4
57	9.76 870	18	9.86 046	26	0.13 954	9.90 823	9	3
58	9.76 887	17	9.86 073	27	0.13 927	9.90 814	9	2
59	9.76 904	17	9.86 100	27	0.13 900	9.90 805	9	1
60	9.76 922	18	9.86 126	26	0.13 874	9.90 796	9	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	27	26	18
1	0.4	0.4	0.3
2	0.9	0.9	0.6
3	1.4	1.3	0.9
4	1.8	1.7	1.2
5	2.2	2.2	1.5
6	2.7	2.6	1.8
7	3.2	3.0	2.1
8	3.6	3.5	2.4
9	4.0	3.9	2.7
10	4.5	4.3	3.0
20	9.0	8.7	6.0
30	13.5	13.0	9.0
40	18.0	17.3	12.0
50	22.5	21.7	15.0

P. P.

	17	10	9	8
1	0.3	0.2	0.2	0.1
2	0.6	0.3	0.3	0.3
3	0.8	0.5	0.4	0.4
4	1.1	0.7	0.6	0.5
5	1.4	0.8	0.8	0.7
6	1.7	1.0	0.9	0.8
7	2.0	1.2	1.0	0.9
8	2.3	1.3	1.2	1.1
9	2.6	1.5	1.4	1.2
10	2.8	1.7	1.5	1.3
20	5.7	3.3	3.0	2.7
30	8.5	5.0	4.5	4.0
40	11.3	6.7	6.0	5.3
50	14.2	8.3	7.5	6.7

36°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′
0	9.76 922		9.86 126		0.13 874	9.90 796		**60**
1	9.76 939	17	9.86 153	27	0.13 847	9.90 787	9	59
2	9.76 957	18	9.86 179	26	0.13 821	9.90 777	10	58
3	9.76 974	17	9.86 206	27	0.13 794	9.90 768	9	57
4	9.76 991	17	9.86 232	26	0.13 768	9.90 759	9	56
5	9.77 009	18	9.86 259	27	0.13 741	9.90 750	9	55
6	9.77 026	17	9.86 285	26	0.13 715	9.90 741	9	54
7	9.77 043	17	9.86 312	27	0.13 688	9.90 731	10	53
8	9.77 061	18	9.86 338	26	0.13 662	9.90 722	9	52
9	9.77 078	17	9.86 365	27	0.13 635	9.90 713	9	51
10	9.77 095	17	9.86 392	27	0.13 608	9.90 704	9	**50**
11	9.77 112	17	9.86 418	26	0.13 582	9.90 694	10	49
12	9.77 130	18	9.86 445	27	0.13 555	9.90 685	9	48
13	9.77 147	17	9.86 471	26	0.13 529	9.90 676	9	47
14	9.77 164	17	9.86 498	27	0.13 502	9.90 667	9	46
15	9.77 181	17	9.86 524	26	0.13 476	9.90 657	10	45
16	9.77 199	18	9.86 551	27	0.13 449	9.90 648	9	44
17	9.77 216	17	9.86 577	26	0.13 423	9.90 639	9	43
18	9.77 233	17	9.86 603	26	0.13 397	9.90 630	9	42
19	9.77 250	17	9.86 630	27	0.13 370	9.90 620	10	41
20	9.77 268	18	9.86 656	26	0.13 344	9.90 611	9	**40**
21	9.77 285	17	9.86 683	27	0.13 317	9.90 602	9	39
22	9.77 302	17	9.86 709	26	0.13 291	9.90 592	10	38
23	9.77 319	17	9.86 736	27	0.13 264	9.90 583	9	37
24	9.77 336	17	9.86 762	26	0.13 238	9.90 574	9	36
25	9.77 353	17	9.86 789	27	0.13 211	9.90 565	9	35
26	9.77 370	17	9.86 815	26	0.13 185	9.90 555	10	34
27	9.77 387	17	9.86 842	27	0.13 158	9.90 546	9	33
28	9.77 405	18	9.86 868	26	0.13 132	9.90 537	9	32
29	9.77 422	17	9.86 894	26	0.13 106	9.90 527	10	31
30	9.77 439	17	9.86 921	27	0.13 079	9.90 518	9	**30**
31	9.77 456	17	9.86 947	26	0.13 053	9.90 509	9	29
32	9.77 473	17	9.86 974	27	0.13 026	9.90 499	10	28
33	9.77 490	17	9.87 000	26	0.13 000	9.90 490	9	27
34	9.77 507	17	9.87 027	27	0.12 973	9.90 480	10	26
35	9.77 524	17	9.87 053	26	0.12 947	9.90 471	9	25
36	9.77 541	17	9.87 079	26	0.12 921	9.90 462	9	24
37	9.77 558	17	9.87 106	27	0.12 894	9.90 452	10	23
38	9.77 575	17	9.87 132	26	0.12 868	9.90 443	9	22
39	9.77 592	17	9.87 158	26	0.12 842	9.90 434	9	21
40	9.77 609	17	9.87 185	27	0.12 815	9.90 424	10	**20**
41	9.77 626	17	9.87 211	26	0.12 789	9.90 415	9	19
42	9.77 643	17	9.87 238	27	0.12 762	9.90 405	10	18
43	9.77 660	17	9.87 264	26	0.12 736	9.90 396	9	17
44	9.77 677	17	9.87 290	26	0.12 710	9.90 386	10	16
45	9.77 694	17	9.87 317	27	0.12 682	9.90 377	9	15
46	9.77 711	17	9.87 343	26	0.12 657	9.90 368	9	14
47	9.77 728	17	9.87 369	26	0.12 631	9.90 358	10	13
48	9.77 744	16	9.87 396	27	0.12 604	9.90 349	9	12
49	9.77 761	17	9.87 422	26	0.12 578	9.90 339	10	11
50	9.77 778	17	9.87 448	26	0.12 552	9.90 330	9	**10**
51	9.77 795	17	9.87 475	27	0.12 525	9.90 320	10	9
52	9.77 812	17	9.87 501	26	0.12 499	9.90 311	9	8
53	9.77 829	17	9.87 527	26	0.12 473	9.90 301	10	7
54	9.77 846	17	9.87 554	27	0.12 446	9.90 292	9	6
55	9.77 862	16	9.87 580	26	0.12 420	9.90 282	10	5
56	9.77 879	17	9.87 606	26	0.12 394	9.90 273	9	4
57	9.77 896	17	9.87 633	27	0.12 367	9.90 263	10	3
58	9.77 913	17	9.87 659	26	0.12 341	9.90 254	9	2
59	9.77 930	17	9.87 685	26	0.12 315	9.90 244	10	1
60	9.77 946	16	9.87 711	26	0.12 289	9.90 235	9	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	27	26
1	0.4	0.4
2	0.9	0.9
3	1.4	1.3
4	1.8	1.7
5	2.2	2.2
6	2.7	2.6
7	3.2	3.0
8	3.6	3.5
9	4.0	3.9
10	4.5	4.3
20	9.0	8.7
30	13.5	13.0
40	18.0	17.3
50	22.5	21.7

	18	17	16
1	0.3	0.3	0.3
2	0.6	0.6	0.5
3	0.9	0.8	0.8
4	1.2	1.1	1.1
5	1.5	1.4	1.3
6	1.8	1.7	1.6
7	2.1	2.0	1.9
8	2.4	2.3	2.1
9	2.7	2.6	2.4
10	3.0	2.8	2.7
20	6.0	5.7	5.3
30	9.0	8.5	8.0
40	12.0	11.3	10.7
50	15.0	14.2	13.3

	10	9
1	0.2	0.2
2	0.3	0.3
3	0.5	0.4
4	0.7	0.6
5	0.8	0.8
6	1.0	0.9
7	1.2	1.0
8	1.3	1.2
9	1.5	1.4
10	1.7	1.5
20	3.3	3.0
30	5.0	4.5
40	6.7	6.0
50	8.3	7.5

P. P.

53°

37°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.		P. P.	
0	9.77 946		9.87 711		0.12 289	9.90 235		**60**		
1	9.77 963	17	9.87 738	27	0.12 262	9.90 225	10	59		
2	9.77 980	17	9.87 764	26	0.12 236	9.90 216	9	58		
3	9.77 997	17	9.87 790	26	0.12 210	9.90 206	10	57		
4	9.78 013	16	9.87 817	27	0.12 183	9.90 197	9	56		
5	9.78 030	17	9.87 843	26	0.12 157	9.90 187	10	55		
6	9.78 047	17	9.87 869	26	0.12 131	9.90 178	9	54		
7	9.78 063	16	9.87 895	26	0.12 105	9.90 168	10	53	**27**	**26**
8	9.78 080	17	9.87 922	27	0.12 078	9.90 159	9	52	1 0.4	0.4
9	9.78 097	17	9.87 948	26	0.12 052	9.90 149	10	51	2 0.9	0.9
10	9.78 113	16	9.87 974	26	0.12 026	9.90 139	10	**50**	3 1.4	1.3
11	9.78 130	17	9.88 000	26	0.12 000	9.90 130	9	49	4 1.8	1.7
12	9.78 147	17	9.88 027	27	0.11 973	9.90 120	10	48	5 2.2	2.2
13	9.78 163	16	9.88 053	26	0.11 947	9.90 111	9	47	6 2.7	2.6
14	9.78 180	17	9.88 079	26	0.11 921	9.90 101	10	46	7 3.2	3.0
15	9.78 197	17	9.88 105	26	0.11 895	9.90 091	10	45	8 3.6	3.5
16	9.78 213	16	9.88 131	26	0.11 869	9.90 082	9	44	9 4.0	3.9
17	9.78 230	17	9.88 158	27	0.11 842	9.90 072	10	43	10 4.5	4.3
18	9.78 246	16	9.88 184	26	0.11 816	9.90 063	9	42	20 9.0	8.7
19	9.78 263	17	9.88 210	26	0.11 790	9.90 053	10	41	30 13.5	13.0
20	9.78 280	17	9.88 236	26	0.11 764	9.90 043	10	**40**	40 18.0	17.3
21	9.78 296	16	9.88 262	26	0.11 738	9.90 034	9	39	50 22.5	21.7
22	9.78 313	17	9.88 289	27	0.11 711	9.90 024	10	38		
23	9.78 329	16	9.88 315	26	0.11 685	9.90 014	10	37		
24	9.78 346	17	9.88 341	26	0.11 659	9.90 005	9	36		
25	9.78 362	16	9.88 367	26	0.11 633	9.89 995	10	35	**17**	**16**
26	9.78 379	17	9.88 393	26	0.11 607	9.89 985	10	34	1 0.3	0.3
27	9.78 395	16	9.88 420	27	0.11 580	9.89 976	9	33	2 0.6	0.5
28	9.78 412	17	9.88 446	26	0.11 554	9.89 966	10	32	3 0.8	0.8
29	9.78 428	16	9.88 472	26	0.11 528	9.89 956	10	31	4 1.1	1.1
30	9.78 445	17	9.88 498	26	0.11 502	9.89 947	9	**30**	5 1.4	1.3
31	9.78 461	16	9.88 524	26	0.11 476	9.89 937	10	29	6 1.7	1.6
32	9.78 478	17	9.88 550	26	0.11 450	9.89 927	10	28	7 2.0	1.9
33	9.78 494	16	9.88 577	27	0.11 423	9.89 918	9	27	8 2.3	2.1
34	9.78 510	16	9.88 603	26	0.11 397	9.89 908	10	26	9 2.6	2.4
35	9.78 527	17	9.88 629	26	0.11 371	9.89 898	10	25	10 2.8	2.7
36	9.78 543	16	9.88 655	26	0.11 345	9.89 888	10	24	20 5.7	5.3
37	9.78 560	17	9.88 681	26	0.11 319	9.89 879	9	23	30 8.5	8.0
38	9.78 576	16	9.88 707	26	0.11 293	9.89 869	10	22	40 11.3	10.7
39	9.78 592	16	9.88 733	26	0.11 267	9.89 859	10	21	50 14.2	13.3
40	9.78 609	17	9.88 759	26	0.11 241	9.89 849	10	**20**		
41	9.78 625	16	9.88 786	27	0.11 214	9.89 840	9	19		
42	9.78 642	17	9.88 812	26	0.11 188	9.89 830	10	18		
43	9.78 658	16	9.88 838	26	0.11 162	9.89 820	10	17	**10**	**9**
44	9.78 674	16	9.88 864	26	0.11 136	9.89 810	10	16	1 0.2	0.2
45	9.78 691	17	9.88 890	26	0.11 110	9.89 801	9	15	2 0.3	0.3
46	9.78 707	16	9.88 916	26	0.11 084	9.89 791	10	14	3 0.5	0.4
47	9.78 723	16	9.88 942	26	0.11 058	9.89 781	10	13	4 0.7	0.6
48	9.78 739	16	9.88 968	26	0.11 032	9.89 771	10	12	5 0.8	0.8
49	9.78 756	17	9.88 994	26	0.11 006	9.89 761	10	11	6 1.0	0.9
50	9.78 772	16	9.89 020	26	0.10 980	9.89 752	9	**10**	7 1.2	1.0
51	9.78 788	16	9.89 046	26	0.10 954	9.89 742	10	9	8 1.3	1.2
52	9.78 805	17	9.89 073	27	0.10 927	9.89 732	10	8	9 1.5	1.4
53	9.78 821	16	9.89 099	26	0.10 901	9.89 722	10	7	10 1.7	1.5
54	9.78 837	16	9.89 125	26	0.10 875	9.89 712	10	6	20 3.3	3.0
55	9.78 853	16	9.89 151	26	0.10 849	9.89 702	10	5	30 5.0	4.5
56	9.78 869	16	9.89 177	26	0.10 823	9.89 693	9	4	40 6.7	6.0
57	9.78 886	17	9.89 203	26	0.10 797	9.89 683	10	3	50 8.3	7.5
58	9.78 902	16	9.89 229	26	0.10 771	9.89 673	10	2		
59	9.78 918	16	9.89 255	26	0.10 745	9.89 663	10	1		
60	9.78 934	16	9.89 281	26	0.10 719	9.89 653	10	**0**		
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′	P. P.	

38°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′
0	9.78 934		9.89 281		0.10 719	9.89 653		**60**
1	9.78 950	16	9.89 307	26	0.10 693	9.89 643	10	59
2	9.78 967	17	9.89 333	26	0.10 667	9.89 633	10	58
3	9.78 983	16	9.89 359	26	0.10 641	9.89 624	9	57
4	9.78 999	16	9.89 385	26	0.10 615	9.89 614	10	56
5	9.79 015		9.89 411	26	0.10 589	9.89 604	10	55
6	9.79 031	16	9.89 437	26	0.10 563	9.89 594	10	54
7	9.79 047	16	9.89 463	26	0.10 537	9.89 584	10	53
8	9.79 063	16	9.89 489	26	0.10 511	9.89 574	10	52
9	9.79 079	16	9.89 515	26	0.10 485	9.89 564	10	51
10	9.79 095		9.89 541	26	0.10 459	9.89 554	10	**50**
11	9.79 111	16	9.89 567	26	0.10 433	9.89 544	10	49
12	9.79 128	17	9.89 593	26	0.10 407	9.89 534	10	48
13	9.79 144	16	9.89 619	26	0.10 381	9.89 524	10	47
14	9.79 160	16	9.89 645	26	0.10 355	9.89 514	10	46
15	9.79 176	16	9.89 671	26	0.10 329	9.89 504	9	45
16	9.79 192	16	9.89 697	26	0.10 303	9.89 495	10	44
17	9.79 208	16	9.89 723	26	0.10 277	9.89 485	10	43
18	9.79 224	16	9.89 749	26	0.10 251	9.89 475	10	42
19	9.79 240	16	9.89 775	26	0.10 225	9.89 465	10	41
20	9.79 256		9.89 801	26	0.10 199	9.89 455	10	**40**
21	9.79 272	16	9.89 827	26	0.10 173	9.89 445	10	39
22	9.79 288	16	9.89 853	26	0.10 147	9.89 435	10	38
23	9.79 304	16	9.89 879	26	0.10 121	9.89 425	10	37
24	9.79 319	15	9.89 905	26	0.10 095	9.89 415	10	36
25	9.79 335	16	9.89 931	26	0.10 069	9.89 405	10	35
26	9.79 351	16	9.89 957	26	0.10 043	9.89 395	10	34
27	9.79 367	16	9.89 983	26	0.10 017	9.89 385	10	33
28	9.79 383	16	9.90 009	26	0.09 991	9.89 375	10	32
29	9.79 399	16	9.90 035	26	0.09 965	9.89 364	11	31
30	9.79 415		9.90 061	25	0.09 939	9.89 354	10	**30**
31	9.79 431	16	9.90 086	26	0.09 914	9.89 344	10	29
32	9.79 447	16	9.90 112	26	0.09 888	9.89 334	10	28
33	9.79 463	16	9.90 138	26	0.09 862	9.89 324	10	27
34	9.79 478	15	9.90 164	26	0.09 836	9.89 314	10	26
35	9.79 494	16	9.90 190	26	0.09 810	9.89 304	10	25
36	9.79 510	16	9.90 216	26	0.09 784	9.89 294	10	24
37	9.79 526	16	9.90 242	26	0.09 758	9.89 284	10	23
38	9.79 542	16	9.90 268	26	0.09 732	9.89 274	10	22
39	9.79 558	16	9.90 294	26	0.09 706	9.89 264	10	21
40	9.79 573	15	9.90 320	26	0.09 680	9.89 254	10	**20**
41	9.79 589	16	9.90 346	26	0.09 654	9.89 244	10	19
42	9.79 605	16	9.90 371	25	0.09 629	9.89 233	11	18
43	9.79 621	16	9.90 397	26	0.09 603	9.89 223	10	17
44	9.79 636	15	9.90 423	26	0.09 577	9.89 213	10	16
45	9.79 652	16	9.90 449	26	0.09 551	9.89 203	10	15
46	9.79 668	16	9.90 475	26	0.09 525	9.89 193	10	14
47	9.79 684	16	9.90 501	26	0.09 499	9.89 183	10	13
48	9.79 699	15	9.90 527	26	0.09 473	9.89 173	10	12
49	9.79 715	16	9.90 553	25	0.09 447	9.89 162	11	11
50	9.79 731	16	9.90 578	26	0.09 422	9.89 152	10	**10**
51	9.79 746	15	9.90 604	26	0.09 396	9.89 142	10	9
52	9.79 762	16	9.90 630	26	0.09 370	9.89 132	10	8
53	9.79 778	16	9.90 656	26	0.09 344	9.89 122	10	7
54	9.79 793	15	9.90 682	26	0.09 318	9.89 112	10	6
55	9.79 809	16	9.90 708	26	0.09 292	9.89 101	11	5
56	9.79 825	16	9.90 734	26	0.09 266	9.89 091	10	4
57	9.79 840	15	9.90 759	25	0.09 241	9.89 081	10	3
58	9.79 856	16	9.90 785	26	0.09 215	9.89 071	10	2
59	9.79 872	16	9.90 811	26	0.09 189	9.89 060	11	1
60	9.79 887	15	9.90 837	26	0.09 163	9.89 050	10	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	26	25
1	0.4	0.4
2	0.9	0.8
3	1.3	1.2
4	1.7	1.7
5	2.2	2.1
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

	17	16	15
1	0.3	0.3	0.2
2	0.6	0.5	0.5
3	0.8	0.8	0.8
4	1.1	1.1	1.0
5	1.4	1.3	1.2
6	1.7	1.6	1.5
7	2.0	1.9	1.8
8	2.3	2.1	2.0
9	2.6	2.4	2.2
10	2.8	2.7	2.5
20	5.7	5.3	5.0
30	8.5	8.0	7.5
40	11.3	10.7	10.0
50	14.2	13.3	12.5

	11	10	9
1	0.2	0.2	0 2
2	0.4	0.3	0.3
3	0.6	0.5	0.4
4	0.7	0.7	0.6
5	0.9	0.8	0.8
6	1.1	1.0	0.9
7	1.3	1.2	1.0
8	1.5	1.3	1.2
9	1.6	1.5	1.4
10	1.8	1.7	1.5
20	3.7	3.3	3.0
30	5.5	5.0	4.5
40	7.3	6.7	6.0
50	9.2	8.3	7.5

51°

39°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.79 887		9.90 837		0.09 163	9.89 050		**60**
1	9.79 903	16	9.90 863	26	0.09 137	9.89 040	10	59
2	9.79 918	15	9.90 889	26	0.09 111	9.89 030	10	58
3	9.79 934	16	9.90 914	25	0.09 086	9.89 020	10	57
4	9.79 950	16	9.90 940	26	0.09 060	9.89 009	11	56
		15		26			10	
5	9.79 965		9.90 966		0.09 034	9.88 999		55
6	9.79 981	16	9.90 992	26	0.09 008	9.88 989	10	54
7	9.79 996	15	9.91 018	26	0.08 982	9.88 978	11	53
8	9.80 012	16	9.91 043	25	0.08 957	9.88 968	10	52
9	9.80 027	15	9.91 069	26	0.08 931	9.88 958	10	51
		16		26			10	
10	9.80 043		9.91 095		0.08 905	9.88 948		**50**
11	9.80 058	15	9.91 121	26	0.08 879	9.88 937	11	49
12	9.80 074	16	9.91 147	26	0.08 853	9.88 927	10	48
13	9.80 089	15	9.91 172	25	0.08 828	9.88 917	10	47
14	9.80 105	16	9.91 198	26	0.08 802	9.88 906	11	46
		15		26			10	
15	9.80 120		9.91 224		0.08 776	9.88 896		45
16	9.80 136	16	9.91 250	26	0.08 750	9.88 886	10	44
17	9.80 151	15	9.91 276	26	0.08 724	9.88 875	11	43
18	9.80 166	15	9.91 301	25	0.08 699	9.88 865	10	42
19	9.80 182	16	9.91 327	26	0.08 673	9.88 855	10	41
		15		26			11	
20	9.80 197		9.91 353		0.08 647	9.88 844		**40**
21	9.80 213	16	9.91 379	26	0.08 621	9.88 834	10	39
22	9.80 228	15	9.91 404	25	0.08 596	9.88 824	10	38
23	9.80 244	16	9.91 430	26	0.08 570	9.88 813	11	37
24	9.80 259	15	9.91 456	26	0.08 544	9.88 803	10	36
		15		26			10	
25	9.80 274		9.91 482		0.08 518	9.88 793		35
26	9.80 290	16	9.91 507	25	0.08 493	9.88 782	11	34
27	9.80 305	15	9.91 533	26	0.08 467	9.88 772	10	33
28	9.80 320	15	9.91 559	26	0.08 441	9.88 761	11	32
29	9.80 336	16	9.91 585	26	0.08 415	9.88 751	10	31
		15		25			10	
30	9.80 351		9.91 610		0.08 390	9.88 741		**30**
31	9.80 366	15	9.91 636	26	0.08 364	9.88 730	11	29
32	9.80 382	16	9.91 662	26	0.08 338	9.88 720	10	28
33	9.80 397	15	9.91 688	26	0.08 312	9.88 709	11	27
34	9.80 412	15	9.91 713	25	0.08 287	9.88 699	10	26
		16		26			11	
35	9.80 428		9.91 739		0.08 261	9.88 688		25
36	9.80 443	15	9.91 765	26	0.08 235	9.88 678	10	24
37	9.80 458	15	9.91 791	26	0.08 209	9.88 668	10	23
38	9.80 473	15	9.91 816	25	0.08 184	9.88 657	11	22
39	9.80 489	16	9.91 842	26	0.08 158	9.88 647	10	21
		15		26			11	
40	9.80 504		9.91 868		0.08 132	9.88 636		**20**
41	9.80 519	15	9.91 893	25	0.08 107	9.88 626	10	19
42	9.80 534	15	9.91 919	26	0.08 081	9.88 615	11	18
43	9.80 550	16	9.91 945	26	0.08 055	9.88 605	10	17
44	9.80 565	15	9.91 971	26	0.08 029	9.88 594	11	16
		15		25			10	
45	9.80 580		9.91 996		0.08 004	9.88 584		15
46	9.80 595	15	9.92 022	26	0.07 978	9.88 573	11	14
47	9.80 610	15	9.92 048	26	0.07 952	9.88 563	10	13
48	9.80 625	15	9.92 073	25	0.07 927	9.88 552	11	12
49	9.80 641	16	9.92 099	26	0.07 901	9.88 542	10	11
		15		26			11	
50	9.80 656		9.92 125		0.07 875	9.88 531		**10**
51	9.80 671	15	9.92 150	25	0.07 850	9.88 521	10	9
52	9.80 686	15	9.92 176	26	0.07 824	9.88 510	11	8
53	9.80 701	15	9.92 202	26	0.07 798	9.88 499	11	7
54	9.80 716	15	9.92 227	25	0.07 773	9.88 489	10	6
		15		26			11	
55	9.80 731		9.92 253		0.07 747	9.88 478		5
56	9.80 746	15	9.92 279	26	0.07 721	9.88 468	10	4
57	9.80 762	16	9.92 304	25	0.07 696	9.88 457	11	3
58	9.80 777	15	9.92 330	26	0.07 670	9.88 447	10	2
59	9.80 792	15	9.92 356	26	0.07 644	9.88 436	11	1
		15		25			11	
60	9.80 807		9.92 381		0.07 619	9.88 425		**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	26	**25**
1	0.4	0.4
2	0.9	0.8
3	1.3	1.2
4	1.7	1.7
5	2.2	2.1
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

	16	**15**
1	0.3	0.2
2	0.5	0.5
3	0.8	0.8
4	1.1	1.0
5	1.3	1.2
6	1.6	1.5
7	1.9	1.8
8	2.1	2.0
9	2.4	2.2
10	2.7	2.5
20	5.3	5.0
30	8.0	7.5
40	10.7	10.0
50	13.3	12.5

	11	**10**
1	0.2	0.2
2	0.4	0.3
3	0.6	0.5
4	0.7	0.7
5	0.9	0.8
6	1.1	1.0
7	1.3	1.2
8	1.5	1.3
9	1.6	1.5
10	1.8	1.7
20	3.7	3.3
30	5.5	5.0
40	7.3	6.7
50	9.2	8.3

50°

40°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′
0	9.80 807		9.92 381		0.07 619	9.88 425		**60**
1	9.80 822	15	9.92 407	26	0.07 593	9.88 415	10	59
2	9.80 837	15	9.92 433	26	0.07 567	9.88 404	11	58
3	9.80 852	15	9.92 458	25	0.07 542	9.88 394	10	57
4	9.80 867	15	9.92 484	26	0.07 516	9.88 383	11	56
5	9.80 882	15	9.92 510	26	0.07 490	9.88 372	11	55
6	9.80 897	15	9.92 535	25	0.07 465	9.88 362	10	54
7	9.80 912	15	9.92 561	26	0.07 439	9.88 351	11	53
8	9.80 927	15	9.92 587	26	0.07 413	9.88 340	11	52
9	9.80 942	15	9.92 612	25	0.07 388	9.88 330	10	51
10	9.80 957	15	9.92 638	26	0.07 362	9.88 319	11	**50**
11	9.80 972	15	9.92 663	25	0.07 337	9.88 308	11	49
12	9.80 987	15	9.92 689	26	0.07 311	9.88 298	10	48
13	9.81 002	15	9.92 715	26	0.07 285	9.88 287	11	47
14	9.81 017	15	9.92 740	25	0.07 260	9.88 276	11	46
15	9.81 032	15	9.92 766	26	0.07 234	9.88 266	10	45
16	9.81 047	15	9.92 792	26	0.07 208	9.88 255	11	43
17	9.81 061	14	9.92 817	25	0.07 183	9.88 244	11	43
18	9.81 076	15	9.92 843	26	0.07 157	9.88 234	10	42
19	9.81 091	15	9.92 868	25	0.07 132	9.88 223	11	41
20	9.81 106	15	9.92 894	26	0.07 106	9.88 212	11	**40**
21	9.81 121	15	9.92 920	26	0.07 080	9.88 201	11	39
22	9.81 136	15	9.92 945	25	0.07 055	9.88 191	10	38
23	9.81 151	15	9.92 971	26	0.07 029	9.88 180	11	37
24	9.81 166	15	9.92 996	25	0.07 004	9.88 169	11	36
25	9.81 180	14	9.93 022	26	0.06 978	9.88 158	11	35
26	9.81 195	15	9.93 048	26	0.06 952	9.88 148	10	34
27	9.81 210	15	9.93 073	25	0.06 927	9.88 137	11	33
28	9.81 225	15	9.93 099	26	0.06 901	9.88 126	11	32
29	9.81 240	15	9.93 124	25	0.06 876	9.88 115	11	31
30	9.81 254	14	9.93 150	26	0.06 850	9.88 105	10	**30**
31	9.81 269	15	9.93 175	25	0.06 825	9.88 094	11	29
32	9.81 284	15	9.93 201	26	0.06 799	9.88 083	11	28
33	9.81 299	15	9.93 227	26	0.06 773	9.88 072	11	27
34	9.81 314	15	9.93 252	25	0.06 748	9.88 061	11	26
35	9.81 328	14	9.93 278	26	0.06 722	9.88 051	10	25
36	9.81 343	15	9.93 303	25	0.06 697	9.88 040	11	24
37	9.81 358	15	9.93 329	26	0.06 671	9.88 029	11	23
38	9.81 372	14	9.93 354	25	0.06 646	9.88 018	11	22
39	9.81 387	15	9.93 380	26	0.06 620	9.88 007	11	21
40	9.81 402	15	9.93 406	26	0.06 594	9.87 996	11	**20**
41	9.81 417	15	9.93 431	25	0.06 569	9.87 985	11	19
42	9.81 431	14	9.93 457	26	0.06 543	9.87 975	10	18
43	9.81 446	15	9.93 482	25	0.06 518	9.87 964	11	17
44	9.81 461	15	9.93 508	26	0.06 492	9.87 953	11	16
45	9.81 475	14	9.93 533	25	0.06 467	9.87 942	11	15
46	9.81 490	15	9.93 559	26	0.06 441	9.87 931	11	14
47	9.81 505	15	9.93 584	25	0.06 416	9.87 920	11	13
48	9.81 519	14	9.93 610	26	0.06 390	9.87 909	11	12
49	9.81 534	15	9.93 636	26	0.06 364	9.87 898	11	11
50	9.81 549	15	9.93 661	25	0.06 339	9.87 887	11	**10**
51	9.81 563	14	9.93 687	26	0.06 313	9.87 877	10	9
52	9.81 578	15	9.93 712	25	0.06 288	9.87 866	11	8
53	9.81 592	14	9.93 738	26	0.06 262	9.87 855	11	7
54	9.81 607	15	9.93 763	25	0.06 237	9.87 844	11	6
55	9.81 622	15	9.93 789	26	0.06 211	9.87 833	11	5
56	9.81 636	14	9.93 814	25	0.06 186	9.87 822	11	4
57	9.81 651	15	9.93 840	26	0.06 160	9.87 811	11	3
58	9.81 665	14	9.93 865	25	0.06 135	9.87 800	11	2
59	9.81 680	15	9.93 891	26	0.06 109	9.87 789	11	1
60	9.81 694	14	9.93 916	25	0.06 084	9.87 778	11	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	26	25
1	0.4	0.4
2	0.9	0.8
3	1.3	1.2
4	1.7	1.7
5	2.2	2.1
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

P. P.

	15	14
1	0.2	0.2
2	0.5	0.5
3	0.8	0.7
4	1.0	0.9
5	1.2	1.2
6	1.5	1.4
7	1.8	1.6
8	2.0	1.9
9	2.2	2.1
10	2.5	2.3
20	5.0	4.7
30	7.5	7.0
40	10.0	9.3
50	12.5	11.7

P. P.

	11	10
1	0.2	0.2
2	0.4	0.3
3	0.6	0.5
4	0.7	0.7
5	0.9	0.8
6	1.1	1.0
7	1.3	1.2
8	1.5	1.3
9	1.6	1.5
10	1.8	1.7
20	3.7	3.3
30	5.5	5.0
40	7.3	6.7
50	9.2	8.3

41°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′
0	9.81 694		9.93 916		0.06 084	9.87 778		**60**
1	9.81 709	15	9.93 942	26	0.06 058	9.87 767	11	59
2	9.81 723	14	9.93 967	25	0.06 033	9.87 756	11	58
3	9.81 738	15	9.93 993	26	0.06 007	9.87 745	11	57
4	9.81 752	14	9.94 018	25	0.05 982	9.87 734	11	56
5	9.81 767	15	9.94 044	26	0.05 956	9.87 723	11	55
6	9.81 781	14	9.94 069	25	0.05 931	9.87 712	11	54
7	9.81 796	15	9.94 095	26	0.05 905	9.87 701	11	53
8	9.81 810	14	9.94 120	25	0.05 880	9.87 690	11	52
9	9.81 825	15	9.94 146	26	0.05 854	9.87 679	11	51
10	9.81 839	14	9.94 171	25	0.05 829	9.87 668	11	**50**
11	9.81 854	15	9.94 197	26	0.05 803	9.87 657	11	49
12	9.81 868	14	9.94 222	25	0.05 778	9.87 646	11	48
13	9.81 882	14	9.94 248	26	0.05 752	9.87 635	11	47
14	9.81 897	15	9.94 273	25	0.05 727	9.87 624	11	46
15	9.81 911	14	9.94 299	26	0.05 701	9.87 613	11	45
16	9.81 926	15	9.94 324	25	0.05 676	9.87 601	12	44
17	9.81 940	14	9.94 350	26	0.05 650	9.87 590	11	43
18	9.81 955	15	9.94 375	25	0.05 625	9.87 579	11	42
19	9.81 969	14	9.94 401	26	0.05 599	9.87 568	11	41
20	9.81 983	14	9.94 426	25	0.05 574	9.87 557	11	**40**
21	9.81 998	15	9.94 452	26	0.05 548	9.87 546	11	39
22	9.82 012	14	9.94 477	25	0.05 523	9.87 535	11	38
23	9.82 026	14	9.94 503	26	0.05 497	9.87 524	11	37
24	9.82 041	15	9.94 528	25	0.05 472	9.87 513	11	36
25	9.82 055	14	9.94 554	26	0.05 446	9.87 501	12	35
26	9.82 069	14	9.94 579	25	0.05 421	9.87 490	11	34
27	9.82 084	15	9.94 604	25	0.05 396	9.87 479	11	33
28	9.82 098	14	9.94 630	26	0.05 370	9.87 468	11	32
29	9.82 112	14	9.94 655	25	0.05 345	9.87 457	11	31
30	9.82 126	14	9.94 681	26	0.05 319	9.87 446	11	**30**
31	9.82 141	15	9.94 706	25	0.05 294	9.87 434	12	29
32	9.82 155	14	9.94 732	26	0.05 268	9.87 423	11	28
33	9.82 169	14	9.94 757	25	0.05 243	9.87 412	11	27
34	9.82 184	15	9.94 783	26	0.05 217	9.87 401	11	26
35	9.82 198	14	9.94 808	25	0.05 192	9.87 390	11	25
36	9.82 212	14	9.94 834	26	0.05 166	9.87 378	12	24
37	9.82 226	14	9.94 859	25	0.05 141	9.87 367	11	23
38	9.82 240	14	9.94 884	25	0.05 116	9.87 356	11	22
39	9.82 255	15	9.94 910	26	0.05 090	9.87 345	11	21
40	9.82 269	14	9.94 935	25	0.05 065	9.87 334	11	**20**
41	9.82 283	14	9.94 961	26	0.05 039	9.87 322	12	19
42	9.82 297	14	9.94 986	25	0.05 014	9.87 311	11	18
43	9.82 311	14	9.95 012	26	0.04 988	9.87 300	11	17
44	9.82 326	15	9.95 037	25	0.04 963	9.87 288	12	16
45	9.82 340	14	9.95 062	25	0.04 938	9.87 277	11	15
46	9.82 354	14	9.95 088	26	0.04 912	9.87 266	11	14
47	9.82 368	14	9.95 113	25	0.04 887	9.87 255	11	13
48	9.82 382	14	9.95 139	26	0.04 861	9.87 243	12	12
49	9.82 396	14	9.95 164	25	0.04 836	9.87 232	11	11
50	9.82 410	14	9.95 190	26	0.04 810	9.87 221	11	**10**
51	9.82 424	14	9.95 215	25	0.04 785	9.87 209	12	9
52	9.82 439	15	9.95 240	25	0.04 760	9.87 198	11	8
53	9.82 453	14	9.95 266	26	0.04 734	9.87 187	11	7
54	9.82 467	14	9.95 291	25	0.04 709	9.87 175	12	6
55	9.82 481	14	9.95 317	26	0.04 683	9.87 164	11	5
56	9.82 495	14	9.95 342	25	0.04 658	9.87 153	11	4
57	9.82 509	14	9.95 368	26	0.04 632	9.87 141	12	3
58	9.82 523	14	9.95 393	25	0.04 607	9.87 130	11	2
59	9.82 537	14	9.95 418	25	0.04 582	9.87 119	11	1
60	9.82 551		9.95 444	26	0.04 556	9.87 107	12	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	26	25
1	0.4	0.4
2	0.9	0.8
3	1.3	1.2
4	1.7	1.7
5	2.2	2.1
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

	15	14
1	0.2	0.2
2	0.5	0.5
3	0.8	0.7
4	1.0	0.9
5	1.2	1.2
6	1.5	1.4
7	1.8	1.6
8	2.0	1.9
9	2.2	2.1
10	2.5	2.3
20	5.0	4.7
30	7.5	7.0
40	10.0	9.3
50	12.5	11.7

	12	11
1	0.2	0.2
2	0.4	0.4
3	0.6	0.6
4	0.8	0.7
5	1.0	0.9
6	1.2	1.1
7	1.4	1.3
8	1.6	1.5
9	1.8	1.6
10	2.0	1.8
20	4.0	3.7
30	6.0	5.5
40	8.0	7.3
50	10.0	9.2

42°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	′
0	9.82 551		9.95 444		0.04 556	9.87 107		**60**
1	9.82 565	14	9.95 469	25	0.04 531	9.87 096	11	59
2	9.82 579	14	9.95 495	26	0.04 505	9.87 085	11	58
3	9.82 593	14	9.95 520	25	0.04 480	9.87 073	12	57
4	9.82 607	14	9.95 545	25	0.04 455	9.87 062	11	56
5	9.82 621	14	9.95 571	26	0.04 429	9.87 050	12	55
6	9.82 635	14	9.95 596	25	0.04 404	9.87 039	11	54
7	9.82 649	14	9.95 622	26	0.04 378	9.87 028	11	53
8	9.82 663	14	9.95 647	25	0.04 353	9.87 016	12	52
9	9.82 677	14	9.95 672	25	0.04 328	9.87 005	11	51
10	9.82 691	14	9.95 698	26	0.04 302	9.86 993	12	**50**
11	9.82 705	14	9.95 723	25	0.04 277	9.86 982	11	49
12	9.82 719	14	9.95 748	25	0.04 252	9.86 970	12	48
13	9.82 733	14	9.95 774	26	0.04 226	9.86 959	11	47
14	9.82 747	14	9.95 799	25	0.04 201	9.86 947	12	46
15	9.82 761	14	9.95 825	26	0.04 175	9.86 936	11	45
16	9.82 775	14	9.95 850	25	0.04 150	9.86 924	12	44
17	9.82 788	13	9.95 875	25	0.04 125	9.86 913	11	43
18	9.82 802	14	9.95 901	26	0.04 099	9.86 902	11	42
19	9.82 816	14	9.95 926	25	0.04 074	9.86 890	12	41
20	9.82 830	14	9.95 952	26	0.04 048	9.86 879	11	**40**
21	9.82 844	14	9.95 977	25	0.04 023	9.86 867	12	39
22	9.82 858	14	9.96 002	25	0.03 998	9.86 855	12	38
23	9.82 872	14	9.96 028	26	0.03 972	9.86 844	11	37
24	9.82 885	13	9.96 053	25	0.03 947	9.86 832	12	36
25	9.82 899	14	9.96 078	25	0.03 922	9.86 821	11	25
26	9.82 913	14	9.96 104	26	0.03 896	9.86 809	12	34
27	9.82 927	14	9.96 129	25	0.03 871	9.86 798	11	33
28	9.82 941	14	9.96 155	26	0.03 845	9.86 786	12	32
29	9.82 955	14	9.96 180	25	0.03 820	9.86 775	11	31
30	9.82 968	13	9.96 205	25	0.03 795	9.86 763	12	**30**
31	9.82 982	14	9.96 231	26	0.03 769	9.86 752	11	29
32	9.82 996	14	9.96 256	25	0.03 744	9.86 740	12	28
33	9.83 010	14	9.96 281	25	0.03 719	9.86 728	12	27
34	9.83 023	13	9.96 307	26	0.03 693	9.86 717	11	26
35	9.83 037	14	9.96 332	25	0.03 668	9.86 705	12	25
36	9.83 051	14	9.96 357	25	0.03 643	9.86 694	11	24
37	9.83 065	14	9.96 383	26	0.03 617	9.86 682	12	23
38	9.83 078	13	9.96 408	25	0.03 592	9.86 670	12	22
39	9.83 092	14	9.96 433	25	0.03 567	9.86 659	11	21
40	9.83 106	14	9.96 459	26	0.03 541	9.86 647	12	**20**
41	9.83 120	14	9.96 484	25	0.03 516	9.86 635	12	19
42	9.83 133	13	9.96 510	26	0.03 490	9.86 624	11	18
43	9.83 147	14	9.96 535	25	0.03 465	9.86 612	12	17
44	9.83 161	14	9.96 560	25	0.03 440	9.86 600	12	16
45	9.83 174	13	9.96 586	26	0.03 414	9.86 589	11	15
46	9.83 188	14	9.96 611	25	0.03 389	9.86 577	12	14
47	9.83 202	14	9.96 636	25	0.03 364	9.86 565	12	13
48	9.83 215	13	9.96 662	26	0.03 338	9.86 554	11	12
49	9.83 229	14	9.96 687	25	0.03 313	9.86 542	12	11
50	9.83 242	13	9.96 712	25	0.03 288	9.86 530	12	**10**
51	9.83 256	14	9.96 738	26	0.03 262	9.86 518	12	9
52	9.83 270	14	9.96 763	25	0.03 237	9.86 507	11	8
53	9.83 283	13	9.96 788	25	0.03 212	9.86 495	12	7
54	9.83 297	14	9.96 814	26	0.03 186	9.86 483	12	6
55	9.83 310	13	9.96 839	25	0.03 161	9.86 472	11	5
56	9.83 324	14	9.96 864	25	0.03 136	9.86 460	12	4
57	9.83 338	14	9.96 890	26	0.03 110	9.86 448	12	3
58	9.83 351	13	9.96 915	25	0.03 085	9.86 436	12	2
59	9.83 365	14	9.96 940	25	0.03 060	9.86 425	11	1
60	9.83 378	13	9.96 966	26	0.03 034	9.86 413	12	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	26	25
1	0.4	0.4
2	0.9	0.8
3	1.3	1.2
4	1.7	1.7
5	2.2	2.1
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

P. P.

	14	13
1	0.2	0.2
2	0.5	0.4
3	0.7	0.6
4	0.9	0.9
5	1.2	1.1
6	1.4	1.3
7	1.6	1.5
8	1.9	1.7
9	2.1	2.0
10	2.3	2.2
20	4.7	4.3
30	7.0	6.5
40	9.3	8.7
50	11.7	10.8

P. P.

	12	11
1	0.2	0.2
2	0.4	0.4
3	0.6	0.6
4	0.8	0.7
5	1.0	0.9
6	1.2	1.1
7	1.4	1.3
8	1.6	1.5
9	1.8	1.6
10	2.0	1.8
20	4.0	3.7
30	6.0	5.5
40	8.0	7.3
50	10.0	9.2

47°

43°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.83 378		9.96 966		0.03 034	9.86 413		**60**
1	9.83 392	14	9.96 991	25	0.03 009	9.86 401	12	59
2	9.83 405	13	9.97 016	25	0.02 984	9.86 389	12	48
3	9.83 419	14	9.97 042	26	0.02 958	9.86 377	12	57
4	9.83 432	13	9.97 067	25	0.02 933	9.86 366	11	56
5	9.83 446	14	9.97 092	25	0.02 908	9.86 354	12	55
6	9.83 459	13	9.97 118	26	0.02 882	9.86 342	12	54
7	9.83 473	14	9.97 143	25	0.02 857	9.86 330	12	53
8	9.83 486	13	9.97 168	25	0.02 832	9.86 318	12	52
9	9.83 500	14	9.97 193	25	0.02 807	9.86 306	12	51
10	9.83 513	13	9.97 219	26	0.02 781	9.86 295	11	**50**
11	9.83 527	14	9.97 244	25	0.02 756	9.86 283	12	49
12	9.83 540	13	9.97 269	25	0.02 731	9.86 271	12	48
13	9.83 554	14	9.97 295	26	0.02 705	9.86 259	12	47
14	9.83 567	13	9.97 320	25	0.02 680	9.86 247	12	46
15	9.83 581	14	9.97 345	25	0.02 655	9.86 235	12	45
16	9.83 594	13	9.97 371	26	0.02 629	9.86 223	12	44
17	9.83 608	14	9.97 396	25	0.02 604	9.86 211	12	43
18	9.83 621	13	9.97 421	25	0.02 579	9.86 200	11	42
19	9.83 634	13	9.97 447	26	0.02 553	9.86 188	12	41
20	9.83 648	14	9.97 472	25	0.02 528	9.86 176	12	**40**
21	9.83 661	13	9.97 497	25	0.02 503	9.86 164	12	39
22	9.83 674	13	9.97 523	26	0.02 477	9.86 152	12	38
23	9.83 688	14	9.97 548	25	0.02 452	9.86 140	12	37
24	9.83 701	13	9.97 573	25	0.02 427	9.86 128	12	36
25	9.83 715	14	9.97 598	25	0.02 402	9.86 116	12	35
26	9.83 728	13	9.97 624	26	0.02 376	9.86 104	12	34
27	9.83 741	13	9.97 649	25	0.02 351	9.86 092	12	33
28	9.83 755	14	9.97 674	25	0.02 326	9.86 080	12	32
29	9.83 768	13	9.97 700	26	0.02 300	9.86 068	12	31
30	9.83 781	13	9.97 725	25	0.02 275	9.86 056	12	**30**
31	9.83 795	14	9.97 750	25	0.02 250	9.86 044	12	29
32	9.83 808	13	9.97 776	26	0.02 224	9.86 032	12	28
33	9.83 821	13	9.97 801	25	0.02 199	9.86 020	12	27
34	9.83 834	13	9.97 826	25	0.02 174	9.86 008	12	26
35	9.83 848	14	9.97 851	25	0.02 149	9.85 996	12	25
36	9.83 861	13	9.97 877	26	0.02 123	9.85 984	12	24
37	9.83 874	13	9.97 902	25	0.02 098	9.85 972	12	23
38	9.83 887	13	9.97 927	25	0.02 073	9.85 960	12	22
39	9.83 901	14	9.97 953	26	0.02 047	9.85 948	12	21
40	9.83 914	13	9.97 978	25	0.02 022	9.85 936	12	**20**
41	9.83 927	13	9.98 003	25	0.01 997	9.85 924	12	19
42	9.83 940	13	9.98 029	26	0.01 971	9.85 912	12	18
43	9.83 954	14	9.98 054	25	0.01 946	9.85 900	12	17
44	9.83 967	13	9.98 079	25	0.01 921	9.85 888	12	16
45	9.83 980	13	9.98 104	25	0.01 896	9.85 876	12	15
46	9.83 993	13	9.89 130	26	0.01 870	9.85 864	12	14
47	9.84 006	13	9.98 155	25	0.01 845	9.85 851	13	13
48	9.84 020	14	9.98 180	25	0.01 820	9.85 839	12	12
49	9.84 033	13	9.98 206	26	0.01 794	9.85 827	12	11
50	9.84 046	13	9.98 231	25	0.01 769	9.85 815	12	**10**
51	9.84 059	13	9.98 256	25	0.01 744	9.85 803	12	9
52	9.84 072	13	9.98 281	25	0.01 719	9.85 791	12	8
53	9.84 085	13	9.98 307	26	0.01 693	9.85 779	13	7
54	9.84 098	13	9.98 332	25	0.01 668	9.85 766	12	6
55	9.84 112	14	9.98 357	25	0.01 643	9.85 754	12	5
56	9.84 125	13	9.98 383	26	0.01 617	9.85 742	12	4
57	9.84 138	13	9.98 408	25	0.01 592	9.85 730	12	3
58	9.84 151	13	9.98 433	25	0.01 567	9.85 718	12	2
59	9.84 164	13	9.98 458	25	0.01 542	9.85 706	12	1
60	9.84 177	13	9.98 484	26	0.01 516	9.85 693	13	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	26	25
1	0.4	0.4
2	0.9	0.8
3	1.3	1.2
4	1.7	1.7
5	2.2	2.1
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

	14	13
1	0.2	0.2
2	0.5	0.4
3	0.7	0.6
4	0.9	0.9
5	1.2	1.1
6	1.4	1.3
7	1.6	1.5
8	1.9	1.7
9	2.1	2.0
10	2.3	2.2
20	4.7	4.3
30	7.0	6.5
40	9.3	8.7
50	11.7	10.8

	12	11
1	0.2	0.2
2	0.4	0.4
3	0.6	0.6
4	0.8	0.7
5	1.0	0.9
6	1.2	1.1
7	1.4	1.3
8	1.6	1.5
9	1.8	1.6
10	2.0	1.8
20	4.0	3.7
30	6.0	5.5
40	8.0	7.3
50	10.0	9.2

P. P.

44°

′	L. Sin.	d.	L. Tan.	d. c.	L. Cot.	L. Cos.	d.	
0	9.84 177		9.98 484		0.01 516	9.85 693		**60**
1	9.84 190	13	9.98 509	25	0.01 491	9.85 681	12	59
2	9.84 203	13	9.98 534	25	0.01 466	9.85 669	12	58
3	9.84 216	13	9.98 560	26	0.01 440	9.85 657	12	57
4	9.84 229	13	9.98 585	25	0.01 415	9.85 645	12	56
5	9.84 242	13	9.98 610	25	0.01 390	9.85 632	13	55
6	9.84 255	13	9.98 635	25	0.01 365	9.85 620	12	54
7	9.84 269	14	9.98 661	26	0.01 339	9.85 608	12	53
8	9.84 282	13	9.98 686	25	0.01 314	9.85 596	12	52
9	9.84 295	13	9.98 711	25	0.01 289	9.85 583	13	51
10	9.84 308	13	9.98 737	26	0.01 263	9.85 571	12	**50**
11	9.84 321	13	9.98 762	25	0.01 238	9.85 559	12	49
12	9.84 334	13	9.98 787	25	0.01 213	9.85 547	12	48
13	9.84 347	13	9.98 812	25	0.01 188	9.85 534	13	47
14	9.84 360	13	9.98 838	26	0.01 162	9.85 522	12	46
15	9.84 373	13	9.98 863	25	0.01 137	9.85 510	12	45
16	9.84 385	12	9.98 888	25	0.01 112	9.85 497	13	44
17	9.84 398	13	9.98 913	25	0.01 087	9.85 485	12	43
18	9.84 411	13	9.98 939	26	0.01 061	9.85 473	12	42
19	9.84 424	13	9.98 964	25	0.01 036	9.85 460	13	41
20	9.84 437	13	9.98 989	25	0.01 011	9.85 448	12	**40**
21	9.84 450	13	9.99 015	26	0.00 985	9.85 436	12	39
22	9.84 463	13	9.99 040	25	0.00 960	9.85 423	13	38
23	9.84 476	13	9.99 065	25	0.00 935	9.85 411	12	37
24	9.84 489	13	9.99 090	25	0.00 910	9.85 399	12	36
25	9.84 502	13	9.99 116	26	0.00 884	9.85 386	13	35
26	9.84 515	13	9.99 141	25	0.00 859	9.85 374	12	34
27	9.84 528	13	9.99 166	25	0.00 834	9.85 361	13	33
28	9.84 540	12	9.99 191	25	0.00 809	9.85 349	12	32
29	9.84 553	13	9.99 217	26	0.00 783	9.85 337	12	31
30	9.84 566	13	9.99 242	25	0.00 758	9.85 324	13	**30**
31	9.84 579	13	9.99 267	25	0.00 733	9.85 312	12	29
32	9.84 592	13	9.99 293	26	0.00 707	9.85 299	13	28
33	9.84 605	13	9.99 318	25	0.00 682	9.85 287	12	27
34	9.84 618	13	9.99 343	25	0.00 657	9.85 274	13	26
35	9.84 630	12	9.99 368	25	0.00 632	9.85 262	12	25
36	9.84 643	13	9.99 394	26	0.00 606	9.85 250	12	24
37	9.84 656	13	9.99 419	25	0.00 581	9.85 237	13	23
38	9.84 669	13	9.99 444	25	0.00 556	9.85 225	12	22
39	9.84 682	13	9.99 469	25	0.00 531	9.85 212	13	21
40	9.84 694	12	9.99 495	26	0.00 505	9.85 200	12	**20**
41	9.84 707	13	9.99 520	25	0.00 480	9.85 187	13	19
42	9.84 720	13	9.99 545	25	0.00 455	9.85 175	12	18
43	9.84 733	13	9.99 570	25	0.00 430	9.85 162	13	17
44	9.84 745	12	9.99 596	26	0.00 404	9.85 150	12	16
45	9.84 758	13	9.99 621	25	0.00 379	9.85 137	13	15
46	9.84 771	13	9.99 646	25	0.00 354	9.85 125	12	14
47	9.84 784	13	9.99 672	26	0.00 328	9.85 112	13	13
48	9.84 796	12	9.99 697	25	0.00 303	9.85 100	12	12
49	9.84 809	13	9.99 722	25	0.00 278	9.85 087	13	11
50	9.84 822	13	9.99 747	25	0.00 253	9.85 074	13	**10**
51	9.84 835	13	9.99 773	26	0.00 227	9.85 062	12	9
52	9.84 847	12	9.99 798	25	0.00 202	9.85 049	13	8
53	9.84 860	13	9.99 823	25	0.00 177	9.85 037	12	7
54	9.84 873	13	9.99 848	25	0.00 152	9.85 024	13	6
55	9.84 885	12	9.99 874	26	0.00 126	9.85 012	12	5
56	9.84 898	13	9.99 899	25	0.00 101	9.84 999	13	4
57	9.84 911	13	9.99 924	25	0.00 076	9.84 986	13	3
58	9.84 923	12	9.99 949	25	0.00 051	9.84 974	12	2
59	9.84 936	13	9.99 975	26	0.00 025	9.84 961	13	1
60	9.84 949	13	0.00 000	25	0.00 000	9.84 949	12	**0**
	L. Cos.	d.	L. Cot.	d. c.	L. Tan.	L. Sin.	d.	′

P. P.

	26	25
1	0.4	0.4
2	0.9	0.8
3	1.3	1.2
4	1.7	1.7
5	2.2	2.1
6	2.6	2.5
7	3.0	2.9
8	3.5	3.3
9	3.9	3.8
10	4.3	4.2
20	8.7	8.3
30	13.0	12.5
40	17.3	16.7
50	21.7	20.8

P. P.

	14	13	12
1	0.2	0.2	0.2
2	0.5	0.4	0.4
3	0.7	0.6	0.6
4	0.9	0.9	0.8
5	1.2	1.1	1.0
6	1.4	1.3	1.2
7	1.6	1.5	1.4
8	1.9	1.7	1.6
9	2.1	2.0	1.8
10	2.3	2.2	2.0
20	4.7	4.3	4.0
30	7.0	6.5	6.0
40	9.3	8.7	8.0
50	11.7	10.8	10.0

45°